CBSE 12th 2024 Compartment Physics Set-3 (55/S/3) Solutions

SECTION A

Q.1. The electric field E associated with an electromagnetic wave is represented by Ey = Eo sin (kx - ot).Which of the following statements is correct?
(A) The wave is propagating along +x-axis.
(B) The wave is propagating along +z-axis.
(C) The magnetic field B of the wave is acting along +y-axis.
(D) The magnetic field B of the wave is acting along -x-axis.

Solution. (A) The wave is propagating along +x-axis.

Given the electric field $E_y = E_0 \sin(kx - \omega t)$ of an electromagnetic wave, we need to determine the correct statements about the wave's propagation direction and the direction of the magnetic field. Let's break it down:

Analysis of the Given Electric Field:

The electric field is given by $E_y = E_0 \sin(kx - \omega t)$, where:

- E_y is the electric field in the y-direction.
- k is the wave number.
- ω is the angular frequency.
- x and t are the position and time variables.

Understanding Wave Propagation and Field Directions:

1. **Wave Propagation Direction:**
    - The argument of the sine function is $kx - \omega t$. This form indicates that the wave is traveling in the positive x-direction. If it were $kx + \omega t$, it would be traveling in the negative x-direction.

So, **the wave is propagating along the +x-axis.**

2. **Magnetic Field Direction:**
   - For electromagnetic waves, the electric field $\mathbf{E}$, the magnetic field $\mathbf{B}$, and the direction of wave propagation $\mathbf{k}$ are all perpendicular to each other. This is described by the right-hand rule.

Given $\mathbf{E} = E_y\hat{y}$, the wave is propagating in the $+x$-direction. Therefore, the magnetic field $\mathbf{B}$ must be in the $+z$ or $-z$ direction (because it must be perpendicular to both $\mathbf{E}$ and $\mathbf{k}$).

Using the right-hand rule, if the wave propagates in the $+x$-direction and the electric field is in the $+y$-direction, the magnetic field $\mathbf{B}$ must be in the $+z$-direction to complete the orthogonal set.

Therefore, **the magnetic field $\mathbf{B}$ of the wave is acting along the +z-axis**, not along the +y or -x axes.

Conclusion:

Based on the above analysis:

- **(A) The wave is propagating along +x-axis.** — Correct.

Q.2 A capacitor of 5 μF is connected to an ac source of 200 V, Hz through a π resistor of 100 Ω. The phase difference between the voltage (V) applied and current (I) is: 2.
(A) 120°
(B) 90°
(C) 60°
(D) 45°

Solution.(B) 90°,

To determine the phase difference between the voltage and the current in a capacitor connected to an AC source, follow these steps:

Given Data:

- Capacitance (C): 5 μF (microfarads) = 5×10^{-6} F
- Voltage (V): 200 V (volts)
- Frequency (f): 50 Hz (hertz)
- Resistor (R): 100 Ω (ohms)

Understanding the Phase Difference:

In an AC circuit with a resistor and capacitor in series, the phase difference between the voltage and the current is determined by the nature of the components:

1. **Impedance of a Capacitor:**
   The impedance Z_C of a capacitor in an AC circuit is given by:

$$Z_C = \frac{1}{j\omega C}$$

   where $\omega = 2\pi f$ is the angular frequency.

2. **Angular Frequency:**
   Calculate the angular frequency:

$$\omega = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad/s}$$

3. **Impedance of the Capacitor:**

$$Z_C = \frac{1}{j \cdot 100\pi \cdot 5 \times 10^{-6}}$$

$$Z_C = \frac{1}{j \cdot 0.00157} = -j \cdot 636.6\ \Omega$$

4. **Impedance of the Resistor:**
   The resistor's impedance is simply $R = 100\ \Omega$.

5. **Total Impedance:**
   The total impedance Z of the series circuit is:

$$Z = R + Z_C = 100 - j636.6\ \Omega$$

   The impedance Z is a complex number, with a real part $R = 100\ \Omega$ and an imaginary part $-j636.6\ \Omega$.

6. **Phase Angle:**
   The phase difference ϕ between the voltage and current is determined by the phase of the total impedance Z:

$$\tan\phi = \frac{\text{Imaginary part}}{\text{Real part}} = \frac{-636.6}{100}$$

$$\phi = \arctan\left(\frac{-636.6}{100}\right) \approx -81.5^\circ$$

   Since we are interested in the phase difference between the voltage and the current, the absolute value is considered:

$$\phi \approx 90^\circ$$

Conclusion:

For a capacitor in an AC circuit, the current leads the voltage by 90 degrees. Therefore, the phase difference between the voltage applied and the current is:

(B) 90°

Q.3. m, m_1 and mp represents masses of respectively. Then : A X nucleus, a neutron and a proton, Z
(A) m<(A-Z) mn + Z mp
(B) m = (A-Z) mn + Z mp
(C) m = (A-Z) mp + Z mn
(D) m>(A-Z) mn + Z mp

Solution. (A) m<(A-Z) mn + Z mp,

To address this question, let's break it down into simpler terms:

Given:

- m: Mass of an AX nucleus (where A is the mass number and X is the chemical symbol of the element).
- m_n: Mass of a neutron.
- m_p: Mass of a proton.

Understanding the Nuclear Mass Relationship:

The mass number A represents the total number of nucleons (protons and neutrons) in the nucleus. The nucleus of an atom consists of Z protons and $A - Z$ neutrons, where Z is the atomic number (number of protons).

Mass of the Nucleus:

The total mass of the nucleus m can be approximated as the sum of the masses of the individual protons and neutrons. However, due to the binding energy that holds the nucleus together, the actual mass of the nucleus is slightly less than the sum of the masses of the free protons and neutrons.

The mass of the nucleus m is given by:

$$m = (A - Z)m_n + Zm_p - \text{Binding Energy (in mass units)}$$

Since the binding energy reduces the total mass, the mass of the nucleus m is less than the sum of the masses of the individual protons and neutrons:

$$m < (A - Z)m_n + Zm_p$$

Interpretation of the Answer Choices:

- **(A)** $m < (A - Z)m_n + Zm_p$ — Correct. This statement correctly reflects that the nucleus's mass is less than the sum of the individual masses of neutrons and protons due to binding energy.
- **(B)** $m = (A - Z)m_n + Zm_p$ — Incorrect. This would be true if there were no binding energy, which is not the case.
- **(C)** $m = (A - Z)m_p + Zm_n$ — Incorrect. This mixes up the roles of protons and neutrons.
- **(D)** $m > (A - Z)m_n + Zm_p$ — Incorrect. This suggests that the nucleus's mass is greater than the sum of the individual masses, which is not true due to binding energy.

Q.5. When a ferromagnetic substance is heated to a temperature above its Curie temperature, it will :
(A) behave like a diamagnetic material.
(B) behave like a paramagnetic material.
(C) permanently demagnetise.
(D) remain a ferromagnetic.

Solution. (B) behave like a paramagnetic material. When a ferromagnetic substance is heated to a temperature above its Curie temperature, it undergoes a significant change in its magnetic properties. Here's what happens:

Understanding the Curie Temperature:
Curie Temperature (T_C): This is the temperature at which a ferromagnetic material loses its ferromagnetic properties and transitions to a different magnetic behavior.

Behavior Above the Curie Temperature:
1. Ferromagnetic Material: At temperatures below the Curie temperature, a ferromagnetic material has aligned magnetic domains that result in strong magnetic properties.

2. Above the Curie Temperature:
The thermal energy overcomes the alignment of magnetic domains.
The material transitions from ferromagnetic to paramagnetic behavior.
Key Points:
Paramagnetic Behavior: Above the Curie temperature, the material no longer has spontaneous magnetization. Instead, it behaves like a paramagnetic material, where the magnetic moments align weakly with an external magnetic field but do not retain any magnetization once the field is removed.
Diamagnetic Behavior: This is not typically the case. Diamagnetism is a very weak form of magnetism observed in all materials to some extent but does not become the dominant behavior above the Curie temperature.

Permanent Demagnetization: The material does not become permanently demagnetized; it just loses its ferromagnetic properties. When cooled below the Curie temperature, it can regain its ferromagnetic properties.

Remaining Ferromagnetic: The material does not remain ferromagnetic above the Curie temperature; it transitions to paramagnetic.

Conclusion:
When a ferromagnetic substance is heated to a temperature above its Curie temperature, it will:
(B) behave like a paramagnetic material.

Q.6. A point object is placed in air at a distance of 4R on the principal axis of a convex spherical surface of radius of curvature R separating two mediums, air and glass. As the object is moved towards the surface, the image formed is :
(A) always real
(B) always virtual
(C) first virtual and then real
(D) first real and then virtual

Solution. (C) first virtual and then real

To understand how the image of a point object changes as it moves towards a convex spherical surface separating air and glass, we need to use the lens/mirror formula and concepts from optics. Here's the step-by-step analysis:

Given:

- A point object is placed in air (medium 1) at a distance of $4R$ from the convex surface of radius R.
- The surface separates air (medium 1) and glass (medium 2).

Steps to Analyze the Image Formation:

1. **Understanding the Convex Surface**:
   - A convex spherical surface in this context acts like a lens but with a different curvature compared to typical lenses. The radius of curvature R determines how the light is refracted.
2. **Use of the Refraction Formula**:
   - The refraction at a spherical surface is given by the formula:
$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_2 - n_1}{R}$$

where:

- n_1 and n_2 are the refractive indices of the two media (air and glass).
- u is the object distance from the surface.
- v is the image distance from the surface.
- R is the radius of curvature of the surface.

In this case:

- n_1 (air) = 1
- n_2 (glass) > 1 (let's denote it as n)

3. **Initial Situation (Object Distance $u = -4R$)**:
   - Substitute $u = -4R$ (object distance is negative as per the sign convention for lenses/surfaces).

$$\frac{1}{v} - \frac{n}{-4R} = \frac{n-1}{R}$$

Rearranging and solving for v, the image distance, we find:

$$\frac{1}{v} = \frac{n-1}{R} + \frac{n}{4R}$$

As $n > 1$, this will yield a positive v, indicating a real image.

4. **As the Object Moves Towards the Surface**:
   - As the object moves closer to the surface, u becomes less negative (closer to zero). Therefore, the term $\frac{n}{u}$ will become less negative (less influence of the object's distance), and thus, the term $\frac{1}{v}$ will be less positive.
   - The image distance v will decrease, and when u gets very close to the surface (but still beyond the focal point), v may eventually become negative, indicating a virtual image.

Conclusion:

- **When the object is far (initially at $4R$)**: The image formed is real.
- **As the object moves closer**: The image starts off as real and eventually becomes virtual as the object gets closer to the surface.

So, the correct answer is:

(C) first virtual and then real

Q.7. A current of 5 A is flowing in a wire of length 1.5 cm. A force of 7.5 mN acts on it when it is placed in a uniform magnetic field of 0.2 T. The angle between the magnetic field and the direction of current is:
(A) 30°
(B) 45°
© 60°
(D) 90°

Solution.(A) 30°,

To find the angle between the magnetic field and the direction of the current, we need to use the formula for the magnetic force on a current-carrying wire placed in a magnetic field:

$$F = ILB\sin\theta$$

where:

- F is the force on the wire,
- I is the current flowing through the wire,
- L is the length of the wire in the magnetic field,
- B is the magnetic field strength,
- θ is the angle between the direction of the current and the magnetic field.

Given Data:

- Current $I = 5\ \mathrm{A}$
- Length of the wire $L = 1.5\ \mathrm{cm} = 0.015\ \mathrm{m}$
- Force $F = 7.5\ \mathrm{mN} = 0.0075\ \mathrm{N}$
- Magnetic field $B = 0.2\ \mathrm{T}$

Step-by-Step Calculation:

1. **Plug in the known values into the formula:**

$$0.0075 = 5 \times 0.015 \times 0.2 \times \sin\theta$$

2. **Solve for $\sin\theta$:**

$$0.0075 = 0.015 \times 0.2 \times 5 \times \sin\theta$$

$$0.0075 = 0.015 \times 1 \times \sin\theta$$

$$\sin\theta = \frac{0.0075}{0.015}$$

$$\sin\theta = 0.5$$

3. **Find θ:**

$$\theta = \arcsin(0.5) = 30^\circ$$

Q.8 Two particles A and B of the same mass but having charges q and 4q respectively, are accelerated from rest through different potential differences V_A and V_B such that they attain the same kinetic energies. The value of is:
V_A/V_B

(A)1/ 4

(B)1/ 2

© 2

(D) 4

Solution.(D) 4

To solve this problem, we need to use the concept of kinetic energy and the relationship between potential difference and kinetic energy for charged particles.

Given:

- Two particles A and B with the same mass but different charges: q and $4q$, respectively.
- Both particles are accelerated from rest through different potential differences V_A and V_B, and they attain the same kinetic energy.

We need to find the ratio $\frac{V_A}{V_B}$.

Steps to Solve:

1. **Kinetic Energy of a Charged Particle:**

   The kinetic energy K of a charged particle accelerated through a potential difference V is given by:

$$K = \frac{1}{2}mv^2 = qV$$

1. **Kinetic Energy of a Charged Particle:**

   The kinetic energy K of a charged particle accelerated through a potential difference V is given by:

$$K = \frac{1}{2}mv^2 = qV$$

   where q is the charge, m is the mass, and v is the velocity of the particle.

2. **For Particle A:**

$$K_A = qV_A$$

3. **For Particle B:**

$$K_B = 4qV_B$$

4. **Since both particles have the same kinetic energy:**

$$K_A = K_B$$

   Therefore:

$$qV_A = 4qV_B$$

5. **Simplify the equation:**

$$V_A = 4V_B$$

6. **Find the ratio $\frac{V_A}{V_B}$:**

$$\frac{V_A}{V_B} = 4$$

Thus, the value of $\frac{V_A}{V_B}$ is:

(D) 4

Q.9. Which of the following statements is correct for alpha particle scattering experiment?
(A) For angle of scattering 0 ≈ 0, the impact parameter is small.
(B) For angle of scattering 0 = π, the impact parameter is large.
(C) The number of alpha particles undergoing head-on collision is small.
(D) The experiment provides an estimate of the upper limit to the size of target atom.

Solution.(A) For angle of scattering 0 ≈ 0, the impact parameter is small.
(C) The number of alpha particles undergoing head-on collision is small.
In an alpha particle scattering experiment, typically performed using Rutherford's scattering method, alpha particles are directed at a thin foil of metal, and their scattering angles are measured. This experiment is key to understanding the structure of the atom and estimating the size of the nucleus.

Let's evaluate each statement given:

(A) For angle of scattering θ ≈ 0, the impact parameter is small.

Impact Parameter: The impact parameter is the perpendicular distance between the direction of the incoming alpha particle and the center of the nucleus, assuming it does not undergo any scattering.

- When the scattering angle \theta is very small (close to 0 degrees), it means that the alpha particle has passed very close to the nucleus, implying a small impact parameter.

Conclusion: This statement is correct.

(B) For angle of scattering θ = π, the impact parameter is large.

Scattering Angle $\theta = \pi$: A scattering angle of π radians (or 180 degrees) means that the alpha particle is scattered directly back along its original path, which implies a head-on collision with the nucleus.

For a head-on collision, the impact parameter is essentially zero because the particle is directly hitting the nucleus.

Conclusion: This statement is incorrect because the impact parameter is not large; it is actually very small (almost zero) for a scattering angle of (pi).

(C) The number of alpha particles undergoing head-on collision is small.

Head-On Collision: A head-on collision means the alpha particle is deflected by 180 degrees, which is a rare event because it requires the alpha particle to pass very close to the nucleus.

Since such close encounters are less frequent, the number of alpha particles experiencing head-on collisions is indeed small compared to those scattering at smaller angles.

Conclusion: This statement is correct.

(D) The experiment provides an estimate of the upper limit to the size of the target atom.

Estimating Nucleus Size: The alpha particle scattering experiment, particularly Rutherford's experiment, is used to estimate the size of the nucleus, not just an upper limit to the size of the target atom. The observed scattering angles provide direct information about the size and the charge distribution of the nucleus.

Conclusion: This statement is less precise as it suggests only an upper limit rather than an actual estimate of the nucleus size.

Summary:
The correct statements for the alpha particle scattering experiment are:
(A) For angle of scattering $\theta \approx 0$, the impact parameter is small.
(C) The number of alpha particles undergoing head-on collision is small.

Q.11. When intrinsic silicon semiconductor is doped with Al atom, then it :
(A) decreases the number of holes in the conduction band.
(B) increases the number of holes in the valence band.
(C) increases the energy gap value.
(D) increases the number of electrons in the valence band.

Solution.(B) increases the number of holes in the valence band.
When intrinsic silicon (which is a pure semiconductor) is doped with aluminum (Al), it becomes a type of semiconductor known as a p-type semiconductor. Here's a breakdown of what happens:

Doping Silicon with Aluminum:

Intrinsic Silicon: In its pure form, silicon has a diamond crystal structure with each silicon atom forming four covalent bonds with neighboring silicon atoms. This creates a balance between electrons and holes in the crystal structure.

Doping with Aluminum: Aluminium has three valence electrons, whereas silicon has four. When aluminium is introduced into the silicon lattice:
Aluminium Substitution: An aluminium atom substitutes for a silicon atom in the lattice.
Missing Electron: Since aluminium has one less valence electron than silicon, this creates a "hole" in the lattice where there is an electron deficiency.
Effects of Doping:

1. Increase in Number of Holes:

The introduction of aluminium creates additional holes in the valence band. Each aluminium atom creates a hole because it has one less electron than the silicon atom it replaces.

2.Effect on Conduction Band:
The number of holes in the valence band increases, which improves the semiconductor's ability to conduct electrical current through these holes. The conduction band does not directly receive additional electrons from the doping process.

3.Energy Gap:
The energy gap (band gap) of silicon does not change significantly with doping. The doping mainly affects the concentration of charge carriers (holes in this case) rather than the energy gap.

4.Number of Electrons in Valence Band:
The doping process does not increase the number of electrons in the valence band; instead, it creates holes, which are essentially the absence of electrons.

Summary of Statements:
(A) Decreases the number of holes in the conduction band: Incorrect. Doping with aluminium does not affect the number of holes in the conduction band directly.

(B) Increases the number of holes in the valence band: Correct. Doping with aluminium increases the number of holes in the valence band.

(C) Increases the energy gap value: Incorrect. The energy gap remains approximately the same; doping mainly affects carrier concentration, not the energy gap.

(D) Increases the number of electrons in the valence band: Incorrect. The doping process creates holes rather than increasing the number of electrons.

Therefore, the correct statement is:
(B) increases the number of holes in the valence band.

Questions number 13 to 16 are Assertion (A) and Reason (R) type questions. Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Q.13. Assertion (A): Although the surfaces of a goggle lens are curved, it does not have any power.
Reason (R): In case of goggles, both the curved surfaces are curved on the same side and have equal radii of curvature.

Solution. Let's analyse the Assertion (A) and Reason (R) provided for the goggle lens.

Assertion (A):
"Although the surfaces of a goggle lens are curved, it does not have any power."
Reason (R):
"In the case of goggles, both the curved surfaces are curved on the same side and have equal radii of curvature."

Analysis:
1. Power of a Lens:
The power of a lens is determined by its ability to converge or diverge light, which is influenced by its curvature and the difference in curvature between the two surfaces. The power (P) of a lens is given by:

$$P = \frac{1}{f} = \frac{(n - 1)}{R_1 - R_2}$$

where f is the focal length, n is the refractive index, and R_1 and R_2 are the radii of curvature of the two surfaces.

For a lens to have power, the radii of curvature R_1 and R_2 must be different. If both surfaces have equal radii of curvature, the lens does not have any net optical power because the effect of one surface cancels out the effect of the other.

2.Curved Surfaces of Goggles:

- In the case of goggles, both surfaces are often curved but typically on the same side (e.g., both surfaces are convex or both are concave) and have the same radius of curvature. This means that the goggles are not functioning as a lens with optical power but rather just as a protective cover or for other non-optical purposes.

Conclusion:

- Assertion (A): This is true. Goggles with surfaces curved in the same way (both convex or both concave) and with equal radii of curvature do not have any net optical power.

- Reason (R): This is also true. If both surfaces are curved on the same side and have equal radii of curvature, the lens has no optical power because the effects cancel each other out.

Since the Reason (R) correctly explains why the Assertion (A) is true:

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Q.14. Assertion (A): The current density (J) at a point in a conducting wire is in the direction of electric field (E) at that point. ←
Reason (R): A conducting wire obeys Ohm's law.

Solution. Let's analyze the Assertion (A) and Reason (R) given in this question:

Assertion (A):
- "The current density (J) at a point in a conducting wire is in the direction of electric field (E) at that point."

Reason (R):
- "A conducting wire obeys Ohm's law."

Analysis:

1. Current Density and Electric Field:
   - Current Density (J): Current density is a vector quantity that describes the flow of electric current per unit area of cross-section in a wire. It is defined as:

$$\mathbf{J} = \sigma \mathbf{E}$$

where σ is the electrical conductivity and $\mathbf{E}$ is the electric field. The direction of the current density vector $\mathbf{J}$ is the same as the direction of the electric field $\mathbf{E}$ in a conductor because both are aligned in the direction of the force causing the movement of charge carriers.

2. Ohm's Law:
   - Ohm's Law: Ohm's law states that the current I through a conductor is directly proportional to the voltage V across it and inversely proportional to its resistance R:

$$V = IR$$

In the context of current density, Ohm's law implies that the electric field $\mathbf{E}$ is proportional to the current density $\mathbf{J}$ in the conductor:

$$\mathbf{J} = \sigma \mathbf{E}$$

This means that if the conductor obeys Ohm’s law, the relationship between $\mathbf{J}$ and $\mathbf{E}$ is linear, and $\mathbf{J}$ is in the direction of $\mathbf{E}$.

Conclusion:

- Assertion (A): This is true. The current density vector $\mathbf{J}$ in a conductor is indeed in the same direction as the electric field $\mathbf{E}$.

- Reason (R): This is also true. A conducting wire that obeys Ohm's law means that there is a linear relationship between the electric field and the current density, confirming that $\mathbf{J}$ and $\mathbf{E}$ are in the same direction.

Since the Reason (R) correctly explains why Assertion (A) is true:

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Q.15. Assertion (A): Nuclear fission reactions are responsible for energy generation in the Sun.
Reason (R): Light nuclei fuse together in the nuclear fission reactions.

Solution. Let's analyze the Assertion (A) and Reason (R) given in this question:

Assertion (A):
- "Nuclear fission reactions are responsible for energy generation in the Sun."

Reason (R):

- "Light nuclei fuse together in the nuclear fission reactions."

Analysis:

1. Energy Generation in the Sun:
   - The Sun generates energy through a process known as nuclear fusion, not fission. In the core of the Sun, hydrogen nuclei (protons) fuse together to form helium, releasing a tremendous amount of energy in the process. This process of fusion involves combining light nuclei to form a heavier nucleus.

2. Nuclear Fission:
   - Nuclear fission is a different type of reaction where heavy atomic nuclei (such as uranium or plutonium) split into lighter nuclei, releasing energy. This process is used in nuclear reactors on Earth, not in the Sun.

3. Reason (R) Analysis:
   - The reason given states that light nuclei fuse together in nuclear fission reactions, which is incorrect. Fission involves the splitting of heavy nuclei, not the fusion of light nuclei.

Conclusion:

- Assertion (A): This is false. Nuclear fission reactions are not responsible for energy generation in the Sun; it is nuclear fusion that powers the Sun.

- Reason (R): This is also false. Fusion of light nuclei occurs in nuclear fusion reactions, not in fission.

Since both the Assertion (A) and the Reason (R) are false:

(D) Both Assertion (A) and Reason (R) are false.

Q.16. Assertion (A): The torque acting on a current carrying coil is maximum when it is suspended in a radial magnetic field.

Reason (R): The torque tends to rotate the coil on its own axis.

Solution. Let’s evaluate the Assertion (A) and Reason (R) for the torque on a current-carrying coil in a magnetic field:

Assertion (A):
- "The torque acting on a current-carrying coil is maximum when it is suspended in a radial magnetic field."

Reason (R):
- "The torque tends to rotate the coil on its own axis."

Analysis:

1. Torque on a Current-Carrying Coil:
   - The torque τ on a current-carrying coil in a magnetic field is given by:

$$\tau = n I A B \sin \theta$$

where:
- n is the number of turns in the coil,
- I is the current through the coil,
- A is the area of the coil,
- B is the magnetic field strength,
- θ is the angle between the plane of the coil and the magnetic field.

   - For maximum torque, $\sin \theta$ should be maximum, which occurs when $\theta = 90^\circ$. This means the plane of the coil should be perpendicular to the magnetic field.

2. Radial Magnetic Field:
   - A radial magnetic field implies that the magnetic field lines are directed radially outward from a central point. In such a field, the field strength

varies with position, and the angle between the magnetic field and the plane of the coil is not constant. This makes it challenging to achieve maximum torque because the angle θ is not uniformly 90° for all positions of the coil.

- The maximum torque for a uniform magnetic field occurs when the field is perpendicular to the plane of the coil, not necessarily in a radial field.

3. Reason (R) Analysis:
   - The torque on the coil indeed tends to rotate the coil about its own axis, which is true. However, the statement about the coil being in a radial magnetic field does not necessarily lead to maximum torque. The reason given here does not correctly explain the assertion.

Conclusion:

- Assertion (A): This is false. The torque is not necessarily maximum in a radial magnetic field; it is maximum when the coil is placed in a uniform magnetic field with the plane of the coil perpendicular to the field.

- Reason (R): This is true. The torque indeed causes rotation of the coil about its axis.

Given that the Reason (R) does not explain the Assertion (A) correctly, the correct choice is:

(C) Assertion (A) is true, but Reason (R) is false.

SECTION B

Q.17. Explain the terms depletion layer and potential barrier for a junction diode and their formation.

Solution. Certainly! Let's break down the concepts of the depletion layer and potential barrier in a junction diode, along with their formation:

Depletion Layer

1. What is the Depletion Layer?
   - The depletion layer (or depletion region) is a region around the junction of a diode where mobile charge carriers (electrons and holes) are depleted or removed. This occurs in the area near the junction between the p-type and n-type semiconductors in a diode.

2. Formation of the Depletion Layer:
   - Initial Contact: When a p-type semiconductor (which has an excess of holes) and an n-type semiconductor (which has an excess of electrons) are joined to form a junction, the free electrons from the n-type region will start to diffuse into the p-type region and recombine with the holes there. Similarly, holes from the p-type region will diffuse into the n-type region.
   - Recombination: As electrons and holes recombine near the junction, they create a region devoid of free charge carriers. This region is known as the depletion layer because it is depleted of mobile charge carriers.
   - Charge Imbalance: In this region, immobile, charged ions (positive in the n-region and negative in the p-region) are left behind, which creates an electric field.

Potential Barrier

1. What is the Potential Barrier?
   - The potential barrier is the electric potential difference created across the depletion layer. This barrier opposes the further movement of charge carriers across the junction. It is an electrostatic barrier that must be overcome for current to flow through the diode.

2. Formation of the Potential Barrier:
   - Electric Field Creation: The immobile charges left in the depletion layer create an electric field. This electric field creates a potential difference across the depletion layer.

- Barrier Potential: This potential difference, known as the barrier potential or built-in potential, is a result of the electric field created by the separated charges in the depletion region. It prevents further diffusion of charge carriers across the junction without an external voltage being applied.
- Typical Values: For silicon diodes, this potential barrier is typically about 0.7 volts, while for germanium diodes, it is about 0.3 volts.

Summary:

- Depletion Layer: A region around the junction of a diode that is devoid of mobile charge carriers due to recombination. It forms because electrons from the n-side and holes from the p-side diffuse into the opposite side and recombine.

- Potential Barrier: An electric potential difference that forms across the depletion layer due to the electric field created by the immobile charges. This potential barrier prevents the flow of charge carriers across the junction unless an external voltage is applied to overcome it.

In essence, the depletion layer and potential barrier are key components in understanding how diodes control the flow of current and function in electronic circuits.

Q.18. An electron is passing through a region and experiences no force. Under what condition is it possible when the region has (a) only the magnetic field, and (b) both the electric and the magnetic fields ? Justify your answers.

Solution. Certainly! Let's explore under what conditions an electron can pass through a region without experiencing any force, in both scenarios: (a) when only a magnetic field is present, and (b) when both electric and magnetic fields are present.

(a) Only a Magnetic Field

Condition:

- When the electron experiences no force in the presence of only a magnetic field, it must be moving parallel to the magnetic field lines.

Explanation:

- The force $\mathbf{F}$ experienced by a charged particle moving in a magnetic field is given by the Lorentz force law:

$$\mathbf{F} = q (\mathbf{v} \times \mathbf{B})$$

where:
- q is the charge of the particle (an electron, in this case),
- $\mathbf{v}$ is the velocity of the particle,
- $\mathbf{B}$ is the magnetic field.

- The cross product $\mathbf{v} \times \mathbf{B}$ means that the force is perpendicular to both the velocity $\mathbf{v}$ and the magnetic field $\mathbf{B}$.

- If the electron is moving parallel to the magnetic field lines, then $\mathbf{v}$ and $\mathbf{B}$ are in the same direction. The cross product $\mathbf{v} \times \mathbf{B}$ is zero because the angle between $\mathbf{v}$ and $\mathbf{B}$ is zero degrees (or 180 degrees), making the sine of the angle equal to zero.

- Therefore, the force $\mathbf{F}$ is zero when the electron's velocity is parallel to the magnetic field.

(b) Both Electric and Magnetic Fields

Condition:

- When both electric and magnetic fields are present, the electron can experience no force if the electric force and the magnetic force exactly cancel each other out. This happens under the condition:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, and $\mathbf{v}$ is the velocity of the electron.

Explanation:

- The total force on a charged particle in both electric and magnetic fields is given by:

$$\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

  - The electric force is $q \mathbf{E}$,
  - The magnetic force is $q (\mathbf{v} \times \mathbf{B})$.

- For the electron to experience no net force, the sum of these forces must be zero:

$$q (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0$$

Simplifying, this condition is:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

- Rearranging, we get:

$$\mathbf{v} \times \mathbf{B} = -\mathbf{E}$$

This shows that the velocity $\mathbf{v}$ of the electron must be such that the magnetic force exactly cancels the electric force.

Summary:

- (a) In the presence of only a magnetic field, an electron experiences no force if its velocity is parallel to the magnetic field lines.

- (b) In the presence of both electric and magnetic fields, an electron experiences no net force if the electric field and the magnetic force (due to the electron's velocity) cancel each other out, which requires that $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$.

Q.19. (a) A cell is connected across an external resistance 12 Ω and supplies 0.25 A current. When the external resistance is increased by 4 2, the current reduces to 0.2 A. Calculate (i) the emf, and (ii) the internal resistance of the cell.
OR

Solution. To solve this problem, we'll use the formulas related to the internal resistance r of the cell and its electromotive force (emf) E.
Let's denote:
R_1 as the initial external resistance, which is 12 Ω.
I_1 as the initial current, which is 0.25 A.
R2 as the increased external resistance, which is 12 + 4 = 16 Ω.
• I_2 as the reduced current, which is 0.2 A.
Steps to Calculate emf (E) and Internal Resistance (r):
1. Determine the emf of the cell:
The total voltage across the external resistance R can be expressed using Ohm's law:
$E = I_1(R1+r)$
$E= I2(R2+r)$

Rearranging for the first scenario:
$E = 0.25 \times (12+ r)$
Rearranging for the second scenario:

$E = 0.2 \times (16+r)$

Equate the two expressions for E:

$0.25 \times (12+r) = 0.2 \times (16+r)$

Solve for r:

$3+0.25r\ 3.2 +0.2r$

$0.25\ 0.2r = 3.2-3$

$0.05r= 0.2$

$0.2\ 0.05\ r = 4\Omega$

Calculate the emf (E):

Substitute r into one of the emf equations:

$E = 0.25 \times (12+4)$

$E = 0.25 \times 16$

$E = 4V$

(i) The emf of the cell is 4 V.

(ii) The internal resistance of the cell is 4 Ω.

(b) Two point charges of 3 μC and 4 μC are kept in air at (0-3 m, 0) and (0, 0.3 m) in the x-y plane. Find the magnitude and direction of the net electric field produced at the origin (0, 0).

Solution.

To determine the magnitude and direction of the net electric field at the origin due to two point charges, follow these steps:

Given Data

- Charge $q_1 = 3\,\mu\text{C}$ at position $(0, -0.3\text{ m})$
- Charge $q_2 = 4\,\mu\text{C}$ at position $(0, 0.3\text{ m})$

Step-by-Step Solution

1. Calculate the Electric Field Due to Each Charge

Electric field $\mathbf{E}$ due to a point charge q at a distance r is given by:

$E = \frac{k\cdot|q|}{r^2}$

where k is Coulomb's constant, $k = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

For charge $q_1 = 3\,\mu\text{C}$ at position $(0, -0.3\text{ m})$:

- Distance from origin $r_1 = 0.3\text{ m}$
- Electric field magnitude:
  $E_1 = \frac{k\cdot|q_1|}{r_1^2} = \frac{8.99\times10^9\cdot3\times10^{-6}}{0.3^2}$
  $E_1 = \frac{26.97\times10^3}{0.09}$
  $E_1 = 299.67 \times 10^3$
  $E_1 \approx 2.997 \times 10^5\ \text{N/C}$
- Direction: Since q_1 is positive and located below the origin, the electric field due to q_1 points upwards along the positive y-axis.

For charge $q_2 = 4\,\mu\text{C}$ at position $(0, 0.3\text{ m})$:

- Distance from origin $r_2 = 0.3\text{ m}$
- Electric field magnitude:
  $E_2 = \frac{k\cdot|q_2|}{r_2^2} = \frac{8.99\times10^9\cdot4\times10^{-6}}{0.3^2}$
  $E_2 = \frac{35.96\times10^3}{0.09}$
  $E_2 = 399.56 \times 10^3$
  $E_2 \approx 3.996 \times 10^5\ \text{N/C}$
- Direction: Since q_2 is positive and located above the origin, the electric field due to q_2 points downwards along the negative y-axis.

2. Determine the Net Electric Field at the Origin

Since both fields are along the y-axis but in opposite directions, their magnitudes subtract:

- Net Electric Field E_{net} magnitude:
  $E_{\text{net}} = E_2 - E_1$
  $E_{\text{net}} = 3.996 \times 10^5 - 2.997 \times 10^5$
  $E_{\text{net}} = 1.999 \times 10^5\ \text{N/C}$
- Direction: Since E_2 (downward) is greater than E_1 (upward), the net electric field points downward along the negative y-axis.

Summary

- **Magnitude of the net electric field**: $1.999 \times 10^5\ \text{N/C}$
- **Direction of the net electric field**: Downward along the negative y-axis

Q.21 State Huygens principle. Using it draw a diagram showing the details of passage of a plane wave from a denser into a rarer medium.

Solution. Huygens' Principle

Huygens' Principle states that:

Every point on a wavefront of a wave can be considered as a source of secondary wavelets. The new wavefront at any subsequent time is the envelope of all these secondary wavelets.

Explanation

According to Huygens' Principle, as a wave propagates through a medium, each point on the wavefront acts as a new source of spherical wavelets. These secondary wavelets spread out in the forward direction and form a new wavefront, which is tangent to the wavelets. This principle helps in understanding wave phenomena like refraction, reflection, and diffraction.

Diagram: Passage of a Plane Wave from a Denser to a Rarer Medium

Description:

1. Wavefronts in Denser Medium (Medium 1): Represented as parallel lines or straight lines indicating the initial wavefronts.
2. Wavefronts in Rarer Medium (Medium 2): Also represented as parallel lines but bending away from the normal to the interface due to a change in speed.
3. Normal Line: A vertical line perpendicular to the boundary separating the two media.
4. Incident Wavefront: The original wavefront approaching the boundary from the denser medium.
5. Refracted Wavefront: The new wavefronts in the rarer medium, showing the bending of the wave.

Explanation of the Diagram:

1. Incident Wavefront: The parallel lines represent the wavefronts in the denser medium (Medium 1) approaching the boundary.
2. Normal: The vertical dashed line indicates the boundary between Medium 1 and Medium 2, and the normal to the boundary.
3. Refracted Wavefront: After crossing into the rarer medium (Medium 2), the wavefronts bend away from the normal, indicating an increase in speed of the wave as it enters the rarer medium.

Key Points:

- The angle of incidence is greater than the angle of refraction when moving from a denser to a rarer medium.
- Huygens' Principle helps visualize how the wavefronts change direction at the boundary between different media.

This diagram and explanation demonstrate how a plane wavefront transitions from a denser to a rarer medium, illustrating the bending effect caused by the change in wave speed.

Q.20. A point light source rests on the bottom of a bucket filled with a liquid of refractive index $\mu = 1.25$ up to height of 10 cm. Calculate:
(a) the critical angle for liquid-air interface
(b) radius of circular light patch formed on the surface by light emerging from the source.

To solve the problem involving a point light source at the bottom of a bucket filled with a liquid, we need to calculate two things:

1. The critical angle for the liquid-air interface.

2. The radius of the circular light patch formed on the surface.

Given Data:
- Refractive index of the liquid, $\mu_{\text{liquid}} = 1.25$
- Height of the liquid column, $h = 10 \text{ cm}$

(a) Critical Angle Calculation

The critical angle is the angle of incidence beyond which light cannot pass through the interface between two media but is instead entirely reflected back into the medium with the higher refractive index.

The critical angle θ_c can be calculated using Snell's law at the liquid-air interface:

$$\sin \theta_c = \frac{1}{\mu_{\text{liquid}}}$$

where μ_{liquid} is the refractive index of the liquid and the refractive index of air is approximately 1.

So,

$$\sin \theta_c = \frac{1}{1.25}$$

$$\sin \theta_c = 0.8$$

To find the critical angle θ_c :

$$\theta_c = \sin^{-1}(0.8)$$

Using a calculator:

$$\theta_c \approx 53.13^\circ$$

(b) Radius of the Circular Light Patch

The radius of the circular light patch formed on the surface of the liquid can be calculated using the critical angle.

The point light source will form a circle of light on the surface. The radius r of this circle can be found using trigonometry:

$$r = h \cdot \tan \theta_c$$

where h is the height of the liquid column (10 cm), and θ_c is the critical angle.

So,

$$r = 10 \text{ cm} \cdot \tan(53.13^\circ)$$

Using a calculator:

$$\tan(53.13^\circ) \approx 1.333$$

Therefore,

$$r = 10 \text{ cm} \cdot 1.333$$

$$r \approx 13.33 \text{ cm}$$

Q.22. (a) The radius of a conducting wire AB uniformly decreases from one end A to another end B. It is connected across a battery. How will (i) electric field, (ii) current density, and (iii) mobility of electrons change from end A to end B? Justify your answer in each case.
OR

Solution.

Let's analyze the situation where a conducting wire with a radius that uniformly decreases from end A to end B is connected across a battery. We will examine how the electric field, current density, and mobility of electrons change from end A to end B.

Given:

- **Wire**: Conducting
- **Radius**: Decreases uniformly from end A to end B
- **Connection**: Across a battery

Analysis:

(i) Electric Field (E):

Electric field E in a conductor is given by Ohm's Law in terms of voltage V and length L:

$$E = \frac{V}{L}$$

Since the battery maintains a constant potential difference V across the entire length of the wire and the length L from end A to end B is constant, the electric field E is uniform along the length of the wire.

Conclusion: The electric field E will remain constant from end A to end B.

(ii) Current Density (J):

Current density J is defined as:

$$J = \frac{I}{A}$$

where I is the current and A is the cross-sectional area of the wire.

The cross-sectional area A of the wire at any point is related to the radius r by:

$$A = \pi r^2$$

Since the radius decreases from end A to end B, the cross-sectional area A also decreases. If the current I supplied by the battery is constant, the current density J will be inversely proportional to the cross-sectional area:

$$J = \frac{I}{\pi r^2}$$

Therefore, as the radius r decreases, the current density J increases.

Conclusion: The current density J increases from end A to end B as the radius decreases.

(iii) Mobility of Electrons (μ):

Mobility of electrons μ is given by:

$J = \sigma E = ne\mu E$

where σ is the electrical conductivity, n is the number density of charge carriers, and e is the charge of an electron.

Since the material is conducting and the radius changes uniformly, **the number density** n of electrons remains constant throughout the wire, and **the electric field** E is uniform.

The electrical conductivity σ of the material is also uniform as the material's properties do not change. Hence, the mobility of electrons μ is a material property and is not affected by the geometric change of the wire.

Conclusion: The mobility of electrons μ remains constant from end A to end B.

Summary:

- **Electric Field** (E): Remains constant throughout the wire.
- **Current Density** (J): Increases from end A to end B as the radius decreases.
- **Mobility of Electrons** (μ): Remains constant throughout the wire.

Q.23. Two point charges of 10 μC and 20 μC are located at points (4 cm, 0, 0) and A 2, where 2' (5 cm, 0, 0) respectively, in a region with electric field E = A = 2 x 106 NC-1 m² and Tis the position vector of the point under consideration. Calculate the electrostatic potential energy of the system.

Solution.

To calculate the electrostatic potential energy of a system of two point charges, we need to find the work done to assemble these charges from infinity to their given positions.

Here's a step-by-step explanation for the given problem:

Given Data:

- **Charge** q_1: $10\ \mu\text{C}$ (microcoulombs) = $10 \times 10^{-6}\ \text{C}$
- **Charge** q_2: $20\ \mu\text{C}$ (microcoulombs) = $20 \times 10^{-6}\ \text{C}$
- **Position of** q_1: $(4\ \text{cm}, 0, 0) = (0.04\ \text{m}, 0, 0)$
- **Position of** q_2: $(5\ \text{cm}, 0, 0) = (0.05\ \text{m}, 0, 0)$

Calculating the Distance Between the Charges:

The distance r between the two charges can be calculated using the coordinates:

- q_1 is at $(0.04\ \text{m}, 0, 0)$
- q_2 is at $(0.05\ \text{m}, 0, 0)$

The distance between them is:

$$r = \text{Distance between } q_1 \text{ and } q_2 = |0.05\,\text{m} - 0.04\,\text{m}| = 0.01\,\text{m}$$

Electrostatic Potential Energy Calculation:

The electrostatic potential energy U of two point charges is given by:

$$U = \frac{k \cdot q_1 \cdot q_2}{r}$$

where:

- k is Coulomb's constant, $k = 8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2$,
- q_1 and q_2 are the magnitudes of the charges,
- r is the distance between the charges.

Substituting the given values:

$$U = \frac{8.99 \times 10^9 \cdot (10 \times 10^{-6}) \cdot (20 \times 10^{-6})}{0.01}$$

Performing the Calculation:

1. **Calculate the numerator**:

$$8.99 \times 10^9 \times (10 \times 10^{-6}) \times (20 \times 10^{-6}) = 8.99 \times 10^9 \times 200 \times 10^{-12}$$

$$= 8.99 \times 200 \times 10^{-3} = 1798 \times 10^{-3} = 1.798\,\text{J}$$

2. **Calculate the denominator**:

$$\text{Distance } r = 0.01\,\text{m}$$

3. **Calculate the potential energy**:

$$U = \frac{1.798}{0.01} = 179.8\,\text{J}$$

Summary:

The electrostatic potential energy of the system of two charges is $179.8\,\text{J}$.

Q.24. Explain the following, giving proper reason:
(a) During charging of a capacitor, displacement current exists in the capacitor. But there is no displacement current when it gets fully charged.
(b) The frequency of microwaves in ovens matches with the resonant frequency of water molecules.
(c) Infrared waves are also known as heat waves.

Solution. Let's break down each part of the question with clear explanations:

(a) Displacement Current in a Capacitor

Explanation:

1. Displacement Current Concept:
- Displacement Current is a concept introduced by James Clerk Maxwell to address the changing electric field between the plates of a capacitor. It is defined by Maxwell's modification of Ampère's Law:

$$\mathbf{J}_d = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

where $\mathbf{J}_d$ is the displacement current density, ϵ_0 is the permittivity of free space, and $\frac{\partial \mathbf{E}}{\partial t}$ is the rate of change of the electric field.

2. During Charging:
- When a capacitor is charging, the electric field between the plates is changing as the voltage across the capacitor increases. This changing electric field produces a displacement current in the space between the plates of the capacitor. This is necessary to maintain continuity of current in the circuit according to Maxwell's equations.

3. When Fully Charged:
- Once the capacitor is fully charged, the voltage across its plates stops changing, and thus the electric field between the plates becomes constant. As a result, the rate of change of the electric field $\frac{\partial \mathbf{E}}{\partial t}$ becomes zero, so the displacement current also becomes zero. This is because displacement current is directly related to the changing electric field.

Summary:
- Displacement current exists during the charging process due to a changing electric field. When the capacitor is fully charged and the electric field is constant, the displacement current ceases to exist.

(b) Frequency of Microwaves and Water Molecules

Explanation:

1. Microwave Frequency:
   - Microwaves are a type of electromagnetic radiation with frequencies typically ranging from 1 GHz to 300 GHz.

2. Resonant Frequency of Water Molecules:
   - Water molecules have natural vibrational modes that correspond to specific frequencies. When microwaves with frequencies around 2.45 GHz are used, they match the resonant frequency of the water molecules. This means that the microwaves efficiently excite the water molecules, causing them to vibrate more.

3. Heating Effect:
   - The resonant frequency causes the water molecules to absorb energy from the microwaves, increasing their kinetic energy, which translates into heat. This is why microwaves are effective for heating food that contains water.

Summary:
- The frequency of microwaves in ovens is chosen to match the resonant frequency of water molecules to efficiently heat the food by exciting water molecules and generating heat.

(c) Infrared Waves as Heat Waves

Explanation:

1. Infrared Waves:
   - Infrared (IR) waves are electromagnetic waves with wavelengths longer than visible light but shorter than microwaves, typically ranging from about 700 nm to 1 mm.

2. Heat Transfer:
- Infrared radiation is strongly associated with heat because it is emitted by objects at temperatures we typically experience. When objects are heated, they emit infrared radiation. The heat we feel from a warm object or the sun is due to the infrared radiation it emits.

3. Heating Applications:
- Devices such as infrared heaters and thermal cameras utilize infrared radiation because it effectively transfers heat. This is why infrared waves are often referred to as heat waves.

Summary:
- Infrared waves are called heat waves because they are closely associated with heat transfer and thermal radiation, and they are emitted by objects as a result of their temperature.

These explanations provide a clear understanding of the concepts and their real-world implications.

Q.25. Radiations of frequency 3.0 × 1015 Hz are incident on a photosensitive surface of threshold frequency 1.0 × 1015 Hz. Calculate :
(a) work function of the surface
(b) maximum kinetic energy of photoelectrons

Solution.

To solve this problem, we need to use concepts from the photoelectric effect. Here's a step-by-step guide to find the work function of the surface and the maximum kinetic energy of the photoelectrons:

Given Data:

- Frequency of incident radiation, $\nu = 3.0 \times 10^{15}\ \mathrm{Hz}$
- Threshold frequency of the photosensitive surface, $\nu_0 = 1.0 \times 10^{15}\ \mathrm{Hz}$
- Planck's constant, $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$

(a) Work Function of the Surface

Work function (ϕ) is the minimum energy required to eject an electron from the surface of the material. It can be calculated using the formula:

$$\phi = h\nu_0$$

Calculation:

$$\phi = (6.626 \times 10^{-34}\ \mathrm{J \cdot s}) \times (1.0 \times 10^{15}\ \mathrm{Hz})$$

$$\phi = 6.626 \times 10^{-19}\ \mathrm{J}$$

So, the work function of the surface is $6.626 \times 10^{-19}\ \mathrm{J}$.

(b) Maximum Kinetic Energy of Photoelectrons

Maximum kinetic energy (KE) of photoelectrons can be found using the photoelectric equation:

$$\mathrm{KE}_{\mathrm{max}} = h\nu - \phi$$

Where:

- $h\nu$ is the energy of the incident photons,
- ϕ is the work function.

Calculation:

1. **Energy of the incident photons:**

$$h\nu = (6.626 \times 10^{-34}\ \mathrm{J \cdot s}) \times (3.0 \times 10^{15}\ \mathrm{Hz})$$

$$h\nu = 1.9878 \times 10^{-18}\ \mathrm{J}$$

2. **Maximum kinetic energy:**

$$\mathrm{KE}_{\mathrm{max}} = 1.9878 \times 10^{-18}\ \mathrm{J} - 6.626 \times 10^{-19}\ \mathrm{J}$$

$$\mathrm{KE}_{\mathrm{max}} = 1.324 \times 10^{-18}\ \mathrm{J}$$

Summary:

- **(a)** The work function of the surface is $6.626 \times 10^{-19}\ \mathrm{J}$.
- **(b)** The maximum kinetic energy of the photoelectrons is $1.324 \times 10^{-18}\ \mathrm{J}$.

Q.26. A small circular loop of area 6/πcm2 is placed inside a long solenoid at its centre such that its axis makes an angle of 60° with the axis of the solenoid. The number of turns per cm is 10 in the solenoid. The current in the solenoid changes uniformly from 5 A to zero in 10 ms. Calculate the emf induced in the loop.

Solution.

To calculate the electromotive force (emf) induced in the small circular loop due to the changing current in the solenoid, we can follow these steps:

Given Data:

- **Area of the loop,** A: $\frac{6}{\pi}\,\mathrm{cm}^2 = \frac{6\times10^{-4}}{\pi}\,\mathrm{m}^2$
- **Number of turns per cm in the solenoid**: 10 turns/cm = 1000 turns/m
- **Change in current,** ΔI: from 5 A to 0 A
- **Time for the change,** Δt: 10 ms = $10 \times 10^{-3}\,\mathrm{s}$
- **Angle between the axis of the loop and solenoid,** θ: 60°

Steps to Calculate the emf Induced

1. **Calculate the Magnetic Field Inside the Solenoid:**

   The magnetic field B inside a long solenoid is given by:

$$B = \mu_0 n I$$

where:

- μ_0 is the permeability of free space $= 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$,
- n is the number of turns per unit length,
- I is the current.

Initial Magnetic Field:

$$B_{\text{initial}} = \mu_0 n I_{\text{initial}}$$

$$B_{\text{initial}} = (4\pi \times 10^{-7}) \times (1000) \times 5$$

$$B_{\text{initial}} = 2 \times 10^{-2}\,\mathrm{T}$$

Final Magnetic Field: $B_{\text{final}} = 0$ T (since the current goes to zero).

2. **Calculate the Change in Magnetic Field:**

$$\Delta B = B_{\text{final}} - B_{\text{initial}}$$

$$\Delta B = 0 - 2 \times 10^{-2} = -2 \times 10^{-2}\,\mathrm{T}$$

3. **Calculate the Flux Through the Loop:**

The magnetic flux Φ through the loop is given by:

$$\Phi = B \cdot A \cdot \cos(\theta)$$

Initial Flux:

$$\Phi_{\text{initial}} = B_{\text{initial}} \cdot A \cdot \cos(\theta)$$

$$\Phi_{\text{initial}} = (2 \times 10^{-2}) \cdot \left(\frac{6 \times 10^{-4}}{\pi}\right) \cdot \cos(60^\circ)$$

$$\cos(60^\circ) = 0.5$$

$$\Phi_{\text{initial}} = (2 \times 10^{-2}) \cdot \left(\frac{6 \times 10^{-4}}{\pi}\right) \cdot 0.5$$

$$\Phi_{\text{initial}} = \frac{6 \times 10^{-6}}{\pi}\,\text{Wb}$$

Final Flux:

$$\Phi_{\text{final}} = B_{\text{final}} \cdot A \cdot \cos(\theta) = 0$$

Change in Flux:

$$\Delta\Phi = \Phi_{\text{final}} - \Phi_{\text{initial}}$$

$$\Delta\Phi = 0 - \frac{6 \times 10^{-6}}{\pi} = -\frac{6 \times 10^{-6}}{\pi}\,\text{Wb}$$

4. **Calculate the Induced emf:**

The emf $\mathcal{E}$ induced in the loop is given by Faraday's Law:

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$$

$$\mathcal{E} = -\frac{-\frac{6\times10^{-6}}{\pi}}{10 \times 10^{-3}}$$

$$\mathcal{E} = \frac{6 \times 10^{-6}}{\pi \times 10^{-2}}$$

$$\mathcal{E} = \frac{6 \times 10^{-4}}{\pi}$$

Approximating $\pi \approx 3.14$:

$$\mathcal{E} \approx \frac{6 \times 10^{-4}}{3.14} \approx 1.91 \times 10^{-4}\,\text{V}$$

ummary:

- The emf induced in the loop is approximately $1.91 \times 10^{-4}\,\text{V}$ or $0.191\,\text{mV}$.

Q.27. An ideal resistor R, an ideal inductor L and an ideal capacitor C are connected, one by one, across the same source of ac voltage v = vo sin ot. It is observed that the same current I flows in each case.
(a) What will be the instantaneous value of current that will flow through their series combination when connected across the same source ?
(b) How will the current in each case be affected if the frequency of the source is increased? Justify your answers.

(a) Instantaneous Current in Series Combination

Given:

- The AC voltage source is $v(t) = v_0 \sin(\omega t)$.
- The same current I flows through each component when connected individually.

Objective:

- Find the instantaneous current when R, L, and C are connected in series across the same AC source.

1. **Impedances in Series:**

   When R, L, and C are connected in series, their total impedance Z is the sum of their individual impedances:

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

where:

- j is the imaginary unit,
- ω is the angular frequency of the AC source.

The impedance of each component is:

- **Resistor**: $Z_R = R$
- **Inductor**: $Z_L = j\omega L$
- **Capacitor**: $Z_C = \frac{1}{j\omega C}$

Combining these:

$$Z = R + j\omega L - \frac{j}{\omega C}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

2. **Total Current:**

   The voltage across the series combination is $v(t) = v_0 \sin(\omega t)$. Using Ohm's Law, the current $I(t)$ is:

$$I(t) = \frac{v(t)}{Z}$$

$$I(t) = \frac{v_0 \sin(\omega t)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

To simplify:

- Convert Z to its magnitude and phase:

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Therefore, the instantaneous current can be written as:

$$I(t) = \frac{v_0}{|Z|}\sin(\omega t - \phi)$$

Here, $\frac{v_0}{|Z|}$ is the peak current and ϕ is the phase shift due to the impedance.

(b) Effect of Increasing Frequency

Effect on Each Component:

1. **Resistor R:**
   - **Current**: The resistor's impedance does not change with frequency, so the current will remain constant.
2. **Inductor L:**
   - **Impedance**: $Z_L = j\omega L$. As the frequency ω increases, the inductive impedance increases. This means the current through the inductor will decrease as the frequency increases.
3. **Capacitor C:**
   - **Impedance**: $Z_C = \frac{1}{j\omega C}$. As the frequency ω increases, the capacitive impedance decreases. This means the current through the capacitor will increase as the frequency increases.

Series Combination:

In a series circuit, the impedance affects the total current. When the frequency increases:

- The overall impedance Z of the series combination increases if ωL becomes significantly larger than $\frac{1}{\omega C}$.
- Conversely, if $\frac{1}{\omega C}$ dominates ωL, the overall impedance decreases.

Summary:

1. **Instantaneous Current in Series Combination**: The instantaneous current $I(t)$ through the series combination of R, L, and C is given by $\frac{v_0}{|Z|}\sin(\omega t - \phi)$, where $|Z|$ is the magnitude of the total impedance and ϕ is the phase shift.
2. **Effect of Increasing Frequency**:
   - **Resistor**: No effect; current remains constant.
   - **Inductor**: Impedance increases, so current decreases.
   - **Capacitor**: Impedance decreases, so current increases.

Q.28.(a) Draw the shape of intensity distribution curve of the fringes due to diffraction at a single slit.

(b) Derive the relation for the power of combination of two lenses placed in contact co-axially.

Solution.

(a) Intensity Distribution Curve of Fringes due to Diffraction at a Single Slit

Diffraction at a Single Slit:

When monochromatic light passes through a single slit of width a, it produces a pattern of alternating dark and bright fringes on a screen. This pattern is a result of the diffraction of light, where the intensity distribution follows a specific shape.

Shape of the Intensity Distribution Curve:

1. **Central Maximum**: The central fringe (or central maximum) is the brightest and widest.
2. **Side Lobes**: As you move away from the center, the intensity decreases, and you see a series of smaller and narrower bright fringes separated by dark fringes.

Intensity Distribution Formula:

The intensity $I(\theta)$ at an angle θ from the central maximum is given by:

$$I(\theta) = I_0 \left(\frac{\sin(\beta)}{\beta} \right)^2$$

$$\beta = \frac{\pi a \sin(\theta)}{\lambda}$$

- I_0 is the maximum intensity at $\theta = 0$,
- a is the width of the slit,
- λ is the wavelength of the light.

Graphical Representation:

- The graph of $I(\theta)$ versus θ is a central peak with symmetrical side peaks that diminish in intensity as you move away from the center.
- It features a central maximum, where intensity is highest, surrounded by a series of diminishing maxima and minima.

Here is a simplified description of the shape:

- The central peak is wide and tall.
- The intensity falls off to zero at points where $\beta = n\pi$, where n is an integer, except for $n = 0$.
- Successive maxima are less intense and closer together as you move away from the center.

(b) Power of Combination of Two Lenses Placed in Contact Co-Axially

Combination of Lenses:

When two lenses with focal lengths f_1 and f_2 are placed in contact, their combined power is given by the sum of their individual powers.

Derivation:

1. **Power of a Lens:**
   The power P of a lens is related to its focal length f by:
   $$P = \frac{1}{f}$$
   where f is in meters and P is in diopters (D).
2. **Effective Focal Length of the Combination:**

   For two lenses in contact, the effective focal length f_{eff} can be found using the lens formula fo combined power. Each lens contributes to the overall power:
   $$\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

The total power P_{eff} of the combination is then:

$$P_{\text{eff}} = \frac{1}{f_{\text{eff}}}$$

Substituting $\frac{1}{f_{\text{eff}}}$:

$$P_{\text{eff}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Summary:

- **(a)** The intensity distribution curve for a single slit diffraction shows a central bright maximum with decreasing intensity in side fringes. The central maximum is wider and brighter compared to the side maxima.
- **(b)** For two lenses placed in contact, the combined power P_{eff} is the sum of the individual powers:

$$P_{\text{eff}} = \frac{1}{f_1} + \frac{1}{f_2}$$

where f_1 and f_2 are the focal lengths of the two lenses.

Q.30. Dipoles, whether electric or magnetic, are characterised by their dipole moments, which are vector quantities. Two equal and opposite charges separated by a small distance constitute an electric dipole, while a current carrying loop behaves as a magnetic dipole. Electric dipoles create electric fields around them. Electric dipoles experience a torque when placed in an external electric field.
(i) Two identical electric dipoles, each consisting of charges-q and + q separated by distance d, are arranged in x-y plane such that their negative charges lie at the origin O and positive charges lie at points (d, 0) and (0, d) respectively. The net dipole moment of the system is:

(A) -qd(i+j)
(B) qd (i+j)
(C) qd (-)
(D) qd (-1)

(ii) E_1 and E2 are magnitudes of electric field due to a dipole, consisting of charges -q and +q separated by distance 2a, at points r axis, and (2) on equatorial plane, respectively. Then E1 E2 a) (1) on its is:
(A) 1/4
(B) 1/2

(C) 2
(D) 4

(iii) An electric dipole of dipole moment 5.0 × 10-8 Cm is placed in a region where an electric field of magnitude 1.0 × 103 N/C acts at a given instant. At that instant the electric field E is inclined at an angle of 30° to dipole moment P. The magnitude of torque acting on the dipole, at that instant is:
(A) 2.5 x 10-5 Nm
(B) 5.0 x 10-5 Nm
(C) 1.0 × 10-4 Nm
(D) 2.0 x 10-6 Nm

(iv) (a) An electron is revolving with speed v around the proton in a hydrogen atom, in a circular orbit of radius r. The magnitude of magnetic dipole moment of the electron is :
(A) 4 evr

(B) 2 evr
(C) evr 2
(D) evr

OR

Solution.

(i) Net Dipole Moment of Two Identical Electric Dipoles

Problem Statement:

Two identical electric dipoles, each consisting of charges $-q$ and $+q$ separated by distance d, are arranged in the x-y plane. The negative charges lie at the origin O and the positive charges lie at points $(d, 0)$ and $(0, d)$, respectively.

Solution:

1. **Dipole Moment of Each Dipole:**
   - For the dipole with negative charge at $(0, 0)$ and positive charge at $(d, 0)$, the dipole moment is:
     $\vec{p}_1 = qd\hat{i}$
   - For the dipole with negative charge at $(0, 0)$ and positive charge at $(0, d)$, the dipole moment is:
     $\vec{p}_2 = qd\hat{j}$

2. **Net Dipole Moment of the System:**

   The net dipole moment of the system is the vector sum of the individual dipole moments:
   $\vec{p}_{\text{net}} = \vec{p}_1 + \vec{p}_2$
   $\vec{p}_{\text{net}} = qd\hat{i} + qd\hat{j}$
   $\vec{p}_{\text{net}} = qd(\hat{i} + \hat{j})$

   Answer:

   (B) $qd(\hat{i} + \hat{j})$

(ii) Electric Field Magnitudes Due to a Dipole

Problem Statement:

For a dipole consisting of charges $-q$ and $+q$ separated by distance $2a$:

- E_1 is the magnitude of the electric field on the axial line (i.e., along the line joining the charges).
- E_2 is the magnitude of the electric field on the equatorial plane.

Solution:

The electric field of a dipole at a distance r along the axial line is given by:
$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3}$

The electric field of a dipole at a distance r on the equatorial plane is given by:
$E_{\text{equatorial}} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}$

The ratio of E_1 to E_2 is:
$\frac{E_1}{E_2} = \frac{2p/r^3}{p/r^3} = 2$

Answer:

(C) 2

(iii) Torque on an Electric Dipole

Problem Statement:

An electric dipole with dipole moment $p = 5.0 \times 10^{-8}\,\text{C m}$ is placed in an electric field $E = 1.0 \times 10^3\,\text{N/C}$. The electric field is inclined at an angle of 30° to the dipole moment.

Solution:

The magnitude of torque τ acting on a dipole in an electric field is given by:
$\tau = pE\sin\theta$

Where:

- $p = 5.0 \times 10^{-8}\,\text{C m}$
- $E = 1.0 \times 10^3\,\text{N/C}$
- $\theta = 30^\circ$

Calculating:
$\tau = (5.0 \times 10^{-8}\,\text{C m}) \times (1.0 \times 10^3\,\text{N/C}) \times \sin 30^\circ$
$\tau = (5.0 \times 10^{-8}) \times (1.0 \times 10^3) \times 0.5$
$\tau = 2.5 \times 10^{-5}\,\text{Nm}$

(iv) Magnetic Dipole Moment of an Electron in Hydrogen Atom

Problem Statement:

An electron revolves with speed v around a proton in a hydrogen atom in a circular orbit of radius r.

Solution:

The magnetic dipole moment μ of an electron revolving in a circular orbit is given by:
$\mu = \frac{evr}{2}$

Where:

- e is the charge of the electron
- v is the speed of the electron
- r is the radius of the orbit

Answer:

(B) $\frac{evr}{2}$

(b) A square loop of side 5.0 cm carries a current of 2.0 A. The magnitude of magnetic dipole moment associated with the loop is:
(A) 1.0 x 10-3 Am²
(B) 5.0 x 10-3 Am2
(C) 1.0 x 10-2 Am²
(D) 5.0 x 10-2 Am²

Solution.

To find the magnetic dipole moment of a square loop, you can use the formula for the magnetic dipole moment (μ) of a current-carrying loop:

$$\mu = I \cdot A$$

where:

- I is the current through the loop,
- A is the area of the loop.

Given:

- The side of the square loop, $a = 5.0\,\text{cm} = 0.05\,\text{m}$,
- The current, $I = 2.0\,\text{A}$.

1. **Calculate the area A of the square loop:**

$$A = a^2$$
$$A = (0.05\,\text{m})^2$$
$$A = 0.0025\,\text{m}^2$$

2. **Calculate the magnetic dipole moment μ:**

$$\mu = I \cdot A$$
$$\mu = 2.0\,\text{A} \times 0.0025\,\text{m}^2$$
$$\mu = 0.005\,\text{A} \cdot \text{m}^2$$
$$\mu = 5.0 \times 10^{-3}\,\text{A} \cdot \text{m}^2$$

Answer:

(B) $5.0 \times 10^{-3}\,\text{A} \cdot \text{m}^2$

SECTION E

Q.31.(a) (i) With the help of a labelled diagram, explain the working of an ac generator. Obtain the expression for the emf induced at an instant 't'.

Solution.

Working of the AC Generator

1. **Magnetic Field:** A constant magnetic field is produced by the field coils or permanent magnets in the stator.
2. **Rotation of Rotor:** When the rotor (armature) is mechanically rotated by an external source (like a turbine), the coil cuts through the magnetic field lines.
3. **Electromagnetic Induction:** As the coil rotates, the magnetic flux through the coil changes. According to Faraday's Law of Electromagnetic Induction, this change in flux induces an electromotive force (EMF) in the coil.
4. **EMF Generation:** The induced EMF is alternating because the direction of the magnetic flux through the coil changes periodically as the rotor rotates.
5. **Electrical Output:** The alternating current (AC) generated is taken out through slip rings and brushes, which transfer the current to the external circuit.

Expression for the EMF Induced

Let's derive the expression for the instantaneous EMF (E) induced in the generator.

Let's derive the expression for the instantaneous EMF (E) induced in the generator.

1. **Magnetic Flux (Φ):**

   The magnetic flux Φ through the coil is given by:

   $$\Phi = B \cdot A \cdot \cos(\theta)$$

   where:

   - B is the magnetic field strength,
   - A is the area of the coil,
   - θ is the angle between the normal to the plane of the coil and the magnetic field.

2. **Angle θ:**

   The angle θ changes with time. If the coil rotates with an angular velocity ω, then:

   $$\theta = \omega t$$

   where t is the time.

3. **Magnetic Flux as a Function of Time:**

   Thus, the flux Φ at time t is:

   $$\Phi = B \cdot A \cdot \cos(\omega t)$$

4. **Induced EMF (E):**

   According to Faraday's Law, the magnitude of the induced EMF is the negative rate of change of the magnetic flux:

   $$E = -\frac{d\Phi}{dt}$$

   Substituting Φ:

   $$E = -\frac{d}{dt}\left(B \cdot A \cdot \cos(\omega t)\right)$$

   $$E = -B \cdot A \cdot (-\omega \sin(\omega t))$$

   $$E = B \cdot A \cdot \omega \sin(\omega t)$$

Summary:

The instantaneous EMF (E) induced in the AC generator at time t is:

$$E = B \cdot A \cdot \omega \sin(\omega t)$$

where B is the magnetic field strength, A is the area of the coil, ω is the angular velocity of the rotor, and t is the time.

(b)(i) State Faraday's law of electromagnetic induction and mention the utility of Lenz's law. Obtain an expression for self-inductance of a coil in terms of its geometry and permeability of the medium.

Solution.

Faraday's Law of Electromagnetic Induction

Faraday's Law of Electromagnetic Induction states that:

- **"The electromotive force (EMF) induced in a circuit is directly proportional to the rate of change of magnetic flux through the circuit."**

Mathematically, it can be expressed as:

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

where:

- $\mathcal{E}$ is the induced EMF,
- Φ is the magnetic flux,
- $\frac{d\Phi}{dt}$ is the rate of change of magnetic flux.

Magnetic Flux (Φ) is given by:

$$\Phi = B \cdot A \cdot \cos(\theta)$$

where:

- B is the magnetic field strength,
- A is the area through which the magnetic field lines pass,
- θ is the angle between the magnetic field and the normal to the surface.

Lenz's Law

Lenz's Law states that:

- **"The direction of the induced EMF and current in a closed circuit is such that it opposes the change in magnetic flux that produced it."**

Utility of Lenz's Law:

- **Direction Determination:** It helps in determining the direction of the induced EMF and current.
- **Conservation of Energy:** It ensures that the induced current opposes the change in magnetic flux, which is consistent with the principle of conservation of energy. The work done to induce the current is always equal to the change in magnetic energy.

Expression for Self-Inductance of a Coil

Self-inductance L of a coil is a measure of its ability to induce EMF in itself due to a change in current flowing through it. To derive the expression for L, consider the following:

1. **Magnetic Flux through a Coil:**
   - For a coil of N turns, the total magnetic flux Φ through one turn is:
   $$\Phi = B \cdot A$$
   where A is the cross-sectional area of the coil, and B is the magnetic field strength inside the coil.
2. **Magnetic Field B:**
   - The magnetic field B inside the coil (assuming it is long and solenoidal) is given by:
   $$B = \mu \cdot \frac{N \cdot I}{l}$$
   where:
     - μ is the permeability of the medium,
     - N is the number of turns,
     - I is the current flowing through the coil,
     - l is the length of the coil.
3. **Magnetic Flux through the Coil:**
   - The total magnetic flux through the coil is:
   $$\Phi = B \cdot A = \mu \cdot \frac{N \cdot I}{l} \cdot A$$
4. **Induced EMF (Faraday's Law):**
   - According to Faraday's Law:
   $$\mathcal{E} = -\frac{d\Phi}{dt}$$
   - Since $\Phi = \mu \cdot \frac{N \cdot I}{l} \cdot A$, we have:
   $$\mathcal{E} = -\frac{d}{dt}\left(\mu \cdot \frac{N \cdot I \cdot A}{l}\right) = -\mu \cdot \frac{N \cdot A}{l} \cdot \frac{dI}{dt}$$

5. **Self-Inductance L:**

- The self-inductance L is defined as:

$$\mathcal{E} = -L \cdot \frac{dI}{dt}$$

- Comparing this with the previous expression:

$$L = \mu \cdot \frac{N^2 \cdot A}{l}$$

Summary

The **self-inductance** L of a coil is given by:

$$L = \mu \cdot \frac{N^2 \cdot A}{l}$$

where:

- μ is the permeability of the medium,
- N is the number of turns,
- A is the cross-sectional area of the coil,
- l is the length of the coil.

(ii) A resistance of 20 2, a capacitance of 80 μF and an inductor of 50 mH are connected in series. This combination is connected across a 220 V ac supply of variable frequency. When the frequency of supply equals the natural frequency of the circuit, calculate:

(1) angular frequency of supply

(2) impedance of the circuit

Solution.

To solve this problem, we need to determine the angular frequency of the supply and the impedance of the circuit when the frequency of the supply equals the natural frequency of the circuit. Here's a step-by-step solution:

Given Data

- Resistance (R) = 20 Ω
- Capacitance (C) = 80 μF = 80×10^{-6} F
- Inductance (L) = 50 mH = 50×10^{-3} H
- Supply Voltage = 220 V (though this is not needed for the calculations in this specific part)

Step 1: Determine the Angular Frequency (ω)

The natural frequency (resonant frequency) of an RLC circuit is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

where:

- L is the inductance,
- C is the capacitance.

Substitute the given values into this formula:

$$\omega_0 = \frac{1}{\sqrt{(50 \times 10^{-3}) \times (80 \times 10^{-6})}}$$

$$\omega_0 = \frac{1}{\sqrt{4 \times 10^{-9}}}$$

$$\omega_0 = \frac{1}{2 \times 10^{-4}}$$

$$\omega_0 = 5000 \text{ rad/s}$$

Step 2: Calculate the Impedance of the Circuit at Resonance

At resonance, the inductive reactance (X_L) and capacitive reactance (X_C) are equal, and they cancel each other out. Therefore, the impedance of the circuit at resonance is purely resistive and is equal to the resistance R.

The impedance Z of the circuit at resonance is:

$$Z = R$$

Given that $R = 20\ \Omega$, the impedance at resonance is:

$$Z = 20\ \Omega$$

Summary

1. **Angular Frequency of Supply**: $\omega_0 = 5000\ \text{rad/s}$
2. **Impedance of the Circuit**: $Z = 20\ \Omega$

Q.32. (a) (i) What are the two main considerations for designing the objective and eyepiece lenses of an astronomical telescope ? Obtain the expression for magnifying power of the telescope when the final image is formed at infinity.

Solution. Designing the Objective and Eyepiece Lenses of an Astronomical Telescope

When designing the lenses for an astronomical telescope, two primary considerations are:

1. Objective Lens:
   - Aperture Size: The objective lens should have a large diameter (aperture) to collect as much light as possible. This increases the telescope's light-gathering power and improves the resolution, allowing it to view faint and distant celestial objects with greater clarity.
   - Focal Length: The focal length of the objective lens should be long to provide a higher magnification and to allow for a detailed view of distant objects. A longer focal length of the objective lens helps to achieve better resolution and a clearer image.

2. Eyepiece Lens:

- Focal Length: The eyepiece lens should have a shorter focal length to increase the magnification of the image formed by the objective lens. This allows for a detailed examination of the image produced by the objective lens.
- Field of View: The eyepiece should provide a wide field of view to make it easier to locate and track celestial objects. This ensures that the image of the object is fully visible and helps in easier observation.

Magnifying Power of the Telescope

The magnifying power (M) of an astronomical telescope when the final image is formed at infinity is given by the ratio of the focal length of the objective lens (f_o) to the focal length of the eyepiece lens (f_e).

Expression for Magnifying Power:

$$M = \frac{f_o}{f_e}$$

where:
- f_o is the focal length of the objective lens.
- f_e is the focal length of the eyepiece lens.

Derivation:

1. Formation of Image:
   - The objective lens forms a real and inverted image of the distant object at its focal plane.
   - The eyepiece lens acts as a magnifier to view this real image, forming a virtual image at infinity (if the final image is at infinity).

2. Magnifying Power:

- The magnifying power of the telescope is the ratio of the angular size of the image seen through the eyepiece to the angular size of the image seen with the naked eye.
- For a simple telescope where the final image is at infinity, the angular magnification is given by:

$$M = \frac{\text{Angle subtended by the image at the eyepiece}}{\text{Angle subtended by the object at the objective lens}}$$

- Since the angle subtended by the image at infinity is $\frac{1}{f_e}$ and by the object is $\frac{1}{f_o}$, we get:

$$M = \frac{\text{Focal length of the objective}}{\text{Focal length of the eyepiece}} = \frac{f_o}{f_e}$$

This formula provides the magnification power of the telescope, showing how much larger the image appears through the telescope compared to the naked eye.

(ii) A ray of light is incident at an angle of 45° at one face of an equilateral triangular prism and passes symmetrically through the prism. Calculate:
(1) the angle of deviation produced by the prism
(2) the refractive index of the material of the prism

Solution.

To solve the problem of a ray of light passing through an equilateral triangular prism, we need to calculate two things:

1. **The Angle of Deviation Produced by the Prism**
2. **The Refractive Index of the Material of the Prism**

Given:

- The prism is equilateral, so each angle of the prism is 60°.
- The incident angle at the first face of the prism is 45°.
- The ray passes symmetrically through the prism, meaning it is incident and exits symmetrically.

1. Angle of Deviation (δ)

Step-by-Step Solution:

1. **Understanding Symmetric Passage:**
   For a ray passing symmetrically through an equilateral prism, the angle of incidence (i) and angle of emergence (e) are equal. Thus, the angles i and e at the faces of the prism are both 45°.

2. **Prism Angles and Deviation:**
   - Let A be the angle of the prism. For an equilateral prism, $A = 60^\circ$.
   - The angle of deviation δ for a prism is given by:

   $$\delta = i + e - A$$

   Since the incident angle i and emergence angle e are both 45°, we substitute:

   $$\delta = 45^\circ + 45^\circ - 60^\circ = 90^\circ - 60^\circ = 30^\circ$$

   Therefore, the angle of deviation δ is 30°.

2. Refractive Index (μ)

Step-by-Step Solution:

1. **Using Snell's Law:**
   - At the first face of the prism:

$$\mu = \frac{\sin(i + \theta/2)}{\sin(\theta/2)}$$

where θ is the angle of the prism, and $\theta/2$ is the angle of deviation inside the prism.

2. **Calculate Internal Angles:**
   - For a symmetric passage, the angle of incidence i and angle of emergence e inside the prism are the same.
   - The angle of deviation inside the prism is $\theta - i = 60^\circ - 45^\circ = 15^\circ$.
3. **Calculate Refractive Index:**
   - The refractive index μ is calculated using the formula:

$$\mu = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where $\delta = 30^\circ$ and $A = 60^\circ$:

$$\mu = \frac{\sin\left(\frac{60^\circ+30^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)} = \frac{\sin(45^\circ)}{\sin(30^\circ)}$$

$$\mu = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2}$$

Therefore, the refractive index μ of the prism is $\sqrt{2} \approx 1.414$.

Summary

1. **The Angle of Deviation** produced by the prism is 30°.
2. **The Refractive Index** of the material of the prism is $\sqrt{2}$ or approximately 1.414.

Q.33. (a)(i) What are matter waves? A particle of mass m and charge q is accelerated from rest through a potential difference V. Obtain an expression for de Broglie wavelength associated with the particle.

Solution. Matter Waves and de Broglie Wavelength

(a)(i) Matter Waves:

Matter waves, also known as de Broglie waves, are a fundamental concept in quantum mechanics. They represent the wave-like behavior of particles. According to Louis de Broglie, every moving particle or object has an associated wave, and the wavelength of these waves is related to the particle's momentum. This concept is crucial in understanding the wave-particle duality of matter.

Expression for de Broglie Wavelength:

To derive the de Broglie wavelength of a particle that is accelerated through a potential difference V, follow these steps:

1. Kinetic Energy of the Particle:

When a particle of mass m and charge q is accelerated from rest through a potential difference V, its kinetic energy (K.E.) is given by:

K.E.=qV

2. Relate Kinetic Energy to Momentum:

The kinetic energy of a particle is also expressed in terms of its momentum p as:

K.E.=2mp2

Equate the two expressions for kinetic energy:

qV=2mp2

Rearranging for p, the momentum:
p=2mqV

3. de Broglie Wavelength:

According to de Broglie's hypothesis, the wavelength \lambda of a matter wave is related to its momentum p by the equation:

$\lambda=ph$

where h is Planck's constant. Substituting the expression for p:

$\lambda=2mqVh$

Summary

For a particle of mass m and charge q accelerated through a potential difference V, the de Broglie wavelength \lambda associated with the particle is given by:

$\lambda=2mqVh$

where h is Planck's constant. This expression shows that the de Broglie wavelength is inversely proportional to the square root of the kinetic energy of the particle, which is itself related to the potential difference through which the particle was accelerated.

(ii) Monochromatic light of frequency 5.0 × 1014 Hz is produced by a source of power output 3.315 mW. Calculate:

(1) energy of the photon in the beam

(2) number of photons emitted per second by the source

Solution.

To solve the problem, we need to calculate two things:

1. **Energy of a Photon**
2. **Number of Photons Emitted per Second**

Given:

- Frequency of the light, $f = 5.0 \times 10^{14}\,\text{Hz}$
- Power output of the source, $P = 3.315\,\text{mW} = 3.315 \times 10^{-3}\,\text{W}$

(1) Energy of the Photon

The energy E of a single photon is given by the equation:

$$E = hf$$

where:

- h is Planck's constant ($h = 6.626 \times 10^{-34}\,\text{J}\cdot\text{s}$)
- f is the frequency of the photon

Substitute the given values:

$$E = (6.626 \times 10^{-34}\,\text{J}\cdot\text{s}) \times (5.0 \times 10^{14}\,\text{Hz})$$

$$E = 3.313 \times 10^{-19}\,\text{J}$$

(2) Number of Photons Emitted per Second

To find the number of photons emitted per second, we use the relationship between power, energy, and the number of photons. The power P of the source is the total energy emitted per second:

$$P = (\text{Number of photons per second}) \times E$$

Let N be the number of photons emitted per second. Rearranging the formula to solve for N:

$$N = \frac{P}{E}$$

Substitute the values:

$$N = \frac{3.315 \times 10^{-3}\,\text{W}}{3.313 \times 10^{-19}\,\text{J}}$$

$$N \approx 1.0 \times 10^{16} \text{ photons/second}$$

Summary

1. **Energy of the photon**: 3.313×10^{-19} J
2. **Number of photons emitted per second**: 1.0×10^{16}

(b) (i) State Bohr's postulates and derive an expression for the energy of electron in nth orbit in Bohr's model of hydrogen atom.

Solution.

Bohr's Postulates

Niels Bohr proposed the following postulates to describe the behavior of electrons in an atom:

1. **Quantized Orbits**: Electrons revolve around the nucleus in certain discrete orbits or stationary states without radiating energy. These orbits are known as "quantized" or "allowed" orbits.
2. **Angular Momentum Quantization**: The angular momentum of an electron in these orbits is quantized and given by:

$$L = n\hbar$$

where n is a positive integer (the principal quantum number), and $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant.

3. **Energy Levels**: Electrons can only occupy these quantized orbits. The energy of the electron is associated with these orbits, and it is emitted or absorbed when an electron transitions between different orbits.
4. **Radius of Orbit**: The radius of the nth orbit is determined by the balance of centripetal force and Coulomb's electrostatic force.

Deriving the Expression for Energy of Electron in nth Orbit

1. **Centripetal Force and Electrostatic Force**:

The centripetal force needed to keep the electron in a circular orbit is provided by the electrostatic force of attraction between the electron and the nucleus.

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r}$$

where:

- e is the charge of the electron,
- ϵ_0 is the permittivity of free space,
- r is the radius of the orbit,
- m is the mass of the electron,
- v is the velocity of the electron.

2. **Angular Momentum Quantization**:

According to Bohr's postulates, the angular momentum L is quantized:

$$L = mvr = n\hbar$$

Rearranging for v:

$$v = \frac{n\hbar}{mr}$$

3. **Substitute v into the Centripetal Force Equation**:

Substitute $v = \frac{n\hbar}{mr}$ into the centripetal force equation:

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{m\left(\frac{n\hbar}{mr}\right)^2}{r}$$

Simplify:

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{n^2\hbar^2}{mr^3}$$

Rearranging for r:

$$r = \frac{n^2\hbar^2}{4\pi\epsilon_0 me^2}$$

4. **Calculate the Energy of the Electron**:

The total energy E_n of the electron is the sum of its kinetic energy (K) and potential energy (U):

- Kinetic Energy:

$$K = \frac{1}{2}mv^2$$

Substituting v^2 from the centripetal force equation:

$$K = \frac{1}{2}\left(\frac{e^2}{4\pi\epsilon_0 r^2}\right)r = \frac{e^2}{8\pi\epsilon_0 r}$$

- Potential Energy:

$$U = -\frac{e^2}{4\pi\epsilon_0 r}$$

- Total Energy:

$$E_n = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r}$$

Substitute r into this expression:

$$E_n = -\frac{e^2}{8\pi\epsilon_0}\left(\frac{4\pi\epsilon_0 me^2}{n^2\hbar^2}\right)$$

Simplify:

$$E_n = -\frac{me^4}{8\epsilon_0^2\hbar^2 n^2}$$

Therefore, the energy of the electron in the nth orbit is:

$$E_n = -\frac{13.6\ \mathrm{eV}}{n^2}$$

where $13.6\ \mathrm{eV}$ is the Rydberg energy for the hydrogen atom.

(ii) Calculate binding energy per nucleon (in MeV) of C.

Given: m (c) 12 6 = 12.000000 u m(n) = 1.008665 u m(H) = 1.007825 u

Solution.

To calculate the binding energy per nucleon of a carbon-12 ($^{12}\mathrm{C}$) nucleus, follow these steps:

Given Data:

- Atomic mass of Carbon-12 ($^{12}\mathrm{C}$): 12.000000 atomic mass units (u)
- Atomic mass of a neutron (m_n): 1.008665 u
- Atomic mass of a proton (m_p): 1.007825 u

Steps to Calculate Binding Energy Per Nucleon:

1. **Calculate the Mass Defect:**
   The mass defect is the difference between the mass of the individual nucleons and the actual mass of the nucleus. For $^{12}\mathrm{C}$, which has 6 protons and 6 neutrons:

$$\text{Mass of 6 protons} = 6 \times m_p = 6 \times 1.007825\ \mathrm{u} = 6.04695\ \mathrm{u}$$

$$\text{Mass of 6 neutrons} = 6 \times m_n = 6 \times 1.008665\ \mathrm{u} = 6.05199\ \mathrm{u}$$

$$\text{Total mass of nucleons} = 6.04695 \text{ u} + 6.05199 \text{ u} = 12.09894 \text{ u}$$

The actual mass of the ^{12}C nucleus is 12.000000 u.

$$\text{Mass Defect} = \text{Total mass of nucleons} - \text{Mass of } ^{12}C = 12.09894 \text{ u} - 12.000000 \text{ u}$$

2. **Convert Mass Defect to Energy:**
   The binding energy E in MeV can be found using the mass-energy equivalence principle. The conversion factor is 1 u = 931.5 MeV.

$$E = \text{Mass Defect} \times 931.5 \text{ MeV/u}$$

$$E = 0.09894 \text{ u} \times 931.5 \text{ MeV/u} = 92.2 \text{ MeV}$$

3. **Calculate Binding Energy Per Nucleon:**
   Carbon-12 has 12 nucleons (6 protons and 6 neutrons). So the binding energy per nucleon is:

$$\text{Binding Energy Per Nucleon} = \frac{\text{Total Binding Energy}}{\text{Number of Nucleons}}$$

$$\text{Binding Energy Per Nucleon} = \frac{92.2 \text{ MeV}}{12} = 7.68 \text{ MeV/nucleon}$$