CBSE 12th 2024 Compartment Physics Set-2 (55/S/2) Solutions

SECTION A

Q.1. The relative magnetic permeability of a medium is 0.075. Its magnetic susceptibility will be :
(A) 0.925
(B) - 0.925
(C) 1.075
(D) - 1.075

Solution. (B) - 0.925, To find the magnetic susceptibility χ of a medium given its relative magnetic permeability μ_r, we use the following relationship:

$\mu_r = 1 + \chi$

Where:
μ_r is the relative magnetic permeability
χ is the magnetic susceptibility

Given:
$\mu_r = 0.075$

Substitute μ_r into the equation:

$0.075 = 1 + \chi$

Solve for χ

$\chi = 0.075 - 1$

$\chi = -0.925$

So, the magnetic susceptibility is:

(B) -0.925

Q.2. Two circular loops of areas A and 4A carry currents 21 and I respectively. The magnetic fields at their centers will be in the ratio of:
(A) 3:1
(B) 4:1
(C) 1:1
(D)1:2

Solution. (B) 4:1,

To determine the ratio of the magnetic fields at the centers of two circular loops with different areas and currents, we use the formula for the magnetic field at the center of a circular loop carrying current I:

$$B = \frac{\mu_0 I}{2R}$$

where:

- B is the magnetic field at the center of the loop,
- μ_0 is the permeability of free space,
- I is the current in the loop,
- R is the radius of the loop.

The area A of the loop is related to the radius R by:

$$A = \pi R^2$$
$$R = \sqrt{\frac{A}{\pi}}$$

So the magnetic field can be expressed as:

$$B = \frac{\mu_0 I}{2\sqrt{\frac{A}{\pi}}}$$
$$B = \frac{\mu_0 I\sqrt{\pi}}{2\sqrt{A}}$$

For two loops:

- Loop 1: Area A, Current $I_1 = 2I$
- Loop 2: Area $4A$, Current $I_2 = I$

The magnetic fields at their centers are:

For Loop 1:
$$B_1 = \frac{\mu_0 (2I)}{2\sqrt{\frac{A}{\pi}}} = \frac{\mu_0 I\sqrt{\pi}}{\sqrt{A}}$$

For Loop 2:
$$B_2 = \frac{\mu_0 I}{2\sqrt{\frac{4A}{\pi}}} = \frac{\mu_0 I\sqrt{\pi}}{4\sqrt{A}}$$

Now, find the ratio $\frac{B_1}{B_2}$:

$$\frac{B_1}{B_2} = \frac{\frac{\mu_0 I\sqrt{\pi}}{\sqrt{A}}}{\frac{\mu_0 I\sqrt{\pi}}{4\sqrt{A}}} = \frac{\sqrt{A}}{\frac{\sqrt{A}}{4}} = 4$$

Q.3. A conducting sphere of radius R is given a charge Q. Consider three points A, B and C- A at the centre, B at a distance R/2 from the centre and C on the surface of the sphere. The electric potentials at these points are such that:

(A) VAVB = VC

(B) VAVB VC

(C) VAVB≠ VC

(D) VAVB VC

Solution.

To solve the problem of comparing electric potentials at different points around a charged conducting sphere, let's use the principles of electrostatics.

For a conducting sphere with radius R and total charge Q, the electric potential V at a distance r from the center of the sphere is given by:

- Inside the sphere (i.e., for $r < R$), the potential V is constant and equal to the potential on the surface of the sphere, because the electric field inside a conductor is zero. Hence, $V_{\text{inside}} = \frac{Q}{4\pi\epsilon_0 R}$
- On the surface of the sphere (i.e., for $r = R$), the potential is: $V_{\text{surface}} = \frac{Q}{4\pi\epsilon_0 R}$
- Outside the sphere (i.e., for $r > R$), the potential decreases with distance according to: $V = \frac{Q}{4\pi\epsilon_0 r}$

Now, consider the points:

1. **Point A**: At the center of the sphere ($r = 0$). Since this is inside the sphere, the potential here is: $V_A = \frac{Q}{4\pi\epsilon_0 R}$
2. **Point B**: At a distance $R/2$ from the center. Since this is inside the sphere, the potential here is also: $V_B = \frac{Q}{4\pi\epsilon_0 R}$
3. **Point C**: On the surface of the sphere ($r = R$). The potential here is: $V_C = \frac{Q}{4\pi\epsilon_0 R}$

Since the potential inside the sphere (including at $R/2$) is constant and equal to the potential on the surface, we have: $V_A = V_B = V_C$

Therefore, the electric potentials at points A, B, and C are equal.

So the correct answer is:

B) $V_A = V_B = V_C$

Q.4. The capacitance of a parallel plate capacitor is 10 μF when the distance between its plates is 8 cm. If the distance between the plates is halved, the capacitance will become:

(A) 10 μF

(B) 15 μF

(C) 20 μF

(D) 40 μF

Solution.(C) 20 μF,

To determine the new capacitance of a parallel plate capacitor when the distance between its plates is halved, we need to understand how capacitance changes with plate separation.

The formula for the capacitance C of a parallel plate capacitor is given by:

$$C = \frac{\epsilon_0 \kappa A}{d}$$

where:

- ϵ_0 is the permittivity of free space,
- κ is the dielectric constant of the material between the plates (assuming it's constant in this problem),
- A is the area of the plates,
- d is the distance between the plates.

Initially, the capacitance C_1 is:

$$C_1 = \frac{\epsilon_0 \kappa A}{d_1}$$

where $d_1 = 8$ cm (0.08 meters).

When the distance between the plates is halved, the new distance d_2 is:

$$d_2 = \frac{d_1}{2} = \frac{8\ \text{cm}}{2} = 4\ \text{cm}$$

The new capacitance C_2 is:

$$C_2 = \frac{\epsilon_0 \kappa A}{d_2}$$

Since $d_2 = \frac{d_1}{2}$, we can write:

$$C_2 = \frac{\epsilon_0 \kappa A}{\frac{d_1}{2}} = 2 \cdot \frac{\epsilon_0 \kappa A}{d_1} = 2 \cdot C_1$$

Given that the initial capacitance C_1 is 10 μF, the new capacitance C_2 will be:

$$C_2 = 2 \cdot 10\ \mu\text{F} = 20\ \mu\text{F}$$

So, the capacitance will become:

(C) 20 μF

Q.5. Isotones are nuclides having :
(A) same number of neutrons but different number of protons
(B) same number of protons but different number of neutrons
(C) same number of protons and also same number of neutrons
(D) different number of protons and also different number of neutrons

Solution.(A) same number of neutrons but different number of protons

Isotones are nuclides that have the same number of neutrons but different numbers of protons.

To break it down:

- Isotones: Nuclides with the same number of neutrons (N) but different numbers of protons (Z). This means they are different elements but share the same neutron count.

So, the correct answer is:

(A) same number of neutrons but different number of protons.

Q.6. The magnetic field of an electromagnetic wave is represented as Bx Bo sin (ky wt). It means that the wave propagation direction and wave 0 vector k are respectively:

(A) + z axis, 2π/v

(B) z axis, 2π/ T

(C) + y axis, 2π/ λ

(D) - y axis, λ /2π

Solution. (C) + y axis, 2π/ λ, To determine the direction of wave propagation and the wave vector k from the given magnetic field representation of an electromagnetic wave B = Bo sin(ky wt), let's analyze the equation:
1. Magnetic Field Representation:
B Bo sin(ky - wt)
This equation shows that the magnetic field varies sinusoidally with position y and time t.
2. Wave Vector Direction:
The argument of the sine function (ky wt) suggests that the wave is propagating in the direction of the y-axis. The wave vector k is therefore along the y-axis.

3. Wave Propagation Direction:
The wave vector k points in the direction in which the phase of the wave increases. For the sinusoidal term sin(ky wt), the wave propagates in the positive y-direction.
4. Wave Vector Magnitude:
The magnitude of the wave vector k is related to the wavelength A by:
k= 2π
Therefore, the wave vector k has a magn↓ 2 of 2π/A and points in the positive y-direction.
Combining these observations:
The wave propagation direction is along the + y axis.
The magnitude of the wave vector k is 2π/λ.
Thus, the correct answer is:
(C) + y axis, 2π/λ

Q.7. An inductor of resistance 5 2 and self-inductance 25 mH is connected across an ac source of angular frequency 200 rad s^{-1}. The phase angle between the voltage and the current is :
(A) 30°
(B) 45°
(C) 60°
(D) 90°

To determine the phase angle between the voltage and the current for an inductor connected to an AC source, we need to consider the following:

1. **Inductor Impedance and Phase Angle:**
   The impedance Z_L of an inductor in an AC circuit is given by:
   $$Z_L = \sqrt{R^2 + (X_L)^2}$$
   where R is the resistance and X_L is the inductive reactance.

   The inductive reactance X_L is given by:
   $$X_L = \omega L$$
   where ω is the angular frequency and L is the self-inductance.

2. **Calculating Inductive Reactance:**
   Given:
   - $L = 25 \text{ mH} = 0.025 \text{ H}$
   - $\omega = 200 \text{ rad/s}$

   $$X_L = \omega L = 200 \times 0.025 = 5 \text{ ohms}$$

3. **Impedance of the Inductor:**
   The inductor's impedance Z_L includes both resistance and reactance:
   $$Z_L = \sqrt{R^2 + (X_L)^2}$$
   where $R = 5$ ohms and $X_L = 5$ ohms.
   $$Z_L = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2} \text{ ohms}$$

4. **Phase Angle Calculation:**
   The phase angle ϕ between the voltage and the current is given by:
   $$\tan\phi = \frac{X_L}{R}$$
   Substituting the values:
   $$\tan\phi = \frac{5}{5} = 1$$
   Therefore:
   $$\phi = \tan^{-1}(1) = 45^\circ$$

The phase angle between the voltage and the current for the inductor is:

(B) 45°

Q.8. Which of the following statements is correct for alpha particle scattering experiment?
(A) For angle of scattering 0 ≈ 0, the impact parameter is small.
(B) For angle of scattering 0 = π, the impact parameter is large.
(C) The number of alpha particles undergoing head-on collision is small.

(D) The experiment provides an estimate of the upper limit to the size of target atom.

Solution.(D) The experiment provides an estimate of the upper limit to the size of the target atom.

In an alpha particle scattering experiment, the following key points are relevant:

1. Impact Parameter:
The impact parameter is the perpendicular distance between the path of the incoming alpha particle and the center of the target nucleus.

2.Scattering Angle and Impact Parameter:
When the scattering angle is small (near 0 degrees), the impact parameter is large. This is because the alpha particle is not coming very close to the nucleus, so it does not experience a strong force.
When the scattering angle is large (near 180 degrees or π radians), the impact parameter is small. This is because the alpha particle is scattering directly back, meaning it came very close to the nucleus, experiencing a strong force.

3. Head-On Collisions:
- Head-on collisions are very rare because they require the alpha particle to come very close to the nucleus, which is a low-probability event due to the Coulomb repulsion between the positively charged alpha particle and the nucleus.

4.Estimating Nucleus Size:
The experiment provides an estimate of the size of the target nucleus because the scattering pattern and the distribution of scattering angles give information about the size and charge distribution of the nucleus.

Given these points, the correct statements about the alpha particle scattering experiment are:

(A) For angle of scattering θ ≈ 0, the impact parameter is large.

(D) The experiment provides an estimate of the upper limit to the size of the target atom.**

Therefore, based on the options provided, the correct statement is:

(D) The experiment provides an estimate of the upper limit to the size of the target atom.

Q.9. The ionization energy of the hydrogen atom, in Bohr model, is:
(A) - 3-4 eV
(B) 3-4 eV
(C) - 13.6 eV
(D) 13-6 eV

Solution. (C) - 13.6 eV, In the Bohr model of the hydrogen atom, the ionization energy is the energy required to remove an electron from the atom, transitioning it from the ground state (the lowest energy level) to the point where the electron is free from the atom.

For a hydrogen atom, the ionization energy is given by:

$E=-13.6$ eV

This negative sign indicates that the energy is required to overcome the binding energy of the electron in the atom.

Therefore, the correct answer is (C) - 13.6 eV

Q.10. A point object is placed in air at a distance of 4R on the principal axis of a convex spherical surface of radius of curvature R separating two mediums, air and glass. As the object is moved towards the surface, the image formed is:

(A) always real
(B) always virtual
(C) first virtual and then real
(D) first real and then virtual

Solution. To analyze the behavior of the image formed by a convex spherical surface as a point object is moved towards it, we need to understand how the image formation changes with respect to the object distance in the context of refraction at the spherical surface.

Key Points:

1. Convex Spherical Surface: This is a surface that bulges outward. When dealing with convex surfaces, we use the refraction formula for spherical surfaces:
un1+vn2=Rn2−n1

where \(n_1 \) and \(n_2 \) are the refractive indices of the two media (air and glass, respectively), \(u \) is the object distance, \(v \) is the image distance, and \(R \) is the radius of curvature of the spherical surface.

2. Object Distance: Initially, the object is placed at \(4R \) from the surface in air (where n1 = 1).

3. Image Formation:
When the object is far from the surface (i.e., at (4R) or more), the image is usually formed on the same side as the object in the glass medium and is real.
As the object moves closer to the surface, at certain points, the object may come within the focal length of the convex surface when it is closer than \(R). In such cases, the image might become virtual.

Transition from Real to Virtual: As the object approaches the convex surface (specifically when (u < R), the nature of the image changes from real to virtual because the image moves from one side of the principal axis

to the other, depending on whether the object distance is greater than or less than the focal length of the surface.
Thus, the image formation is:
(D) first real and then virtual

Q.11. Which of the following statements is not true for a p-n junction diode under reverse bias ?
(A) The current is almost independent of the applied voltage.
(B) Holes flow from p-side to n-side.
(C) Electric field in the depletion region increases.
(D) n-side of the junction is connected to +ve terminal and p-side to -ve terminal of the battery.

Solution.(B) Holes flow from p-side to n-side. To determine which statement is not true for a p-n junction diode under reverse bias, let's analyze each option based on the behavior of a p-n junction diode in reverse bias:

1. Statement (A): "The current is almost independent of the applied voltage."
True: In reverse bias, the current through a p-n junction diode is very small (almost negligible) and is primarily due to the leakage current. This current remains nearly constant regardless of the applied reverse voltage, as long as the voltage is below the breakdown voltage.

2. Statement (B): "Holes flow from p-side to n-side."
Not True: In reverse bias, the p-side is connected to the negative terminal of the battery and the n-side is connected to the positive terminal. In this configuration, holes from the p-side are not able to move towards the n-side; instead, the movement of charge carriers is minimal. The main carriers that contribute to the small reverse current are electrons from the n-side that flow towards the p-side, not holes moving in the opposite direction.

3. Statement (C): "Electric field in the depletion region increases."

True: When a p-n junction diode is reverse biased, the depletion region widens as the applied reverse voltage increases. This widening of the depletion region leads to an increase in the electric field within that region.

4.Statement (D):"n-side of the junction is connected to +ve terminal and p-side to -ve terminal of the battery."
True: In reverse bias, the n-side of the diode is connected to the positive terminal of the battery, and the p-side is connected to the negative terminal. This configuration increases the width of the depletion region and decreases the current flow.
Conclusion:
The incorrect statement about a p-n junction diode under reverse bias is:

(B) Holes flow from p-side to n-side.

Q.12. The current sensitivity of a galvanometer does not depend on the :
(A) magnetic field in which the coil is suspended.
(B) current flowing in the coil.
(C) torsional constant of the spring.
(D) area of the coil.

Solution. The current sensitivity of a galvanometer is defined as the amount of deflection per unit current flowing through the coil. It is determined by various factors related to the construction and characteristics of the galvanometer. Let's review each option to identify which one does not affect the current sensitivity:

1. Magnetic Field in which the Coil is Suspended (A):
The current sensitivity of a galvanometer is directly proportional to the magnetic field in which the coil is placed. A stronger magnetic field increases the torque on the coil for a given current, thus increasing sensitivity.

2. Current Flowing in the Coil (B):

The current sensitivity is independent of the actual current flowing through the coil. Instead, sensitivity is the deflection per unit current. Therefore, while the actual deflection depends on the current, the sensitivity itself does not change with different currents.

3. Torsional Constant of the Spring (C):
The torsional constant of the spring (or the suspension fiber) affects the restoring torque experienced by the coil. Sensitivity is inversely proportional to this constant, meaning a higher torsional constant leads to less deflection per unit current, thus decreasing sensitivity.

4.Area of the Coil (D):
The sensitivity is directly proportional to the area of the coil. A larger coil area results in a greater torque for a given current, thereby increasing sensitivity.

Conclusion:
The current sensitivity of a galvanometer does not depend on:
(B) Current flowing in the coil.

Questions number 13 to 16 are Assertion (A) and Reason (R) type questions. Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Q.13. Assertion (A): Although the surfaces of a goggle lens are curved, it does not have any power.
Reason (R): In case of goggles, both the curved surfaces are curved on the same side and have equal radii of curvature.

Solution. To determine the correct answer for the Assertion (A) and Reason (R) type question, let's analyse both statements:

Assertion (A): "Although the surfaces of a goggle lens are curved, it does not have any power."

Reason (R): "In case of goggles, both the curved surfaces are curved on the same side and have equal radii of curvature."

Explanation:

1. **Curved Surfaces and Power of a Lens:**
   - The power of a lens depends on the curvature of its surfaces and the refractive index of the material. If a lens has curved surfaces but they are equal and opposite (i.e., the lens is plano-convex on one side and plano-concave on the other), the lens can have power.

2. **Goggles and Lens Power:**
   - Goggles typically have lenses that are designed to protect the eyes from external elements rather than to correct vision. These lenses can have curved surfaces, but their curvature is often such that the overall lens power is negligible.

3. **Reason (R):**
   - If the curved surfaces of the lens are on the same side and have equal radii of curvature, the lens is likely to be of minimal optical power, such as in the case of protective goggles where the primary function is not vision correction.

Based on this information:

- **Assertion (A)** is generally true because goggles are designed for protection, and their lenses are not intended to correct vision, so they may not have significant optical power.

- **Reason (R)** is also true because if both surfaces are curved on the same side with equal radii of curvature, the lens essentially has minimal power, consistent with the description of protective goggles.

However, **Reason (R)** does not specifically explain why the goggles do not have power; it describes the nature of the curvature without directly linking it to the power or lack thereof.

Thus, the correct option is:

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Q.14. Assertion (A): Nuclear fission reactions are responsible for energy generation in the Sun.
Reason (R): Light nuclei fuse together in the nuclear fission reactions.

Solution. Let's analyse both the Assertion (A) and Reason (R) statements for the given question:

Assertion (A): "Nuclear fission reactions are responsible for energy generation in the Sun."

Reason (R): "Light nuclei fuse together in the nuclear fission reactions."

Explanation:

1. **Nuclear Fission vs. Fusion:**
   - **Nuclear Fission:** This process involves the splitting of heavy nuclei (like uranium or plutonium) into lighter nuclei, releasing energy. This process is responsible for energy generation in nuclear reactors and atomic bombs.
   - **Nuclear Fusion:** This process involves the combining of light nuclei (like hydrogen isotopes) to form heavier nuclei, releasing energy. Fusion is the process responsible for energy generation in the Sun and other stars.

2. **Assertion (A):**
   - The assertion is incorrect because nuclear fission reactions are not responsible for energy generation in the Sun. The Sun's energy comes from nuclear fusion, not fission.

3. **Reason (R):**
   - The reason is incorrect because nuclear fusion involves the combination (or fusing) of light nuclei, not fission. Fission involves the splitting of heavy nuclei.

Based on this analysis:

- **Assertion (A)** is false because the Sun's energy is generated by nuclear fusion, not fission.
- **Reason (R)** is also false because it describes fusion but incorrectly associates it with fission.

Thus, the correct option is:

(D) Both Assertion (A) and Reason (R) are false.

Q.15. Assertion (A): The current density (J) at a point in a conducting wire is in the direction of electric field (E) at that point. ←
Reason (R): A conducting wire obeys Ohm's law.

Solution. Let's analyze the Assertion (A) and Reason (R) for the given question:

Assertion (A): "The current density (J) at a point in a conducting wire is in the direction of electric field (E) at that point."

Reason (R): "A conducting wire obeys Ohm's law."

Explanation:

1. **Current Density and Electric Field:**
   - The current density **J** in a conductor is defined as the amount of current per unit area flowing through the conductor. It is given by the relationship \(\mathbf{J} = \sigma \mathbf{E} \), where \(\sigma \) is the electrical conductivity and \(\mathbf{E} \) is the electric field.
   - In a conductor, the current density **J** is indeed in the same direction as the electric field **E**. This is a fundamental property of how electric currents flow in conductors.

2. **Ohm's Law:**
   - Ohm's Law states that \(\mathbf{J} = \sigma \mathbf{E} \), which is a direct relationship between current density **J** and electric field **E**. This relationship implies that if a material obeys Ohm's Law, the current density is proportional to and in the same direction as the electric field.

Conclusion:

- **Assertion (A)** is true because the current density in a conducting wire is indeed in the direction of the electric field.
- **Reason (R)** is also true because a conducting wire obeys Ohm's Law, which relates the current density to the electric field.

Since **Reason (R)** correctly explains **Assertion (A)** through the relationship described by Ohm's Law:

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Q.16 Assertion (A): The torque acting on a current carrying coil is maximum when it is suspended in a radial magnetic field.
Reason (R): The torque tends to rotate the coil on its own axis.

Solution. To determine the correctness of Assertion (A) and Reason (R) for the given question, let's break down each statement:

Assertion (A): "The torque acting on a current carrying coil is maximum when it is suspended in a radial magnetic field."

Reason (R): "The torque tends to rotate the coil on its own axis."

Analysis:

1. **Torque in a Magnetic Field:**
   - The torque \(\tau \) on a current-carrying coil in a magnetic field is given by \(\tau = \mathbf{N} \cdot \mathbf{B} \cdot \mathbf{I} \cdot \mathbf{A} \cdot \sin(\theta) \), where \(\mathbf{N} \) is the number of turns, \(\mathbf{B} \) is the magnetic field strength, \(\mathbf{I} \) is the current, \(\mathbf{A} \) is the area of the coil, and \(\theta \) is the angle between the magnetic field and the normal to the plane of the coil.
   - For maximum torque, the angle \(\theta \) should be \(90^\circ \) (i.e., \(\sin(\theta) = 1 \)). This occurs when the plane of the coil is perpendicular to the magnetic field.

2. **Radial Magnetic Field:**
   - In a radial magnetic field, the magnetic field lines radiate outwards from a central point. If a current-carrying coil is placed in a radial field, the field is not uniform across the coil, and hence, the torque will not be maximized in this case. A uniform magnetic field is generally required to achieve maximum torque, not a radial field.

3. **Reason (R):** "The torque tends to rotate the coil on its own axis."
   - This is true in general because torque does cause rotational motion. It attempts to align the plane of the coil perpendicular to the magnetic field lines. This rotation aligns the coil to achieve maximum torque.

Conclusion:

- **Assertion (A)** is false because the torque is not maximized in a radial magnetic field; it is maximized in a uniform magnetic field.

- **Reason (R)** is true because torque does indeed cause the coil to rotate to maximize alignment with the magnetic field.

Therefore:

(C) Assertion (A) is true, but Reason (R) is false.

SECTION B

Q.17. (a) A cell is connected across an external resistance 12 Ω and supplies 0.25 A current. When the external resistance is increased by 4 2, the current reduces to 0.2 A. Calculate (i) the emf, and (ii) the internal resistance of the cell.
OR

Solution. To solve this problem, we'll use the formulas related to the internal resistance r of the cell and its electromotive force (emf) E.
Let's denote:
R_1 as the initial external resistance, which is 12 Ω.
I_1 as the initial current, which is 0.25 A.
R2 as the increased external resistance, which is 12 + 4 = 16 Ω.
• I_2 as the reduced current, which is 0.2 A.
Steps to Calculate emf (E) and Internal Resistance (r):
1. Determine the emf of the cell:
The total voltage across the external resistance R can be expressed using Ohm's law:
$E = I_1(R1+r)$
E= I2(R2+r)

Rearranging for the first scenario:
E = 0.25 x (12+ r)
Rearranging for the second scenario:
E = 0.2 × (16+r)
Equate the two expressions for E:

0.25 x (12+r) = 0.2 × (16+r)

Solve for r:
3+0.25r 3.2 +0.2r
0.25 0.2r = 3.2-3
0.05r= 0.2
0.2 0.05 r = 4Ω

Calculate the emf (E):
Substitute r into one of the emf equations:
E = 0.25 x (12+4)
E = 0.25 x 16
E = 4V

(i) The emf of the cell is 4 V.
(ii) The internal resistance of the cell is 4 Ω.

(b) Two point charges of 3 μC and 4 μC are kept in air at (0-3 m, 0) and (0, 0.3 m) in the x-y plane. Find the magnitude and direction of the net electric field produced at the origin (0, 0).

Solution.

To determine the magnitude and direction of the net electric field at the origin due to two point charges, follow these steps:

Given Data

- Charge $q_1 = 3\,\mu\mathrm{C}$ at position $(0, -0.3\ \mathrm{m})$
- Charge $q_2 = 4\,\mu\mathrm{C}$ at position $(0, 0.3\ \mathrm{m})$

Step-by-Step Solution

1. Calculate the Electric Field Due to Each Charge

Electric field $\mathbf{E}$ due to a point charge q at a distance r is given by:
$E = \frac{k \cdot |q|}{r^2}$

where k is Coulomb's constant, $k = 8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}$.

For charge $q_1 = 3\,\mu\mathrm{C}$ at position $(0, -0.3\ \mathrm{m})$:

- Distance from origin $r_1 = 0.3\ \mathrm{m}$
- Electric field magnitude:
  $E_1 = \frac{k \cdot |q_1|}{r_1^2} = \frac{8.99 \times 10^9 \cdot 3 \times 10^{-6}}{0.3^2}$
  $E_1 = \frac{26.97 \times 10^3}{0.09}$

$E_1 = 299.67 \times 10^3$
$E_1 \approx 2.997 \times 10^5\ \mathrm{N/C}$

- Direction: Since q_1 is positive and located below the origin, the electric field due to q_1 points upwards along the positive y-axis.

For charge $q_2 = 4\,\mu\mathrm{C}$ at position $(0, 0.3\ \mathrm{m})$:

- Distance from origin $r_2 = 0.3\ \mathrm{m}$
- Electric field magnitude:
  $E_2 = \frac{k \cdot |q_2|}{r_2^2} = \frac{8.99\times10^9 \cdot 4\times10^{-6}}{0.3^2}$
  $E_2 = \frac{35.96\times10^3}{0.09}$
  $E_2 = 399.56 \times 10^3$
  $E_2 \approx 3.996 \times 10^5\ \mathrm{N/C}$
- Direction: Since q_2 is positive and located above the origin, the electric field due to q_2 points downwards along the negative y-axis.

2. Determine the Net Electric Field at the Origin

Since both fields are along the y-axis but in opposite directions, their magnitudes subtract:

- Net Electric Field E_{net} magnitude:
  $E_{\text{net}} = E_2 - E_1$
  $E_{\text{net}} = 3.996 \times 10^5 - 2.997 \times 10^5$
  $E_{\text{net}} = 1.999 \times 10^5\ \mathrm{N/C}$
- Direction: Since E_2 (downward) is greater than E_1 (upward), the net electric field points downward along the negative y-axis.

Summary

- **Magnitude of the net electric field**: $1.999 \times 10^5\ \mathrm{N/C}$
- **Direction of the net electric field**: Downward along the negative y-axis

Q.18. Differentiate between magnetisation and the susceptibility of a material. What can you say about the susceptibility of paramagnetic and diamagnetic materials ?

Solution. Magnetization vs. Susceptibility

1. Magnetization (M):
Definition: Magnetization is a measure of the magnetic moment per unit volume of a material. It represents the extent to which a material gets magnetised when exposed to an external magnetic field.
Formula: (mathbf{M} = \frac{\mathbf{m}}{V} \), where \(\mathbf{m} \) is the magnetic moment and \(V \) is the volume of the material.

Units:Amperes per metre (A/m) or Tesla (T) in SI units.

2. Magnetic Susceptibility (χ):
Definition: Magnetic susceptibility is a dimensionless quantity that indicates how easily a material can be magnetised by an external magnetic field. It is the ratio of magnetization $\mathbf{M}$ to the applied magnetic field $\mathbf{H}$.
Formula: $\chi = \frac{\mathbf{M}}{\mathbf{H}}$.
Units: It is dimensionless (no units).
Susceptibility of Paramagnetic and Diamagnetic Materials
1. Paramagnetic Materials:
Susceptibility (χ): Positive and small. This means that paramagnetic materials are weakly attracted to an external magnetic field.
Behaviour: The magnetic susceptibility of paramagnetic materials is generally small but positive. This indicates that these materials are weakly attracted by a magnetic field and become more magnetised in the presence of the field. The susceptibility is typically in the range of 10^{-5} to 10^{-1}.

2. Diamagnetic Materials:
Susceptibility (χ): Negative and very small. This means that diamagnetic materials are weakly repelled by an external magnetic field.
Behavior: The magnetic susceptibility of diamagnetic materials is negative, indicating that these materials are repelled by a magnetic field.
Diamagnetic susceptibility is typically very small in magnitude, often around -10^{-5} to -10^{-1}, and does not depend significantly on temperature.
Summary
Magnetization is the magnetic moment per unit volume of a material, whilemagnetic susceptibility measures how much a material becomes magnetised in response to an external magnetic field.
Paramagnetic materials have a positive susceptibility and are attracted to magnetic fields.
Diamagnetic materials have a negative susceptibility and are repelled by magnetic fields.

Q.19. State Huygens principle. Using it draw a diagram showing the details of passage of a plane wave from a denser into a rarer medium.

Solution. ### Huygens' Principle

Huygens' Principle states that:

Every point on a wavefront of a wave can be considered as a source of secondary wavelets. The new wavefront at any subsequent time is the envelope of all these secondary wavelets.

Explanation

According to Huygens' Principle, as a wave propagates through a medium, each point on the wavefront acts as a new source of spherical wavelets. These secondary wavelets spread out in the forward direction and form a new wavefront, which is tangent to the wavelets. This principle helps in understanding wave phenomena like refraction, reflection, and diffraction.

Diagram: Passage of a Plane Wave from a Denser to a Rarer Medium

Description:

1.Wavefronts in Denser Medium (Medium 1): Represented as parallel lines or straight lines indicating the initial wavefronts.

2. Wavefronts in Rarer Medium (Medium 2):Also represented as parallel lines but bending away from the normal to the interface due to a change in speed.

3. Normal Line: A vertical line perpendicular to the boundary separating the two media.

4. Incident Wavefront: The original wavefront approaching the boundary from the denser medium.

5. Refracted Wavefront: The new wavefronts in the rarer medium, showing the bending of the wave.

Explanation of the Diagram:

1. **Incident Wavefront:** The parallel lines represent the wavefronts in the denser medium (Medium 1) approaching the boundary.
2. **Normal:** The vertical dashed line indicates the boundary between Medium 1 and Medium 2, and the normal to the boundary.
3. **Refracted Wavefront:** After crossing into the rarer medium (Medium 2), the wavefronts bend away from the normal, indicating an increase in speed of the wave as it enters the rarer medium.

Key Points:

- The angle of incidence is greater than the angle of refraction when moving from a denser to a rarer medium.
- Huygens' Principle helps visualize how the wavefronts change direction at the boundary between different media.

This diagram and explanation demonstrate how a plane wavefront transitions from a denser to a rarer medium, illustrating the bending effect caused by the change in wave speed.

Q.20. Name the impurity atoms which are doped in an intrinsic semiconductor to convert it into (a) p-type, and (b) n-type semiconductor. Draw energy band diagrams of p-type and n-type semiconductors at temperature $T > 0$ K. Mark the donor and acceptor energy levels, showing the energy difference from the respective bands.

Solution.### Doping in Semiconductors

Doping is the process of adding impurity atoms to an intrinsic semiconductor to modify its electrical properties. The type of impurity determines whether the semiconductor will become **p-type** or **n-type**.

(a) P-type Semiconductor

- **Impurity Atoms:**

  - **Boron (B)**
  - **Gallium (Ga)**
  - **Indium (In)**
  - These are **trivalent** elements (having three valence electrons).

(b) N-type Semiconductor

- **Impurity Atoms:**
  - **Phosphorus (P)**
  - **Arsenic (As)**
  - **Antimony (Sb)**
  - These are **pentavalent** elements (having five valence electrons).

Energy Band Diagrams

1. **P-type Semiconductor**

In a p-type semiconductor, trivalent impurity atoms are added, creating **holes** (positive charge carriers) in the material.

Energy Band Diagram of P-type Semiconductor:

- **Valence Band:** The energy band where valence electrons reside.
- **Conduction Band:** The energy band where electrons are free to move.
- **Acceptor Energy Level (E_a):** The energy level just above the valence band created by the trivalent impurity atoms.

Explanation:

- The **acceptor energy level** is close to the valence band. When the semiconductor is at a temperature above absolute zero, electrons from the valence band will jump to the acceptor level, creating holes in the valence band.

2. **N-type Semiconductor**

In an n-type semiconductor, pentavalent impurity atoms are added, contributing **extra electrons** (negative charge carriers).

- **Conduction Band:** The energy band where electrons are free to move.
- **Valence Band:** The energy band where valence electrons reside.
- **Donor Energy Level (E_d):** The energy level just below the conduction band created by the pentavalent impurity atoms.

Explanation:

- The **donor energy level** is close to the conduction band. When the semiconductor is at a temperature above absolute zero, electrons from the donor level will move to the conduction band, increasing the number of free electrons available for conduction.

Key Points:

- **P-type Semiconductor:** Acceptors introduce holes in the valence band; the acceptor level is near the valence band.
- **N-type Semiconductor:** Donors provide extra electrons that occupy the conduction band; the donor level is near the conduction band.

The diagrams show the positioning of the energy levels relative to the conduction and valence bands, illustrating how doping alters the electrical properties of the semiconductor.

Q.21. A point light source rests on the bottom of a bucket filled with a liquid of refractive index $\mu = 1.25$ up to height of 10 cm. Calculate:
(a) the critical angle for liquid-air interface
(b) radius of circular light patch formed on the surface by light emerging from the source.

To solve the problem involving a point light source at the bottom of a bucket filled with a liquid, we need to calculate two things:

1. **The critical angle for the liquid-air interface.**
2. **The radius of the circular light patch formed on the surface.**

Given Data:
- Refractive index of the liquid, $\mu_{\text{liquid}} = 1.25$
- Height of the liquid column, $h = 10 \text{ cm}$

(a) Critical Angle Calculation

The **critical angle** is the angle of incidence beyond which light cannot pass through the interface between two media but is instead entirely reflected back into the medium with the higher refractive index.

The critical angle θ_c can be calculated using Snell's law at the liquid-air interface:

$$\sin \theta_c = \frac{1}{\mu_{\text{liquid}}}$$

where μ_{liquid} is the refractive index of the liquid and the refractive index of air is approximately 1.

So,

$$\sin \theta_c = \frac{1}{1.25}$$

$$\sin \theta_c = 0.8$$

To find the critical angle θ_c:

$$\theta_c = \sin^{-1}(0.8)$$

Using a calculator:

$$\theta_c \approx 53.13^\circ$$

(b) Radius of the Circular Light Patch

The radius of the circular light patch formed on the surface of the liquid can be calculated using the critical angle.

The point light source will form a circle of light on the surface. The radius r of this circle can be found using trigonometry:

$$r = h \cdot \tan \theta_c$$

where h is the height of the liquid column (10 cm), and θ_c is the critical angle.

So,

$$r = 10 \text{ cm} \cdot \tan(53.13^\circ)$$

Using a calculator:

$$\tan(53.13^\circ) \approx 1.333$$

Therefore,

$$r = 10 \text{ cm} \cdot 1.333$$

$$r \approx 13.33 \text{ cm}$$

SECTION C

Q.22. A particle of mass m and charge q is moving in a magnetic field B with a velocity. Discuss, giving reasons, the shape of its trajectory when the angle between and Bis: ←
(a) 0°
(b) 90°
(c) 120°

Solution.

When a charged particle moves in a magnetic field, the shape of its trajectory depends on the angle between its velocity and the magnetic field. Here's how the trajectory changes based on the angle between the velocity vector ($\vec{v}$) and the magnetic field ($\vec{B}$):

(a) When the angle is 0°

Angle: 0°
Meaning: The velocity vector $\vec{v}$ is parallel to the magnetic field $\vec{B}$.
Trajectory: In this case, the magnetic field does not exert a force on the particle. According to the Lorentz force law, the magnetic force on a charged particle is given by:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

The cross product $\vec{v} \times \vec{B}$ is zero when $\vec{v}$ and $\vec{B}$ are parallel. Hence, the force is zero, and the particle continues to move in a straight line along its original path without any change in direction.

Shape of Trajectory: **Straight Line**

(b) When the angle is 90°

Angle: 90°
Meaning: The velocity vector $\vec{v}$ is perpendicular to the magnetic field $\vec{B}$.
Trajectory: In this case, the magnetic force is always perpendicular to the velocity of the particle. The force provides the centripetal acceleration needed for circular motion. The particle will move in a circular path with the magnetic field as the center of the circle. The radius r of this circular path can be calculated using the following formula:

$$r = \frac{mv}{qB}$$

where m is the mass of the particle, v is its velocity, q is the charge, and B is the magnetic field strength.

Shape of Trajectory: **Circle**

(b) When the angle is 90°

Angle: 90°

Meaning: The velocity vector $\vec{v}$ is perpendicular to the magnetic field $\vec{B}$.

Trajectory: In this case, the magnetic force is always perpendicular to the velocity of the particle. The force provides the centripetal acceleration needed for circular motion. The particle will move in a circular path with the magnetic field as the center of the circle. The radius r of this circular path can be calculated using the following formula:

$r = \frac{mv}{qB}$

where m is the mass of the particle, v is its velocity, q is the charge, and B is the magnetic field strength.

Shape of Trajectory: **Circle**

Q.23. (a) A ray of light is incident on a surface separating air from a denser medium A of refractive index μ_1. It is then made incident on the parallel surface of another medium B of refractive index μ_2 at the same angle of incidence. If the angle of refraction in the two media are 30° and 35° respectively, then in which one of the two media (A or B) will light travel faster and why ?

(b) The intensity of the two interfering waves in Young's double slit experiment is Io each. Find the intensity at a point on the screen where

λ , and (ii) path difference between the interfering waves is (i) 2, λ 3

Solution.

(a) Comparing the Speed of Light in Two Media

Situation: Light is incident on two different media, A and B, from air. The angle of refraction in medium A (with refractive index μ_1) is 30° and in medium B (with refractive index μ_2) is 35°. We need to determine in which medium the light travels faster.

Solution:

1. **Use Snell's Law**:
   Snell's Law relates the angle of incidence θ_i and the angle of refraction θ_r with the refractive indices of the two media. It is given by:

$$\frac{\sin(\theta_i)}{\sin(\theta_r)} = \frac{\mu_2}{\mu_1}$$

   Since the light is incident from air (refractive index $\mu_{\text{air}} \approx 1$), we simplify this to:

$$\sin(\theta_i) = \mu_1 \sin(30^\circ) \text{ and } \sin(\theta_i) = \mu_2 \sin(35^\circ)$$

2. **Calculate Refractive Indices**:

For medium A:

$$\mu_1 = \frac{\sin(\theta_i)}{\sin(30^\circ)}$$

For medium B:

$$\mu_2 = \frac{\sin(\theta_i)}{\sin(35^\circ)}$$

By substituting $\sin(30^\circ) = 0.5$ and $\sin(35^\circ) \approx 0.574$:

$$\mu_1 = \frac{\sin(\theta_i)}{0.5}$$

$$\mu_2 = \frac{\sin(\theta_i)}{0.574}$$

Since $\sin(\theta_i)$ is the same for both cases, the values of μ_1 and μ_2 are inversely proportional to their sine values. Thus:

$$\mu_1 > \mu_2$$

3. **Speed of Light**:
The speed of light v in a medium is given by:

$$v = \frac{c}{\mu}$$

where c is the speed of light in vacuum. Therefore, a lower refractive index means a higher speed of light in the medium. Since $\mu_1 > \mu_2$, light travels faster in medium B.

Conclusion: Light travels faster in medium B because it has a lower refractive index (μ_2) compared to medium A (μ_1).

(b) Intensity of Interfering Waves in Young's Double Slit Experiment

Given: The intensity of the two interfering waves is I_0 each.

To Find: The intensity at a point on the screen where:

1. Path difference is $\lambda/3$
2. Path difference is 2λ

Solution:

1. **Intensity Formula for Interference**:

The resultant intensity I at a point due to two interfering waves is given by:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

where I_1 and I_2 are the intensities of the two waves and δ is the phase difference between them. For equal intensities $I_1 = I_2 = I_0$, the formula becomes:

$$I = 2I_0 + 2I_0\cos\delta$$

where $\cos\delta = \cos\left(\frac{2\pi \text{path difference}}{\lambda}\right)$.

2. **For Path Difference $\lambda/3$:**

$$\delta = \frac{2\pi(\lambda/3)}{\lambda} = \frac{2\pi}{3}$$

$$\cos\delta = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$I = 2I_0 + 2I_0\left(-\frac{1}{2}\right) = 2I_0 - I_0 = I_0$$

Intensity: $I = I_0$

3. **For Path Difference 2λ:**

$$\delta = \frac{2\pi(2\lambda)}{\lambda} = 4\pi$$

$$\cos\delta = \cos(4\pi) = 1$$

$$I = 2I_0 + 2I_0 \cdot 1 = 4I_0$$

Intensity: $I = 4I_0$

24. In photoelectric effect experiment, show the variation of
(a) photocurrent with collector plate potential for a given surface for different intensities of incident radiation. Do the curves meet at any point ? If so, why?

(b) photocurrent with intensity of radiation incident on a surface keeping the frequency and plate potential fixed.

Solution. Let's explore the variations observed in a photoelectric effect experiment for the given parameters:

(a) Variation of Photocurrent with Collector Plate Potential for Different Intensities of Incident Radiation

Experiment Setup:
- **Collector Plate Potential**: Potential applied to the collector plate in the photoelectric effect experiment.
- **Incident Radiation Intensity**: The brightness or power of the incident light.

Graph Description:
- **X-Axis**: Collector Plate Potential (V)
- **Y-Axis**: Photocurrent (I)

For Different Intensities:
- When the intensity of the incident radiation is increased, the photocurrent also increases. This is because more photons hitting the surface means more electrons are ejected, resulting in a higher current.

Curves Explanation:
1. **Low Intensity**: At low intensities, the photocurrent increases linearly with the collector plate potential until it reaches a maximum value, after which it remains constant.
2. **Higher Intensities**: For higher intensities, the photocurrent also increases linearly with the collector plate potential. However, the maximum photocurrent achieved is higher compared to the low-intensity case.

Intersection of Curves:
- **Meeting Point**: All curves (for different intensities) eventually meet at the same maximum potential (when the collector plate potential is sufficiently high). This is because once the collector plate potential reaches a value where it can stop all emitted photoelectrons (saturation), the photocurrent will not increase further regardless of the intensity of incident radiation. This maximum potential is known as the "saturation potential."

Reason:
- The curves meet at this point because increasing the collector plate potential beyond this value does not increase the photocurrent further. This is because all photoelectrons emitted by the surface have already been collected. Hence, saturation current is independent of the intensity of light once it has reached the maximum value.

(b) Variation of Photocurrent with Intensity of Radiation Incident on a Surface (Keeping Frequency and Plate Potential Fixed)

Experiment Setup:
Intensity of Incident Radiation: The brightness or power of the light.
Frequency of Incident Radiation: Fixed, above the threshold frequency.
Collector Plate Potential: Fixed, sufficient to collect all photoelectrons.

Graph Description:
X-Axis: Intensity of Incident Radiation (I)
Y-Axis: Photocurrent (I)

Variation:
Linear Relationship: The photocurrent increases linearly with the intensity of the incident radiation. This is because increasing the intensity means more photons are incident on the surface, leading to more ejected photoelectrons and thus a higher photocurrent.

Reason:
Photocurrent and Photon Flux: Photocurrent is directly proportional to the number of photoelectrons ejected, which is proportional to the number of incident photons. Since each photon with energy greater than the work function ejects one electron, increasing the photon flux (intensity) increases the number of photoelectrons. The constant frequency ensures that every photon has enough energy to overcome the work function, so the increase in intensity directly results in a proportional increase in photocurrent.

Q.25. Two point charges of 10 μC and 20 μC are located at points (4 cm, 0, 0) and A 2' (5 cm, 0, 0) respectively, in a region with electric field E = A = 2 x where 106 NC-1 m² and is the position vector of the point under consideration. Calculate the electrostatic potential energy of the system.

Solution.

To calculate the electrostatic potential energy of the system with two point charges and a given electric field, follow these steps:

Problem Details

1. **Charges**:
   - $q_1 = 10\,\mu\text{C}$
   - $q_2 = 20\,\mu\text{C}$
2. **Positions**:
   - Charge q_1 is located at $(4\,\text{cm}, 0, 0)$ or $(0.04\,\text{m}, 0, 0)$.
   - Charge q_2 is located at $(5\,\text{cm}, 0, 0)$ or $(0.05\,\text{m}, 0, 0)$.
3. **Electric Field**:
   - The electric field is given by $\mathbf{E} = A\mathbf{r}$, where $A = 2 \times 10^6\,\text{N/C}\cdot\text{m}^2$ and $\mathbf{r}$ is the position vector of the point under consideration.

1. **Calculate the Distance Between Charges**:
   The distance r between the charges q_1 and q_2 is simply the difference in their x-coordinates:

$$r = 0.05\,\text{m} - 0.04\,\text{m} = 0.01\,\text{m}$$

2. **Compute the Electrostatic Potential Energy**:
   The formula for the electrostatic potential energy U of two point charges q_1 and q_2 separated by a distance r in a vacuum is:

$$U = \frac{k \cdot q_1 \cdot q_2}{r}$$

   where k is Coulomb's constant:

$$k = 8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$$

   Substitute the values:

$$U = \frac{8.99 \times 10^9 \times 10 \times 10^{-6} \times 20 \times 10^{-6}}{0.01}$$

$$U = \frac{8.99 \times 10^9 \times 200 \times 10^{-12}}{0.01}$$

$$U = \frac{8.99 \times 200 \times 10^{-3}}{0.01}$$

$$U = \frac{1798 \times 10^{-3}}{0.01}$$

$$U = 179.8\,\text{J}$$

Answer

The electrostatic potential energy of the system of two point charges is $179.8\,\text{J}$.

Q.26. A current of 1 A flows through an inductor connected to a 200 V dc source. When it is connected to 200 V, 50 Hz source, only 0.5 A current flows. Calculate the self-inductance of the inductor.

Solution.

To calculate the self-inductance of the inductor given the information about its behavior in both DC and AC circuits, follow these steps:

Given Data

1. **DC Source**:
   - Current $I_{\text{DC}} = 1\text{ A}$
   - Voltage $V_{\text{DC}} = 200\text{ V}$
2. **AC Source**:
   - Frequency $f = 50\,\text{Hz}$
   - Current $I_{\text{AC}} = 0.5\text{ A}$
   - Voltage $V_{\text{AC}} = 200\text{ V}$

1. **Calculate the Self-Inductance from the DC Circuit**:
   For DC circuits, the voltage across an inductor is given by:

$$V_{\text{DC}} = L \cdot \frac{dI}{dt}$$

Since DC current is constant, the inductor behaves like a short circuit after a steady state is reached. Therefore, the voltage V_{DC} is due to the resistive part, which is effectively zero for an ideal inductor.

In the DC steady state:

$$V_{\text{DC}} = I_{\text{DC}} \cdot R$$

Since $V_{\text{DC}} = 200\text{ V}$ and $I_{\text{DC}} = 1\text{ A}$:

$$R = \frac{V_{\text{DC}}}{I_{\text{DC}}} = \frac{200}{1} = 200\,\Omega$$

For an ideal inductor, R is zero, implying any resistance observed here is due to the actual circuit resistance or approximation.

2. **Calculate the Self-Inductance from the AC Circuit:**
For AC circuits, the voltage across the inductor is related to the inductive reactance X_L:

$$V_{AC} = I_{AC} \cdot X_L$$

where X_L is given by:

$$X_L = 2\pi f L$$

Rearranging for L:

$$L = \frac{V_{AC}}{I_{AC} \cdot 2\pi f}$$

Substitute the values:

$$L = \frac{200}{0.5 \cdot 2\pi \cdot 50}$$

$$L = \frac{200}{0.5 \cdot 314.16}$$

$$L = \frac{200}{157.08}$$

$$L \approx 1.27\,\mathrm{H}$$

Answer

The self-inductance of the inductor is approximately $1.27\,\mathrm{H}$.

Q.27. Two circular coils of radius R each and having equal number of turns N, carry equal currents I in the same direction. They are placed coaxially at a distance 2√3 R. Find the magnitude and direction of the net magnetic field produced at the midpoint of the line joining their centres.

Solution.

To find the net magnetic field at the midpoint of the line joining the centers of two coaxial circular coils, we need to follow these steps:

Given Data:

- Radius of each coil: R
- Number of turns in each coil: N
- Current in each coil: I
- Distance between the coils: $2\sqrt{3}R$

Objective:

- Find the magnitude and direction of the net magnetic field at the midpoint of the line joining the centers of the coils.

Solution:

1. **Magnetic Field Due to a Single Coil at Its Center:**

   For a single circular coil of radius R, carrying a current I with N turns, the magnetic field B_{center} at the center of the coil is given by:

$$B_{\text{center}} = \frac{\mu_0 N I}{2R}$$

   where μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$).

2. **Magnetic Field at the Midpoint of the Line Joining the Centers:**

   Let's denote the coils as Coil 1 and Coil 2. They are placed coaxially at a distance $2\sqrt{3}R$ apart. We are interested in the magnetic field at the midpoint between the two coils. The distance from the midpoint to the center of each coil is:

$$d = \frac{2\sqrt{3}R}{2} = \sqrt{3}R$$

For a circular coil, the magnetic field at a point on the axis at a distance z from the center of the coil is given by:

$$B_{\text{axis}} = \frac{\mu_0 N I R^2}{2(R^2 + z^2)^{3/2}}$$

Substituting $z = \sqrt{3}R$:

$$B_{\text{axis}} = \frac{\mu_0 N I R^2}{2(\sqrt{3}R^2 + R^2)^{3/2}} = \frac{\mu_0 N I R^2}{2(4R^2)^{3/2}} = \frac{\mu_0 N I R^2}{2 \cdot 8R^3} = \frac{\mu_0 N I}{16R}$$

Therefore, the magnetic field at the midpoint due to each coil is:

$$B_{\text{mid}} = \frac{\mu_0 N I}{16R}$$

3. **Net Magnetic Field:**

   Since the currents in both coils are in the same direction, the magnetic fields at the midpoint due to each coil will add up.

   The magnetic fields are in the same direction along the axis of the coils, so the net magnetic field at the midpoint is:

$$B_{\text{net}} = B_{\text{mid, Coil 1}} + B_{\text{mid, Coil 2}} = 2 \times \frac{\mu_0 N I}{16R} = \frac{\mu_0 N I}{8R}$$

Answer:

Magnitude of the net magnetic field at the midpoint of the line joining the centers of the two coils:

$$B_{\text{net}} = \frac{\mu_0 N I}{8R}$$

Direction of the net magnetic field:

The direction of the net magnetic field is along the axis of the coils, in the direction of the current in the coils.

Q.28. (a) The radius of a conducting wire AB uniformly decreases from one end A to another end B. It is connected across a battery. How will (i) electric field, (ii) current density, and (iii) mobility of electrons change from end A to end B? Justify your answer in each case.
OR

Solution.

Let's analyze the situation where a conducting wire with a radius that uniformly decreases from end A to end B is connected across a battery. We will examine how the electric field, current density, and mobility of electrons change from end A to end B.

Given:

- **Wire**: Conducting
- **Radius**: Decreases uniformly from end A to end B
- **Connection**: Across a battery

Analysis:

(i) Electric Field (E):

Electric field E in a conductor is given by Ohm's Law in terms of voltage V and length L:
$E = \frac{V}{L}$

Since the battery maintains a constant potential difference V across the entire length of the wire and the length L from end A to end B is constant, the electric field E is uniform along the length of the wire.

Conclusion: The electric field E will remain constant from end A to end B.

(ii) Current Density (J):

Current density J is defined as:
$J = \frac{I}{A}$
where I is the current and A is the cross-sectional area of the wire.

The cross-sectional area A of the wire at any point is related to the radius r by:
$A = \pi r^2$

Since the radius decreases from end A to end B, the cross-sectional area A also decreases. If the current I supplied by the battery is constant, the current density J will be inversely proportional to the cross-sectional area:
$J = \frac{I}{\pi r^2}$

Therefore, as the radius r decreases, the current density J increases.

Conclusion: The current density J increases from end A to end B as the radius decreases.

(iii) Mobility of Electrons (μ):

Mobility of electrons μ is given by:
$J = \sigma E = ne\mu E$
where σ is the electrical conductivity, n is the number density of charge carriers, and e is the charge of an electron.

Since the material is conducting and the radius changes uniformly, **the number density** n of electrons remains constant throughout the wire, and **the electric field** E is uniform.

The electrical conductivity σ of the material is also uniform as the material's properties do not change. Hence, the mobility of electrons μ is a material property and is not affected by the geometric change of the wire.

Conclusion: The mobility of electrons μ remains constant from end A to end B.

Summary:

- **Electric Field** (E): Remains constant throughout the wire.
- **Current Density** (J): Increases from end A to end B as the radius decreases.
- **Mobility of Electrons** (μ): Remains constant throughout the wire.

Q.30. Dipoles, whether electric or magnetic, are characterised by their dipole moments, which are vector quantities. Two equal and opposite charges separated by a small distance constitute an electric dipole, while a current carrying loop behaves as a magnetic dipole. Electric dipoles create electric fields around them. Electric dipoles experience a torque when placed in an external electric field.
(i) Two identical electric dipoles, each consisting of charges-q and + q separated by distance d, are arranged in x-y plane such that their negative charges lie at the origin O and positive charges lie at points (d, 0) and (0, d) respectively. The net dipole moment of the system is:

(A) -qd(i+j)
(B) qd (i+j)
(C) qd (-)
(D) qd (-1)

(ii) E_1 and E2 are magnitudes of electric field due to a dipole, consisting of charges -q and +q separated by distance 2a, at points r axis, and (2) on equatorial plane, respectively. Then E1 E2 a) (1) on its is:
(A) 1/4
(B) 1/2

(C) 2
(D) 4

(iii) An electric dipole of dipole moment 5.0 × 10-8 Cm is placed in a region where an electric field of magnitude 1.0 × 103 N/C acts at a given instant. At that instant the electric field E is inclined at an angle of 30° to dipole moment P. The magnitude of torque acting on the dipole, at that instant is:
(A) 2.5 x 10-5 Nm
(B) 5.0 x 10-5 Nm
(C) 1.0 × 10-4 Nm
(D) 2.0 x 10-6 Nm

(iv) (a) An electron is revolving with speed v around the proton in a hydrogen atom, in a circular orbit of radius r. The magnitude of magnetic dipole moment of the electron is :
(A) 4 evr

(B) 2 evr
(C) evr 2
(D) evr

OR

Solution.

(i) Net Dipole Moment of Two Identical Electric Dipoles

Problem Statement:

Two identical electric dipoles, each consisting of charges $-q$ and $+q$ separated by distance d, are arranged in the x-y plane. The negative charges lie at the origin O and the positive charges lie at points $(d, 0)$ and $(0, d)$, respectively.

Solution:

1. **Dipole Moment of Each Dipole:**
   - For the dipole with negative charge at $(0, 0)$ and positive charge at $(d, 0)$, the dipole moment is:
     $\vec{p}_1 = qd\hat{i}$
   - For the dipole with negative charge at $(0, 0)$ and positive charge at $(0, d)$, the dipole moment is:
     $\vec{p}_2 = qd\hat{j}$

2. **Net Dipole Moment of the System:**

   The net dipole moment of the system is the vector sum of the individual dipole moments:
   $\vec{p}_{\text{net}} = \vec{p}_1 + \vec{p}_2$
   $\vec{p}_{\text{net}} = qd\hat{i} + qd\hat{j}$
   $\vec{p}_{\text{net}} = qd(\hat{i} + \hat{j})$

   Answer:

   (B) $qd(\hat{i} + \hat{j})$

(ii) Electric Field Magnitudes Due to a Dipole

Problem Statement:

For a dipole consisting of charges $-q$ and $+q$ separated by distance $2a$:

- E_1 is the magnitude of the electric field on the axial line (i.e., along the line joining the charges).
- E_2 is the magnitude of the electric field on the equatorial plane.

Solution:

The electric field of a dipole at a distance r along the axial line is given by:
$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$

The electric field of a dipole at a distance r on the equatorial plane is given by:
$E_{\text{equatorial}} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$

The ratio of E_1 to E_2 is:
$\frac{E_1}{E_2} = \frac{2p/r^3}{p/r^3} = 2$

Answer:

(C) 2

(iii) Torque on an Electric Dipole

Problem Statement:

An electric dipole with dipole moment $p = 5.0 \times 10^{-8}\,\text{C m}$ is placed in an electric field $E = 1.0 \times 10^3\,\text{N/C}$. The electric field is inclined at an angle of 30° to the dipole moment.

Solution:

The magnitude of torque τ acting on a dipole in an electric field is given by:
$\tau = pE\sin\theta$

Where:

- $p = 5.0 \times 10^{-8}\,\text{C m}$
- $E = 1.0 \times 10^3\,\text{N/C}$
- $\theta = 30^\circ$

Calculating:
$\tau = (5.0 \times 10^{-8}\,\text{C m}) \times (1.0 \times 10^3\,\text{N/C}) \times \sin 30^\circ$
$\tau = (5.0 \times 10^{-8}) \times (1.0 \times 10^3) \times 0.5$
$\tau = 2.5 \times 10^{-5}\,\text{Nm}$

(iv) Magnetic Dipole Moment of an Electron in Hydrogen Atom

Problem Statement:

An electron revolves with speed v around a proton in a hydrogen atom in a circular orbit of radius r.

Solution:

The magnetic dipole moment μ of an electron revolving in a circular orbit is given by:
$\mu = \frac{evr}{2}$

Where:

- e is the charge of the electron
- v is the speed of the electron
- r is the radius of the orbit

Answer:

(B) $\frac{evr}{2}$

(b) A square loop of side 5.0 cm carries a current of 2.0 A. The magnitude of magnetic dipole moment associated with the loop is:
(A) 1.0 x 10-3 Am²
(B) 5.0 x 10-3 Am2
(C) 1.0 x 10-2 Am²
(D) 5.0 x 10-2 Am²

Solution.

To find the magnetic dipole moment of a square loop, you can use the formula for the magnetic dipole moment (μ) of a current-carrying loop:

$$\mu = I \cdot A$$

where:

- I is the current through the loop,
- A is the area of the loop.

Given:

- The side of the square loop, $a = 5.0\,\text{cm} = 0.05\,\text{m}$,
- The current, $I = 2.0\,\text{A}$.

1. **Calculate the area A of the square loop:**

$$A = a^2$$
$$A = (0.05\,\text{m})^2$$
$$A = 0.0025\,\text{m}^2$$

2. **Calculate the magnetic dipole moment μ:**

$$\mu = I \cdot A$$
$$\mu = 2.0\,\text{A} \times 0.0025\,\text{m}^2$$
$$\mu = 0.005\,\text{A} \cdot \text{m}^2$$
$$\mu = 5.0 \times 10^{-3}\,\text{A} \cdot \text{m}^2$$

Answer:

(B) $5.0 \times 10^{-3}\,\text{A} \cdot \text{m}^2$

SECTION E

Q.31.(a) (i) With the help of a labelled diagram, explain the working of an ac generator. Obtain the expression for the emf induced at an instant 't'.

Solution.

Working of the AC Generator

1. **Magnetic Field:** A constant magnetic field is produced by the field coils or permanent magnets in the stator.
2. **Rotation of Rotor:** When the rotor (armature) is mechanically rotated by an external source (like a turbine), the coil cuts through the magnetic field lines.
3. **Electromagnetic Induction:** As the coil rotates, the magnetic flux through the coil changes. According to Faraday's Law of Electromagnetic Induction, this change in flux induces an electromotive force (EMF) in the coil.
4. **EMF Generation:** The induced EMF is alternating because the direction of the magnetic flux through the coil changes periodically as the rotor rotates.
5. **Electrical Output:** The alternating current (AC) generated is taken out through slip rings and brushes, which transfer the current to the external circuit.

Expression for the EMF Induced

Let's derive the expression for the instantaneous EMF (E) induced in the generator.

Let's derive the expression for the instantaneous EMF (E) induced in the generator.

1. **Magnetic Flux (Φ):**

   The magnetic flux Φ through the coil is given by:

   $$\Phi = B \cdot A \cdot \cos(\theta)$$

   where:

   - B is the magnetic field strength,
   - A is the area of the coil,
   - θ is the angle between the normal to the plane of the coil and the magnetic field.

2. **Angle θ:**

   The angle θ changes with time. If the coil rotates with an angular velocity ω, then:

   $$\theta = \omega t$$

   where t is the time.

3. **Magnetic Flux as a Function of Time:**

   Thus, the flux Φ at time t is:

   $$\Phi = B \cdot A \cdot \cos(\omega t)$$

4. **Induced EMF (E):**

   According to Faraday's Law, the magnitude of the induced EMF is the negative rate of change of the magnetic flux:

   $$E = -\frac{d\Phi}{dt}$$

   Substituting Φ:

   $$E = -\frac{d}{dt}\left(B \cdot A \cdot \cos(\omega t)\right)$$

   $$E = -B \cdot A \cdot (-\omega \sin(\omega t))$$

$$E = B \cdot A \cdot \omega \sin(\omega t)$$

Summary:

The instantaneous EMF (E) induced in the AC generator at time t is:

$$E = B \cdot A \cdot \omega \sin(\omega t)$$

where B is the magnetic field strength, A is the area of the coil, ω is the angular velocity of the rotor, and t is the time.

(b)(i) State Faraday's law of electromagnetic induction and mention the utility of Lenz's law. Obtain an expression for self-inductance of a coil in terms of its geometry and permeability of the medium.

Solution.

Faraday's Law of Electromagnetic Induction

Faraday's Law of Electromagnetic Induction states that:

- **"The electromotive force (EMF) induced in a circuit is directly proportional to the rate of change of magnetic flux through the circuit."**

Mathematically, it can be expressed as:

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

where:

- $\mathcal{E}$ is the induced EMF,
- Φ is the magnetic flux,
- $\frac{d\Phi}{dt}$ is the rate of change of magnetic flux.

Magnetic Flux (Φ) is given by:

$$\Phi = B \cdot A \cdot \cos(\theta)$$

where:

- B is the magnetic field strength,
- A is the area through which the magnetic field lines pass,
- θ is the angle between the magnetic field and the normal to the surface.

Lenz's Law

Lenz's Law states that:

- **"The direction of the induced EMF and current in a closed circuit is such that it opposes the change in magnetic flux that produced it."**

Utility of Lenz's Law:

- **Direction Determination:** It helps in determining the direction of the induced EMF and current.
- **Conservation of Energy:** It ensures that the induced current opposes the change in magnetic flux, which is consistent with the principle of conservation of energy. The work done to induce the current is always equal to the change in magnetic energy.

Expression for Self-Inductance of a Coil

Self-inductance L of a coil is a measure of its ability to induce EMF in itself due to a change in current flowing through it. To derive the expression for L, consider the following:

1. **Magnetic Flux through a Coil:**
   - For a coil of N turns, the total magnetic flux Φ through one turn is:

   $$\Phi = B \cdot A$$

   where A is the cross-sectional area of the coil, and B is the magnetic field strength inside the coil.

2. **Magnetic Field B:**
   - The magnetic field B inside the coil (assuming it is long and solenoidal) is given by:

   $$B = \mu \cdot \frac{N \cdot I}{l}$$

   where:

   - μ is the permeability of the medium,
   - N is the number of turns,
   - I is the current flowing through the coil,
   - l is the length of the coil.

3. **Magnetic Flux through the Coil:**
   - The total magnetic flux through the coil is:

   $$\Phi = B \cdot A = \mu \cdot \frac{N \cdot I}{l} \cdot A$$

4. **Induced EMF (Faraday's Law):**
   - According to Faraday's Law:

   $$\mathcal{E} = -\frac{d\Phi}{dt}$$

   - Since $\Phi = \mu \cdot \frac{N \cdot I}{l} \cdot A$, we have:

   $$\mathcal{E} = -\frac{d}{dt}\left(\mu \cdot \frac{N \cdot I \cdot A}{l}\right) = -\mu \cdot \frac{N \cdot A}{l} \cdot \frac{dI}{dt}$$

5. **Self-Inductance L:**

- The self-inductance L is defined as:

$$\mathcal{E} = -L \cdot \frac{dI}{dt}$$

- Comparing this with the previous expression:

$$L = \mu \cdot \frac{N^2 \cdot A}{l}$$

Summary

The **self-inductance** L of a coil is given by:

$$L = \mu \cdot \frac{N^2 \cdot A}{l}$$

where:

- μ is the permeability of the medium,
- N is the number of turns,
- A is the cross-sectional area of the coil,
- l is the length of the coil.

(ii) A resistance of 20 2, a capacitance of 80 μF and an inductor of 50 mH are connected in series. This combination is connected across a 220 V ac supply of variable frequency. When the frequency of supply equals the natural frequency of the circuit, calculate:

(1) angular frequency of supply

(2) impedance of the circuit

Solution.

To solve this problem, we need to determine the angular frequency of the supply and the impedance of the circuit when the frequency of the supply equals the natural frequency of the circuit. Here's a step-by-step solution:

Given Data

- Resistance (R) = 20 Ω
- Capacitance (C) = 80 μF = 80×10^{-6} F
- Inductance (L) = 50 mH = 50×10^{-3} H
- Supply Voltage = 220 V (though this is not needed for the calculations in this specific part)

Step 1: Determine the Angular Frequency (ω)

The natural frequency (resonant frequency) of an RLC circuit is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

where:

- L is the inductance,
- C is the capacitance.

Substitute the given values into this formula:

$$\omega_0 = \frac{1}{\sqrt{(50 \times 10^{-3}) \times (80 \times 10^{-6})}}$$

$$\omega_0 = \frac{1}{\sqrt{4 \times 10^{-9}}}$$

$$\omega_0 = \frac{1}{2 \times 10^{-4}}$$

$$\omega_0 = 5000 \text{ rad/s}$$

Step 2: Calculate the Impedance of the Circuit at Resonance

At resonance, the inductive reactance (X_L) and capacitive reactance (X_C) are equal, and they cancel each other out. Therefore, the impedance of the circuit at resonance is purely resistive and is equal to the resistance R.

The impedance Z of the circuit at resonance is:

$$Z = R$$

Given that $R = 20\ \Omega$, the impedance at resonance is:

$$Z = 20\ \Omega$$

Summary

1. **Angular Frequency of Supply**: $\omega_0 = 5000\ \text{rad/s}$
2. **Impedance of the Circuit**: $Z = 20\ \Omega$

Q.32. (a) (i) What are the two main considerations for designing the objective and eyepiece lenses of an astronomical telescope ? Obtain the expression for magnifying power of the telescope when the final image is formed at infinity.

Solution. Designing the Objective and Eyepiece Lenses of an Astronomical Telescope

When designing the lenses for an astronomical telescope, two primary considerations are:

1. **Objective Lens:**
   - **Aperture Size**: The objective lens should have a large diameter (aperture) to collect as much light as possible. This increases the telescope's light-gathering power and improves the resolution, allowing it to view faint and distant celestial objects with greater clarity.
   - **Focal Length**: The focal length of the objective lens should be long to provide a higher magnification and to allow for a detailed view of distant objects. A longer focal length of the objective lens helps to achieve better resolution and a clearer image.

2. **Eyepiece Lens:**

- **Focal Length**: The eyepiece lens should have a shorter focal length to increase the magnification of the image formed by the objective lens. This allows for a detailed examination of the image produced by the objective lens.
- **Field of View**: The eyepiece should provide a wide field of view to make it easier to locate and track celestial objects. This ensures that the image of the object is fully visible and helps in easier observation.

Magnifying Power of the Telescope

The magnifying power (M) of an astronomical telescope when the final image is formed at infinity is given by the ratio of the focal length of the objective lens (f_o) to the focal length of the eyepiece lens (f_e).

Expression for Magnifying Power:

$$
M = \frac{f_o}{f_e}
$$

where:
- f_o is the focal length of the objective lens.
- f_e is the focal length of the eyepiece lens.

Derivation:

1. Formation of Image:
The objective lens forms a real and inverted image of the distant object at its focal plane.
The eyepiece lens acts as a magnifier to view this real image, forming a virtual image at infinity (if the final image is at infinity).

2. Magnifying Power:

The magnifying power of the telescope is the ratio of the angular size of the image seen through the eyepiece to the angular size of the image seen with the naked eye.
For a simple telescope where the final image is at infinity, the angular magnification is given by:

$$M = \frac{\text{Angle subtended by the image at the eyepiece}}{\text{Angle subtended by the object at the objective lens}}$$

Since the angle subtended by the image at infinity is $\frac{1}{f_e}$ and by the object is $\frac{1}{f_o}$, we get:

$$M = \frac{\text{Focal length of the objective}}{\text{Focal length of the eyepiece}} = \frac{f_o}{f_e}$$

This formula provides the magnification power of the telescope, showing how much larger the image appears through the telescope compared to the naked eye.

(ii) A ray of light is incident at an angle of 45° at one face of an equilateral triangular prism and passes symmetrically through the prism. Calculate:
(1) the angle of deviation produced by the prism
(2) the refractive index of the material of the prism

Solution.

To solve the problem of a ray of light passing through an equilateral triangular prism, we need to calculate two things:

1. **The Angle of Deviation Produced by the Prism**
2. **The Refractive Index of the Material of the Prism**

Given:

- The prism is equilateral, so each angle of the prism is 60°.
- The incident angle at the first face of the prism is 45°.
- The ray passes symmetrically through the prism, meaning it is incident and exits symmetrically.

1. Angle of Deviation (δ)

Step-by-Step Solution:

1. **Understanding Symmetric Passage:**
   For a ray passing symmetrically through an equilateral prism, the angle of incidence (i) and angle of emergence (e) are equal. Thus, the angles i and e at the faces of the prism are both 45°.

2. **Prism Angles and Deviation:**
   - Let A be the angle of the prism. For an equilateral prism, $A = 60^\circ$.
   - The angle of deviation δ for a prism is given by:

   $$\delta = i + e - A$$

   Since the incident angle i and emergence angle e are both 45°, we substitute:

   $$\delta = 45^\circ + 45^\circ - 60^\circ = 90^\circ - 60^\circ = 30^\circ$$

   Therefore, the angle of deviation δ is 30°.

2. Refractive Index (μ)

Step-by-Step Solution:

1. **Using Snell's Law:**
   - At the first face of the prism:

$$\mu = \frac{\sin(i + \theta/2)}{\sin(\theta/2)}$$

where θ is the angle of the prism, and $\theta/2$ is the angle of deviation inside the prism.

2. **Calculate Internal Angles:**
   - For a symmetric passage, the angle of incidence i and angle of emergence e inside the prism are the same.
   - The angle of deviation inside the prism is $\theta - i = 60^\circ - 45^\circ = 15^\circ$.
3. **Calculate Refractive Index:**
   - The refractive index μ is calculated using the formula:

$$\mu = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where $\delta = 30^\circ$ and $A = 60^\circ$:

$$\mu = \frac{\sin\left(\frac{60^\circ + 30^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)} = \frac{\sin(45^\circ)}{\sin(30^\circ)}$$

$$\mu = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2}$$

Therefore, the refractive index μ of the prism is $\sqrt{2} \approx 1.414$.

Summary

1. **The Angle of Deviation** produced by the prism is 30°.
2. **The Refractive Index** of the material of the prism is $\sqrt{2}$ or approximately 1.414.

Q.33. (a)(i) What are matter waves? A particle of mass m and charge q is accelerated from rest through a potential difference V. Obtain an expression for de Broglie wavelength associated with the particle.

Solution. Matter Waves and de Broglie Wavelength

(a)(i) Matter Waves:

Matter waves, also known as de Broglie waves, are a fundamental concept in quantum mechanics. They represent the wave-like behavior of particles. According to Louis de Broglie, every moving particle or object has an associated wave, and the wavelength of these waves is related to the particle's momentum. This concept is crucial in understanding the wave-particle duality of matter.

Expression for de Broglie Wavelength:

To derive the de Broglie wavelength of a particle that is accelerated through a potential difference \(V\), follow these steps:

1. Kinetic Energy of the Particle:

When a particle of mass \(m\) and charge \(q\) is accelerated from rest through a potential difference \(V\), its kinetic energy (\(K.E.\)) is given by:

K.E.=qV

2.Relate Kinetic Energy to Momentum:
The kinetic energy of a particle is also expressed in terms of its momentum (p) as:

K.E.=2mp2

Equate the two expressions for kinetic energy:

qV=2mp2

Rearranging for \(p\), the momentum:
p=2mqV

3.de Broglie Wavelength:
According to de Broglie's hypothesis, the wavelength \(\lambda\) of a matter wave is related to its momentum \(p\) by the equation:
λ=ph
where \(h\) is Planck's constant. Substituting the expression for \(p\):

λ=2mqVh

Summary
For a particle of mass \(m\) and charge \(q\) accelerated through a potential difference \(V\), the de Broglie wavelength \(\lambda\) associated with the particle is given by:
λ=2mqVh

where \(h\) is Planck's constant. This expression shows that the de Broglie wavelength is inversely proportional to the square root of the kinetic energy of the particle, which is itself related to the potential difference through which the particle was accelerated.

(ii) Monochromatic light of frequency 5.0 × 1014 Hz is produced by a source of power output 3.315 mW. Calculate:
(1) energy of the photon in the beam
(2) number of photons emitted per second by the source

Solution.

To solve the problem, we need to calculate two things:

1. **Energy of a Photon**
2. **Number of Photons Emitted per Second**

Given:

- Frequency of the light, $f = 5.0 \times 10^{14}\,\mathrm{Hz}$
- Power output of the source, $P = 3.315\,\mathrm{mW} = 3.315 \times 10^{-3}\,\mathrm{W}$

(1) Energy of the Photon

The energy E of a single photon is given by the equation:

$$E = hf$$

where:

- h is Planck's constant ($h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s}$)
- f is the frequency of the photon

Substitute the given values:

$$E = (6.626 \times 10^{-34}\,\mathrm{J \cdot s}) \times (5.0 \times 10^{14}\,\mathrm{Hz})$$

$$E = 3.313 \times 10^{-19}\,\mathrm{J}$$

(2) Number of Photons Emitted per Second

To find the number of photons emitted per second, we use the relationship between power, energy, and the number of photons. The power P of the source is the total energy emitted per second:

$$P = (\text{Number of photons per second}) \times E$$

Let N be the number of photons emitted per second. Rearranging the formula to solve for N:

$$N = \frac{P}{E}$$

Substitute the values:

$$N = \frac{3.315 \times 10^{-3}\,\mathrm{W}}{3.313 \times 10^{-19}\,\mathrm{J}}$$

$$N \approx 1.0 \times 10^{16}\,\text{photons/second}$$

Summary

1. **Energy of the photon**: $3.313 \times 10^{-19}\,\mathrm{J}$
2. **Number of photons emitted per second**: 1.0×10^{16}

(b) (i) State Bohr's postulates and derive an expression for the energy of electron in nth orbit in Bohr's model of hydrogen atom.

Solution.

Bohr's Postulates

Niels Bohr proposed the following postulates to describe the behavior of electrons in an atom:

1. **Quantized Orbits**: Electrons revolve around the nucleus in certain discrete orbits or stationary states without radiating energy. These orbits are known as "quantized" or "allowed" orbits.
2. **Angular Momentum Quantization**: The angular momentum of an electron in these orbits is quantized and given by:

$$L = n\hbar$$

   where n is a positive integer (the principal quantum number), and $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant.
3. **Energy Levels**: Electrons can only occupy these quantized orbits. The energy of the electron is associated with these orbits, and it is emitted or absorbed when an electron transitions between different orbits.
4. **Radius of Orbit**: The radius of the nth orbit is determined by the balance of centripetal force and Coulomb's electrostatic force.

Deriving the Expression for Energy of Electron in nth Orbit

1. **Centripetal Force and Electrostatic Force**:

   The centripetal force needed to keep the electron in a circular orbit is provided by the electrostatic force of attraction between the electron and the nucleus.

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r}$$

   where:

   - e is the charge of the electron,
   - ϵ_0 is the permittivity of free space,
   - r is the radius of the orbit,
   - m is the mass of the electron,
   - v is the velocity of the electron.

2. **Angular Momentum Quantization**:

According to Bohr's postulates, the angular momentum L is quantized:

$$L = mvr = n\hbar$$

Rearranging for v:

$$v = \frac{n\hbar}{mr}$$

3. **Substitute v into the Centripetal Force Equation**:

Substitute $v = \frac{n\hbar}{mr}$ into the centripetal force equation:

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{m\left(\frac{n\hbar}{mr}\right)^2}{r}$$

Simplify:

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{n^2\hbar^2}{mr^3}$$

Rearranging for r:

$$r = \frac{n^2\hbar^2}{4\pi\epsilon_0 me^2}$$

4. **Calculate the Energy of the Electron**:

The total energy E_n of the electron is the sum of its kinetic energy (K) and potential energy (U):

- Kinetic Energy:

$$K = \frac{1}{2}mv^2$$

Substituting v^2 from the centripetal force equation:

$$K = \frac{1}{2}\left(\frac{e^2}{4\pi\epsilon_0 r^2}\right)r = \frac{e^2}{8\pi\epsilon_0 r}$$

- Potential Energy:

$$U = -\frac{e^2}{4\pi\epsilon_0 r}$$

- Total Energy:

$$E_n = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r}$$

Substitute r into this expression:

$$E_n = -\frac{e^2}{8\pi\epsilon_0}\left(\frac{4\pi\epsilon_0 m e^2}{n^2\hbar^2}\right)$$

Simplify:

$$E_n = -\frac{me^4}{8\epsilon_0^2\hbar^2 n^2}$$

Therefore, the energy of the electron in the nth orbit is:

$$E_n = -\frac{13.6 \text{ eV}}{n^2}$$

where 13.6 eV is the Rydberg energy for the hydrogen atom.

(ii) Calculate binding energy per nucleon (in MeV) of C.

Given: m (c) 12 6 = 12.000000 u m(n) = 1.008665 u m(H) = 1.007825 u

Solution.

To calculate the binding energy per nucleon of a carbon-12 (^{12}C) nucleus, follow these steps:

Given Data:

- Atomic mass of Carbon-12 (^{12}C): 12.000000 atomic mass units (u)
- Atomic mass of a neutron (m_n): 1.008665 u
- Atomic mass of a proton (m_p): 1.007825 u

Steps to Calculate Binding Energy Per Nucleon:

1. **Calculate the Mass Defect:**
   The mass defect is the difference between the mass of the individual nucleons and the actual mass of the nucleus. For ^{12}C, which has 6 protons and 6 neutrons:

$$\text{Mass of 6 protons} = 6 \times m_p = 6 \times 1.007825 \text{ u} = 6.04695 \text{ u}$$

$$\text{Mass of 6 neutrons} = 6 \times m_n = 6 \times 1.008665 \text{ u} = 6.05199 \text{ u}$$

$$\text{Total mass of nucleons} = 6.04695 \text{ u} + 6.05199 \text{ u} = 12.09894 \text{ u}$$

The actual mass of the ^{12}C nucleus is 12.000000 u.

$$\text{Mass Defect} = \text{Total mass of nucleons} - \text{Mass of } ^{12}\text{C} = 12.09894 \text{ u} - 12.000000 \text{ u}$$

2. **Convert Mass Defect to Energy:**
   The binding energy E in MeV can be found using the mass-energy equivalence principle. The conversion factor is 1 u = 931.5 MeV.

$$E = \text{Mass Defect} \times 931.5 \text{ MeV/u}$$

$$E = 0.09894 \text{ u} \times 931.5 \text{ MeV/u} = 92.2 \text{ MeV}$$

3. **Calculate Binding Energy Per Nucleon:**
   Carbon-12 has 12 nucleons (6 protons and 6 neutrons). So the binding energy per nucleon is:

$$\text{Binding Energy Per Nucleon} = \frac{\text{Total Binding Energy}}{\text{Number of Nucleons}}$$

$$\text{Binding Energy Per Nucleon} = \frac{92.2 \text{ MeV}}{12} = 7.68 \text{ MeV/nucleon}$$