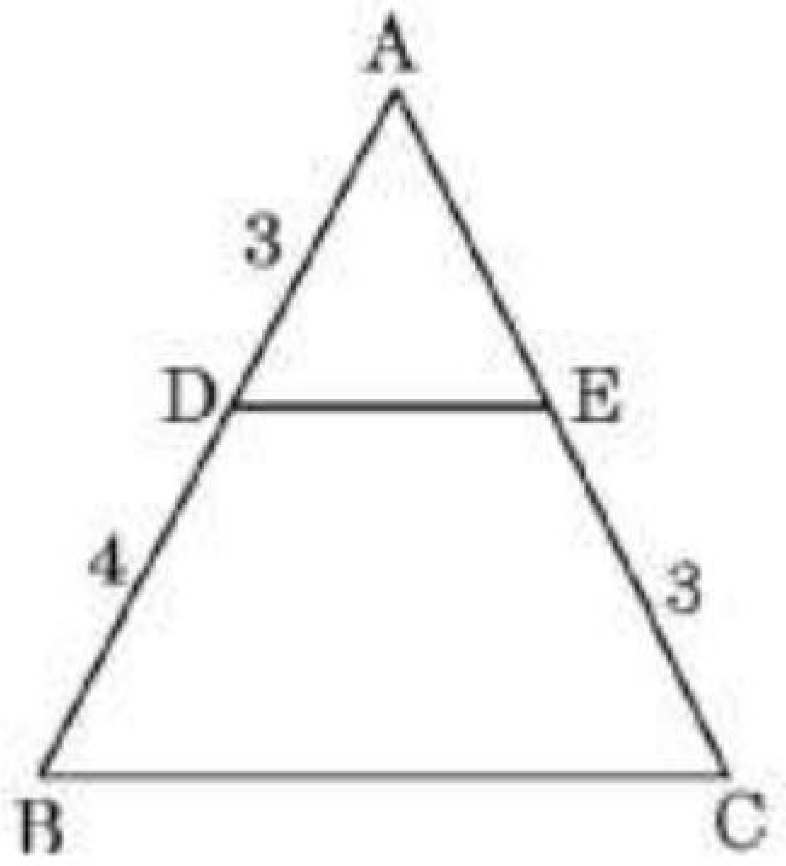
MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 30/C/1)

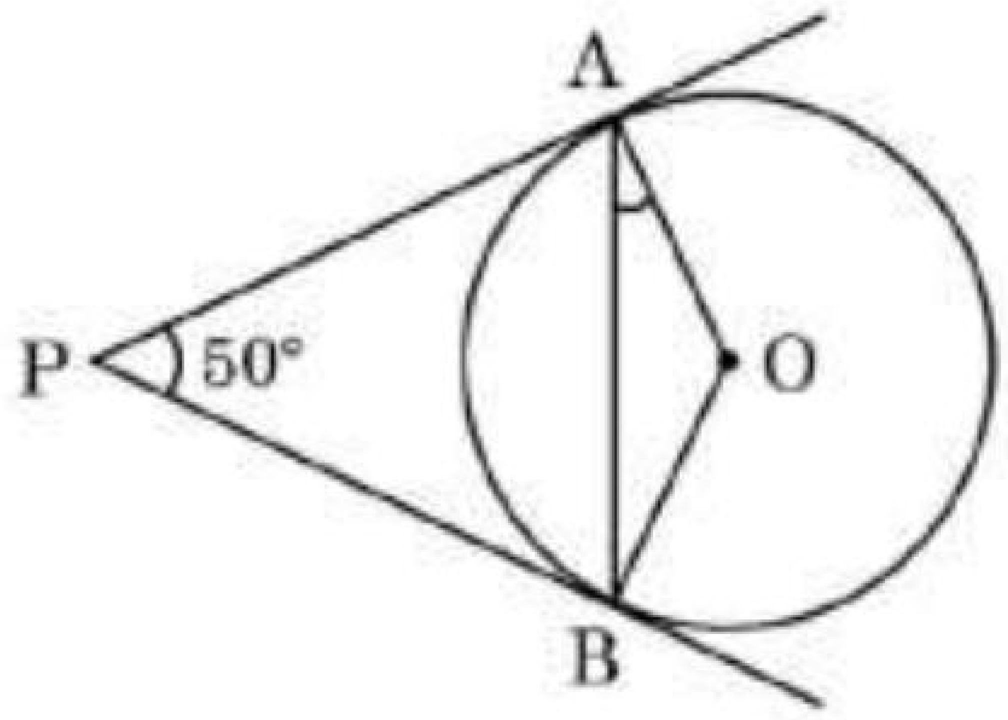
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each	
1.	The values of k for which the equation $4x^2 + kx + 9 = 0$ has real and equal roots are : (a) ± 11 (b) ± 12 (c) ± 6 (d) ± 3	
Sol.	(b) ± 12	1
2.	The distance of the point (4, 7) from the x-axis is : (a) 7 units (b) 5 units (c) 4 units (d) 10 units	
Sol.	(a) 7 units	1
3.	In a family of two children, the probability of having at least one girl is : (a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$	
Sol.	(c) $\frac{3}{4}$	1
4.	The condition for which the pair of equations $ax + 2y = 7$ and $3x + by = 16$ represent parallel lines is : (a) $ab = \frac{7}{16}$ (b) $ab = 6$ (c) $ab = 3$ (d) $ab = 2$	
Sol.	(b) $ab = 6$	1

These answers are meant to be used by evaluators.

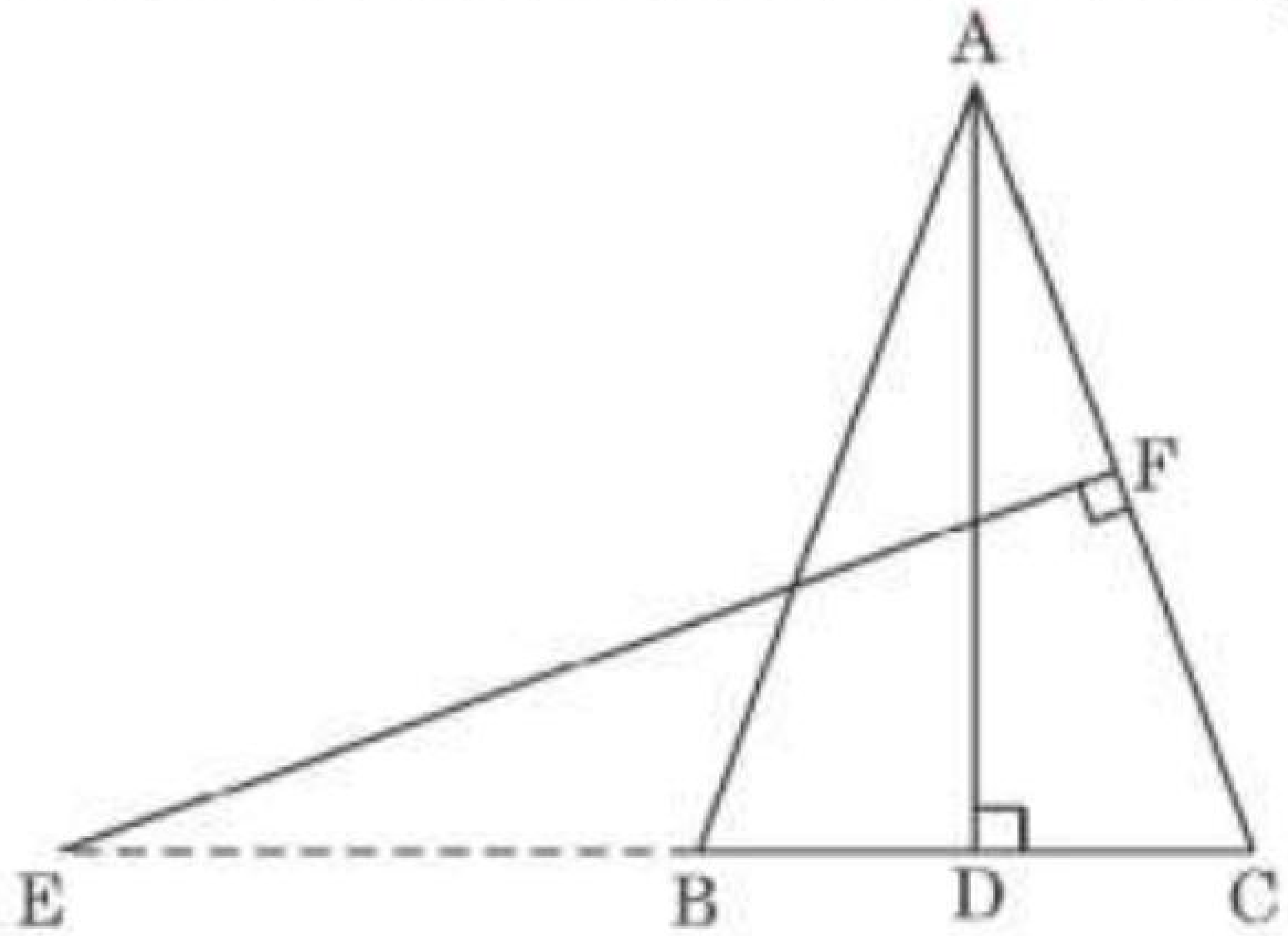
5.	The zeroes of the polynomial $3x^2 + 11x - 4$ are : (a) $\frac{1}{2}, -4$ (b) $\frac{1}{4}, -3$ (c) $\frac{1}{3}, -4$ (d) $\frac{1}{3}, 4$	
Sol.	(c) $\frac{1}{3}, -4$	1
6.	$\cot^2\theta - \frac{1}{\sin^2\theta}$ is equal to : (a) 1 (b) 2 (c) -2 (d) -1	
Sol.	(d) – 1	1
7.	The coordinates of the point A, where AB is the diameter of the circle whose centre is (3, – 2) and B (7, 4) is : (a) (– 1, – 8) (b) (– 1, 8) (c) (1, 8) (d) (1, – 8)	
Sol.	(a) (– 1, – 8)	1
8.	If x, 2x + 9, 4x + 3 are three consecutive terms of an A.P., then the value of x is : (a) 3 (b) 10 (c) 13 (d) 15	
Sol.	(d) 15	1
9.	The height of a tower is 20 m. The length of its shadow made on the level ground when the Sun's altitude is 60°, is : (a) $\frac{20}{\sqrt{3}}$ m (b) $\frac{20}{3}$ m (c) $20\sqrt{3}$ m (d) 20 m	
Sol.	(a) $\frac{20}{\sqrt{3}}$ m	1

10.	In the given figure, DE ∥ BC and all measurements are given in centimetres. The length of AE is :  (a) 2 cm (b) 2·25 cm (c) 2·5 cm (d) 2·75 cm	
Sol.	(b) 2.25 cm	1
11.	A vertical pole 10 m long casts a shadow of length 5 m on the ground. At the same time, a tower casts a shadow of length 12·5 m on the ground. The height of the tower is : (a) 20 m (b) 22 m (c) 25 m (d) 24 m	
Sol.	(c) 25 m	1
12.	Using empirical relationship, the mode of a distribution whose mean is 7·2 and the median 7·1, is : (a) 6·2 (b) 6·3 (c) 6·5 (d) 6·9	
Sol.	(d) 6.9	1
13.	OACB is a quadrant of a circle with centre O and radius 7 cm where ACB is the arc. Then the perimeter of the quadrant is : (a) 15 cm (b) 50 cm (c) 25 cm (d) 44 cm	
Sol.	(c) 25 cm	1

*These answers are meant to be used by evaluators.

14.	In the figure, PA and PB are two tangents to the circle with centre O such that $\angle APB = 50°$. Then, the measure of $\angle OAB$ is :  (a) 25° (b) 50° (c) 75° (d) 100°	
Sol.	(a) 25°	1
15.	The length of the tangent drawn from a point P, whose distance from the centre of a circle is 25 cm, and the radius of the circle is 7 cm, is : (a) 22 cm (b) 24 cm (c) 25 cm (d) 28 cm	
Sol.	(b) 24 cm	1
16.	If a bicycle wheel makes 5000 revolutions in moving 11 km, then the diameter of the wheel is : (a) 65 cm (b) 35 cm (c) 70 cm (d) 50 cm	
Sol.	(c) 70 cm	1
17.	Lali tosses two different coins simultaneously. The probability that she gets at most one head is : (a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{7}$	
Sol.	(b) $\frac{3}{4}$	1

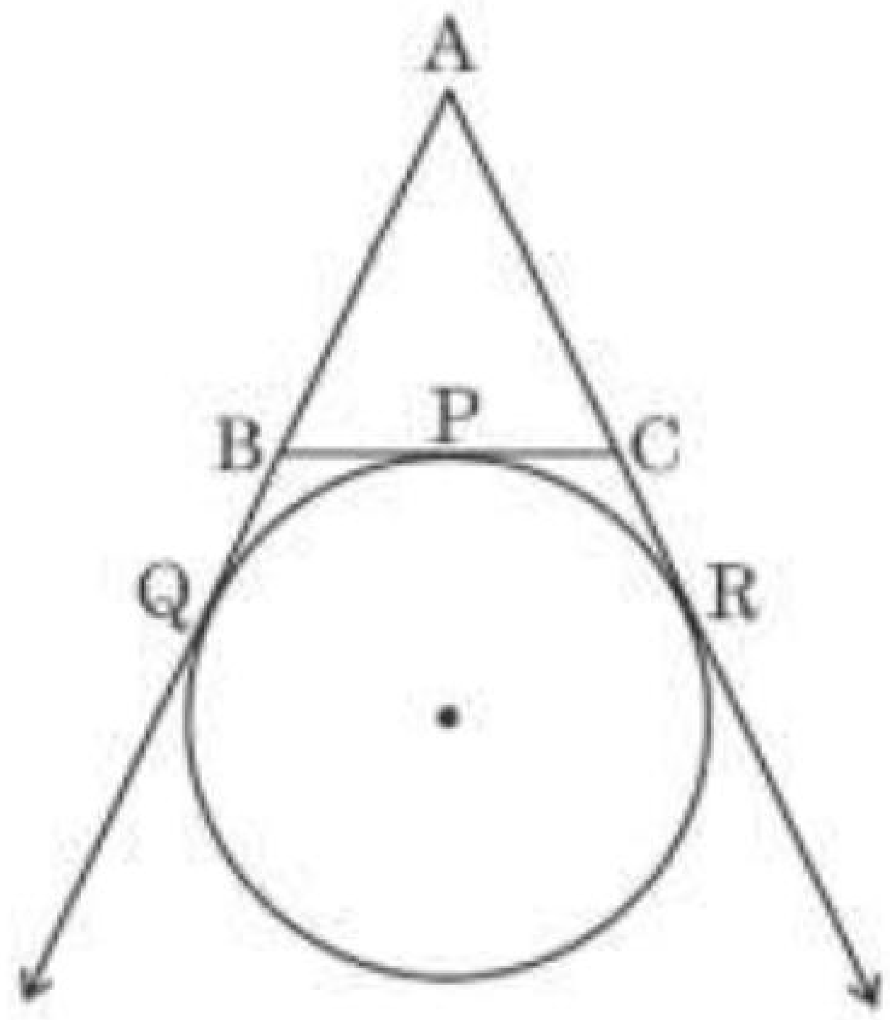
18.	A number is chosen from the numbers 1, 2, 3 and denoted as x, and a number is chosen from the numbers 1, 4, 9 and denoted as y. Then $P(xy < 9)$ is : (a) $\frac{1}{9}$ (b) $\frac{3}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{9}$	
Sol.	(c) $\frac{5}{9}$	1
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* Two players, Sania and Ashnam play a tennis match. The probability of Sania winning the match is 0·79 and that of Ashnam winning the match is 0·21. *Reason (R) :* The sum of probabilities of two complementary events is 1.	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1
20.	*Assertion (A) :* A fair die is thrown once. The probability of getting a prime number is $\frac{1}{2}$. *Reason (R) :* A natural number is a prime number if it has only two factors.	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1

	SECTION B This section comprises very short answer (VSA) type questions of 2 marks each.	
21. (a)	If $\sqrt{2}$ is given as an irrational number, then prove that $(5 - 2\sqrt{2})$ is an irrational number.	
Sol.	Let us assume that $5 - 2\sqrt{2}$ be a rational number. $\therefore 5 - 2\sqrt{2} = \frac{p}{q}$, where p and q are integers and q ≠ 0. $\Rightarrow \sqrt{2} = \frac{5q - p}{2q}$ RHS is a rational number. So, LHS is also a rational number which contradict the given fact that $\sqrt{2}$ is an irrational number. So, our assumption is wrong. Hence, $5 - 2\sqrt{2}$ is an irrational number.	1 ½ ½
	OR	
21. (b)	Check whether 6^n can end with the digit 0 for any natural number n.	
Sol.	If the number 6^{n} ends with the digit 0, then it should be divisible by 2 and 5. But prime factorisation of 6^{n} is $(2 \times 3)^{\mathrm{n}}$. ∴ Prime factorisation of 6^{n} does not contain prime number 5. Hence, 6^{n} can't end with the digit 0.	 1 1
22.	In the figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that Δ ABD ~ Δ ECF.  	

These answers are meant to be used by evaluators.

Sol.	In Δ ABC, AB = AC (Given) ∴ ∠ACB = ∠ABC ----- ① In Δ ABD and Δ ECF ∠ADB = ∠EFC (each 90°) ∠ABD = ∠ACD (from ①) ∴ Δ ABD ~ Δ ECF (AA rule)	1 ½ ½
23. (a)	Show that the points (– 3, – 3), (3, 3) and $(-3\sqrt{3}, 3\sqrt{3})$ are the vertices of an equilateral triangle.	
Sol.	Let A (– 3, – 3), B (3, 3) and C $(-3\sqrt{3}, 3\sqrt{3})$ be the given points. Using distance formula AB = $\sqrt{(3+3)^2+(3+3)^2} = 6\sqrt{2}$ units BC = $\sqrt{(-3\sqrt{3}-3)^2+(3\sqrt{3}-3)^2} = 6\sqrt{2}$ units CA = $\sqrt{(-3+3\sqrt{3})^2+(-3-3\sqrt{3})^2} = 6\sqrt{2}$ units As AB = BC = CA, so the given points are the vertices of an equilateral triangle.	½ ½ ½ ½
	OR	
23(b).	Prove that A(4, 3), B(6, 4), C(5, 6), D(3, 5) are the vertices of a square ABCD.	
Sol.	AB = $\sqrt{(6-4)^2+(4-3)^2} = \sqrt{5}$ units BC = $\sqrt{(5-6)^2+(6-4)^2} = \sqrt{5}$ units CD = $\sqrt{(3-5)^2+(5-6)^2} = \sqrt{5}$ units DA = $\sqrt{(4-3)^2+(3-5)^2} = \sqrt{5}$ units AC = $\sqrt{(5-4)^2+(6-3)^2} = \sqrt{10}$ units BD = $\sqrt{(3-6)^2+(5-4)^2} = \sqrt{10}$ units As AB = BC = CD = DA and AC = BD, so ABCD is a square.	1 ½ ½

These answers are meant to be used by evaluators.

24.	A circle is touching the side BC of a Δ ABC at the point P and touching AB and AC produced at points Q and R respectively. Prove that AQ = $\frac{1}{2}$ (Perimeter of Δ ABC).  	
Sol.	Perimeter of Δ ABC = AB + BC + CA = AB + BP + CP + CA = AB + BQ + CR + CA [BP = BQ ; CP = CR] = AQ + AR = AQ + AQ [AQ = AR] = 2 AQ ∴ AQ = $\frac{1}{2}$ (Perimeter of Δ ABC)	 ½ ½ ½ ½
25.	Find the ratio in which the point (– 1, k) divides the line segment joining the points (– 3, 10) and (6, – 8). Hence, find the value of k.	
Sol.	Let C (– 1, k) be divides the line segment joining the points A (– 3, 10) and B (6, – 8) in the ratio m : 1. Using section formula $-1 = \frac{-3+6m}{m+1}$ $\Rightarrow m = \frac{2}{7}$ Hence, required ratio is 2 : 7 $k = \frac{10\times7-8\times2}{2+7} = 6$	 1 1

These answers are meant to be used by evaluators.

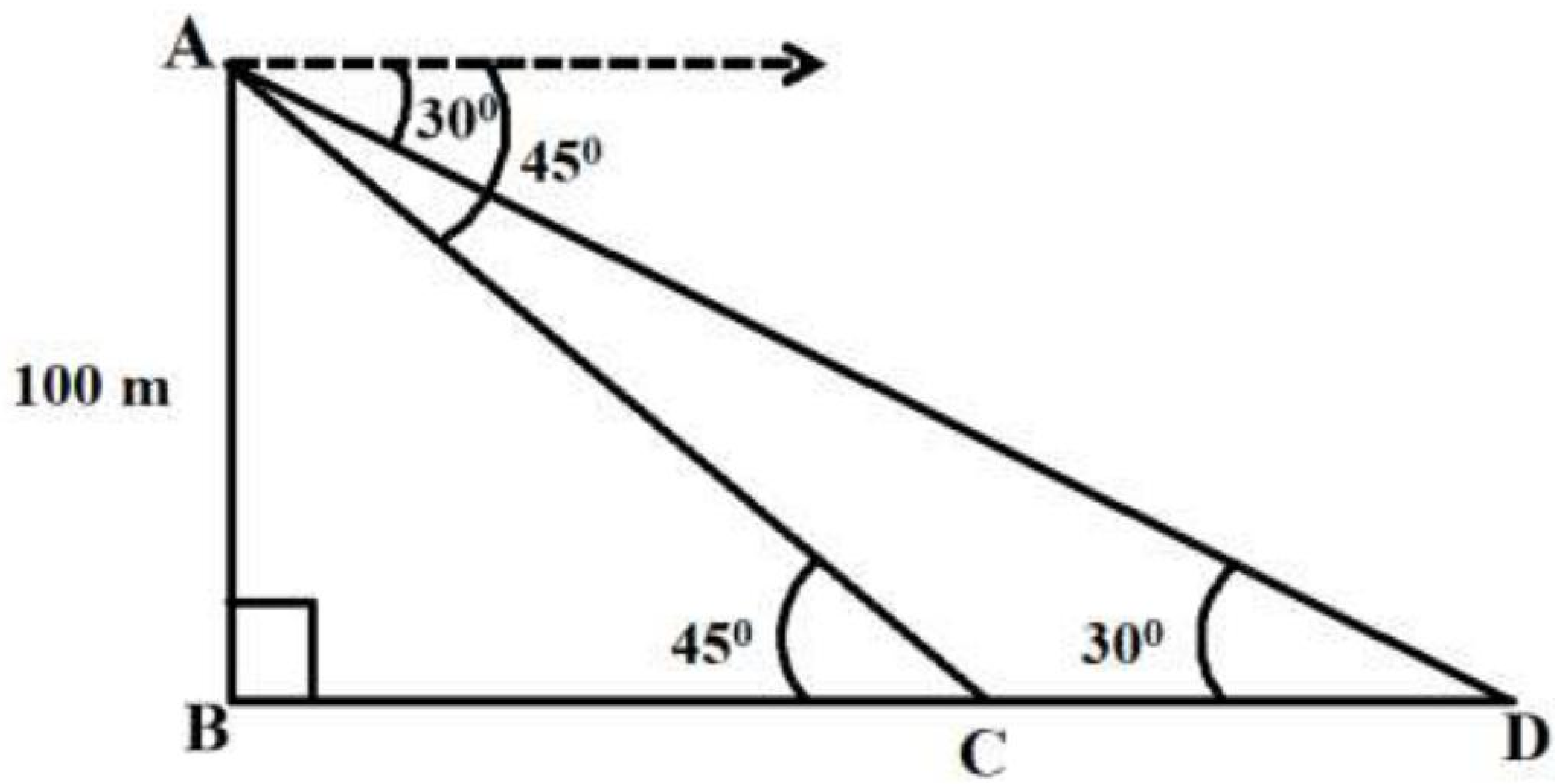
	SECTION C This section comprises of Short Answer (SA) type questions of 3 marks each.	
26.	The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the present age of the father.	
Sol.	Let the present age of the father be ‘x’ years and the sum of present ages of his two children be ‘y’ years A.T.Q. $x = 2y$ ----- ① $x + 20 = y + 40$ ----- ② Solving ① and ②, we get $x = 40$ Hence, the present age of the father is 40 years.	 1 1 1
27.	Two water taps together can fill a tank in $3\frac{1}{3}$ hours. The tap of larger diameter takes 5 hours less than the smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.	
Sol.	Let the time taken by the tap of smaller diameter to fill the tank separately be ‘x’ hours and the time taken by the tap of larger diameter to fill the tank separately be $(x – 5)$ hours. A.T.Q. $\frac{1}{x} + \frac{1}{x-5} = \frac{3}{10}$ $\Rightarrow 3x^2 – 35x + 50 = 0$ $\Rightarrow (x – 10)(3x – 5) = 0$ $\Rightarrow x = 10$ or $x = \frac{5}{3}$ But $x = \frac{5}{3}$ is not possible, so $x = 10$ ∴ time taken by the tap of smaller diameter to fill the tank separately is 10 hours and time taken by the tap of larger diameter to fill the tank separately is $10 – 5 = 5$ hours	 1 1 ½ ½
28.	State and prove Basic Proportionality theorem.	
Sol.	Correct statement of Basic Proportionality Correct figure, given, to prove and construction Correct proof	½ 1 1½

These answers are meant to be used by evaluators.

29 (a).	Find the sum of all integers between 50 and 500, which are divisible by 7.	
Sol.	56, 63, …, 497 Here a = 56 and d = 7 Let $a_n = 497$ $\Rightarrow 56 + (n - 1) \times 7 = 497$ $\Rightarrow n = 64$ $S_{64} = \frac{64}{2} \times (56 + 497) = 17696$	1 ½ ½ 1
	OR	
29 (b).	How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3 ? Also, find their sum.	
Sol.	11, 15, …, 299 Here a = 11 and d = 4 Let $a_n = 299$ $\Rightarrow 11 + (n - 1) \times 4 = 299$ $\Rightarrow n = 73$ $S_{73} = \frac{73}{2} \times (11 + 299) = 11315$	1 ½ ½ 1
30.	Draw the graph of the following equations : $x + y = 5$, $x - y = 5$, and (i) find the solution of the equations from the graph. (ii) shade the triangular region formed by the lines and the y-axis.	
Sol.	Correct graph of line for equation $x + y = 5$. Correct graph of line for equation $x - y = 5$. (i) (5, 0) (ii) Correct shade the required triangular region.	1 1 ½ ½
31 (a).	Find the area of the minor and the major sectors of a circle with radius 6 cm, if the angle subtended by the minor arc at the centre is 60°. (Use $\pi = 3{\cdot}14$)	
Sol.	Area of minor sector = $\frac{3.14 \times (6)^2 \times 60°}{360°}$ $= 18.84$ Hence, area of minor sector is 18.84 cm^2	1 ½

These answers are meant to be used by evaluators.

	Area of major sector = Area of circle – Area of minor sector $= 3.14 \times (6)^2 - 18.84$ $= 94.2$ Hence, area of major sector is 94.2 cm^2	 1 ½
	OR	
31 (b).	If a chord of a circle of radius 10 cm subtends an angle of 60° at the centre of the circle, find the area of the corresponding minor segment of the circle. (Use $\pi = 3{\cdot}14$ and $\sqrt{3} = 1{\cdot}73$)	
Sol.	Area of minor segment $= \frac{3.14 \times (10)^2 \times 60°}{360°} - \frac{1}{2} \times (10)^2 \times \frac{\sqrt{3}}{2}$ $= \frac{314}{6} - \frac{173}{4}$ $= 9\frac{1}{12}$ or 9.08 Hence, area of minor segment is 9.08 cm^2.	2 ½ ½
	SECTION D This section comprises of Long Answer (LA) type questions of 5 marks each.	
32 (a).	A tent is in the shape of a right circular cylinder up to a height of 3 m and then a right circular cone, with a maximum height of 13·5 m above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 m.	
Sol.	Height of conical part = 13.5 – 3 = 10.5 m Slant height $= \sqrt{(14)^2 + (10.5)^2} = 17.5$ m SA of tent = CSA of conical part + CSA of cylindrical part $= \left(\frac{22}{7} \times 14 \times 17.5\right) + \left(2 \times \frac{22}{7} \times 14 \times 3\right)$ $= 1034\ m^2$ Cost of painting @ ₹ 2 per m^2 = 1034 × 2 = ₹ 2068	½ 1 2 ½ 1
	OR	
32 (b).	A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere of same radius. If the radius of the hemisphere is 4·2 cm and the total height of the toy is 10·2 cm, find the volume of the wooden toy. Also, find the total surface area of the toy.	

Sol.	Height of conical part = 10.2 – 4.2 = 6 cm Volume of toy = Volume of conical part + Volume of hemispherical part $= \left(\frac{1}{3} \times \frac{22}{7} \times (4.2)^2 \times 6\right) + \left(\frac{2}{3} \times \frac{22}{7} \times (4.2)^3\right)$ = 266.112 Hence, Volume of toy is 266.112 cm^3 Slant height of conical part = $\sqrt{(4.2)^2 + (6)^2} \approx 7.32$ cm TSA of the toy = CSA of hemispherical part + CSA of conical part $= \left(2 \times \frac{22}{7} \times (4.2)^2\right) + \left(\frac{22}{7} \times 4.2 \times 7.32\right)$ = 207.504 Hence, TSA of toy is 207.504 cm^2	½ 1 1 1 1 ½
33.	As observed from the top of a lighthouse, 100 m above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 45°. Determine the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1{\cdot}732$)	
Sol.	Correct figure.  In Δ ABC $\frac{100}{BC} = \tan 45° = 1$ $\Rightarrow$ BC = 100 --- (1) In Δ ABD $\frac{100}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$ $\Rightarrow$ BD = $100\sqrt{3}$	2 ½ ½ ½

These answers are meant to be used by evaluators.

$\Rightarrow 100 + CD = 100\sqrt{3}$ ½

$\Rightarrow CD = 100\sqrt{3} - 100 = 100\,(1.732 - 1) = 73.2$ 1

Hence, distance travelled by the ship during the period of observation is 73.2 m

34. A survey regarding the heights (in cm) of 50 girls of class X of a school was conducted and the following data was obtained :

Height (in cm)	Number of girls
120 – 130	2
130 – 140	8
140 – 150	12
150 – 160	20
160 – 170	8
Total	50

Find the mean and mode of the above data.

Sol.

Height (in cm)	No. of girls	x_i	u_i	f_iu_i
120 – 130	2	125	– 2	– 4
130 – 140	8	135	– 1	– 8
140 – 150	12	145 = a	0	0
150 – 160	20	155	1	20
160 – 170	8	165	2	16
Total	50			24

Correct table 1½

Mean $= 145 + \frac{24}{50} \times 10$ 1

$= 149.8$ ½

∴ mean height is 149.8 cm

Modal class is 150 – 160 ½

Mode $= 150 + \frac{(20-12)}{(2\times20-12-8)} \times 10$ 1

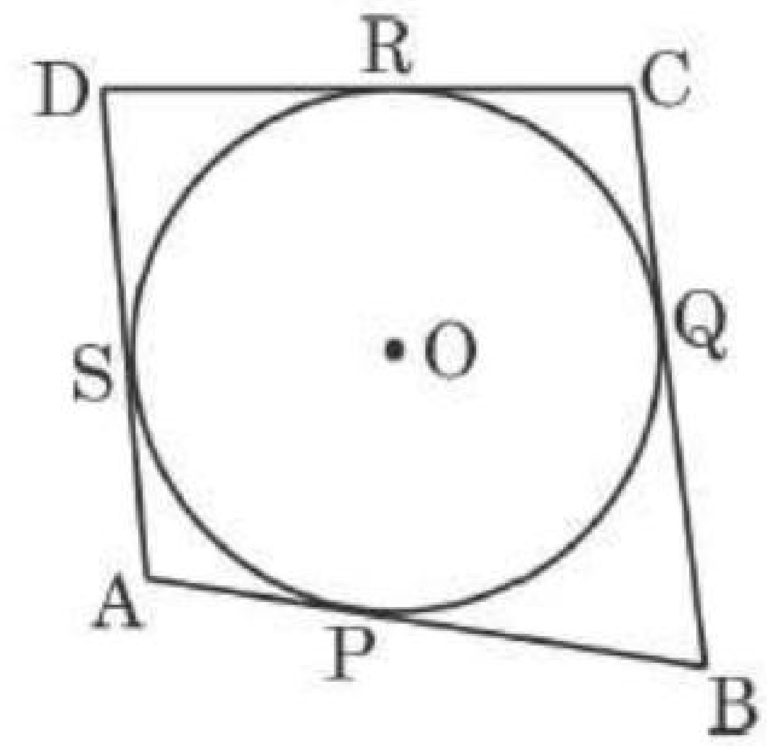
$= 154$ ½

∴ modal height is 154 cm

35 (a).	(i) Prove that : $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$ (ii) Evaluate : $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$	
	(i) LHS = $\sqrt{1+\tan^2\theta+1+\cot^2\theta}$ $= \sqrt{\tan^2\theta+\cot^2\theta+2\times\tan\theta\times\cot\theta}$ $= \sqrt{(\tan\theta+\cot\theta)^2}$ $= \tan\theta+\cot\theta$ = RHS (ii) $\dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}$ $= \dfrac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})} \times \dfrac{\sqrt{2}}{\sqrt{2}}$ $= \dfrac{\sqrt{6}}{4\,(1+\sqrt{3})} \times \dfrac{(1-\sqrt{3})}{(1-\sqrt{3})}$ $= \dfrac{3\sqrt{2}-\sqrt{6}}{8}$	1 1 ½ 1 ½ ½ ½
	OR	
35 (b).	If x $\sin^3\theta$ + y $\cos^3\theta$ = $\sin\theta\cos\theta$ and x $\sin\theta$ = y $\cos\theta$, prove that $x^2 + y^2 = 1$.	
Sol.	Given, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ $\Rightarrow x\sin\theta(\sin^2\theta) + y\cos\theta(\cos^2\theta) = \sin\theta\cos\theta$ $\Rightarrow x\sin\theta(\sin^2\theta) + x\sin\theta(\cos^2\theta) = \sin\theta\cos\theta$ $\Rightarrow x\sin\theta(\sin^2\theta+\cos^2\theta) = \sin\theta\cos\theta$ $\Rightarrow x = \cos\theta$ Given, $x\sin\theta = y\cos\theta$ $\Rightarrow \cos\theta\sin\theta = y\cos\theta$ $\Rightarrow y = \sin\theta$ LHS = $x^2 + y^2 = (\cos\theta)^2 + (\sin\theta)^2 = 1$ = RHS	 1 1 1 1 1
	SECTION E This section comprises of 3 case-study based questions of 4 marks each.	

These answers are meant to be used by evaluators.

36. In a park, four poles are standing at positions A, B, C and D around the circular fountain such that the cloth joining the poles AB, BC, CD and DA touches the circular fountain at P, Q, R and S respectively as shown in the figure.



Based on the above information, answer the following questions :

(i) If O is the centre of the circular fountain, then $\angle$ OSA = ...

(ii) If AB = AD, then write the name of the figure ABCD.

(iii) (a) If DR = 7 cm and AD = 11 cm, then find the length of AP.

OR

(iii) (b) If O is the centre of the circular fountain with $\angle$ QCR = 60°, then find the measure of $\angle$ QOR.

Sol.

(i) 90° — 1

(ii) AB + DC = BC + DA

Given, AB = AD

$\Rightarrow$ BC = DC

So, ABCD is a Kite — 1

(iii) (a) DS = DR = 7 cm — ½

AD = 11 cm — ½

7 + SA = 11 — ½

$\Rightarrow$ SA = 4 cm

$\therefore$ AP = SA = 4 cm — ½

OR

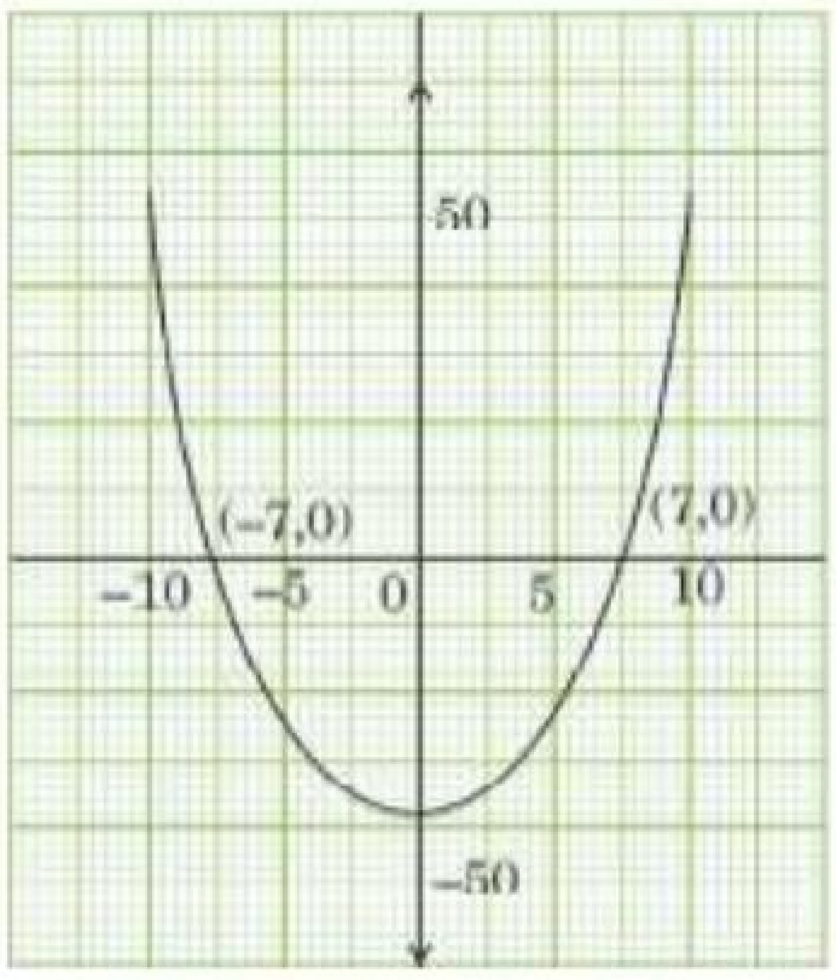
(b) $\angle QOR = 180° - 60°$ — 1

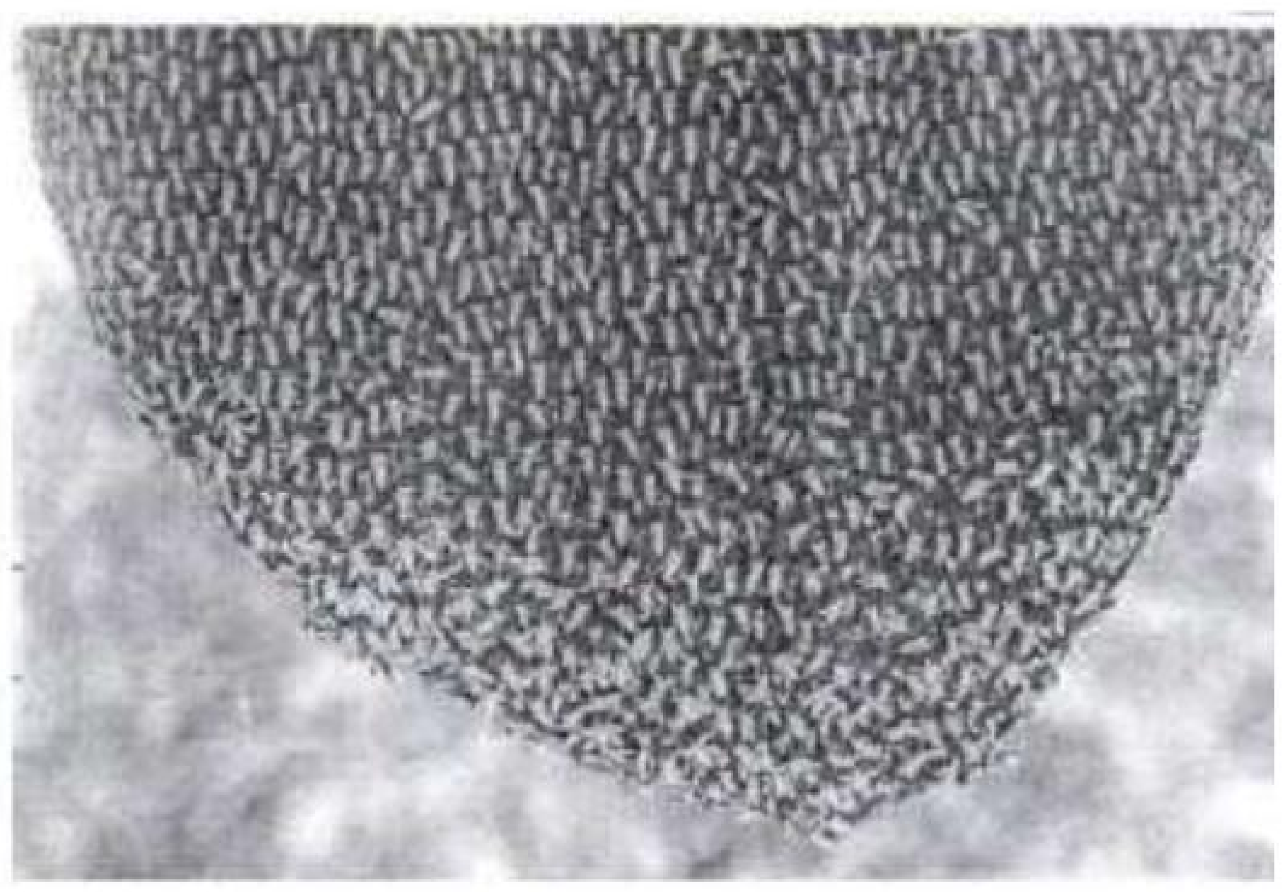
$= 120°$ — 1

These answers are meant to be used by evaluators.

37.

While playing in a garden, Samaira saw a honeycomb and asked her mother what is that. Her mother replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is a mathematical structure. The mathematical representation of the honeycomb is shown in the graph.





Based on the above information, answer the following questions :

(i) How many zeroes are there for the polynomial represented by the graph given ?

(ii) Write the zeroes of the polynomial.

(iii) (a) If the zeroes of a polynomial $x^2 + (a + 1)x + b$ are 2 and -3, then determine the values of a and b.

OR

(iii) (b) If the square of difference of the zeroes of the polynomial $x^2 + px + 45$ is 144, then find the value of p.

Sol.

(i) Two — 1

(ii) 7 and -7 — 1

(iii) (a) $-(a + 1) = 2 + (-3) \Rightarrow a = 0$ — 1

$b = 2 \times (-3) \Rightarrow b = -6$ — 1

OR

(b) Let α and β be the zeroes of given polynomial

Here, $\alpha + \beta = -p$ and $\alpha\beta = 45$ — ½

$(\alpha - \beta)^2 = 144$ — ½

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 144$

$\Rightarrow (-p)^2 - 4 \times 45 = 144$ — ½

$\Rightarrow p = \pm 18$ — ½

These answers are meant to be used by evaluators.

38.	February 14 is celebrated as International Book Giving Day and many countries in the world celebrate this day. Some people in India also started celebrating this day and donated the following number of books of various subjects to a public library : History = 96, Science = 240, Mathematics = 336. These books have to be arranged in minimum number of stacks such that each stack contains books of only one subject and the number of books on each stack is the same. Based on the above information, answer the following questions : (i) How many books are arranged in each stack ? (ii) How many stacks are used to arrange all the Mathematics books ? (iii) (a) Determine the total number of stacks that will be used for arranging all the books. **OR** (iii) (b) If the thickness of each book of History, Science and Mathematics is 1·8 cm, 2·2 cm and 2·5 cm respectively, then find the height of each stack of History, Science and Mathematics books.	
Sol.	(i) HCF (96, 240, 336) = 48 (ii) Number of stacks = $\frac{336}{48} = 7$ (iii) (a) Total number of stacks = $\frac{96}{48} + \frac{240}{48} + \frac{336}{48}$ $= 14$ OR (b) Height of each stack of History = 48 × 1.8 = 86.4 cm Height of each stack of Science = 48 × 2.2 = 105.6 cm Height of each stack of Mathematics = 48 × 2.5 = 120 cm	1 1 1 1 1 mark for 1 correct answer, 1½ mark for two correct answer and 2 marks for all correct answers.

These answers are meant to be used by evaluators.