Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only) Senior School Certificate Examination, 2024** **MATHEMATICS PAPER CODE 65/2/2**

General Instructions:	
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left- hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without cancelling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**

These answers are meant to be used by evaluators.

10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a</u> note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ______(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totalling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totalling on the title page. • Wrong totalling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

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Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each	
1.	The number of solutions of differential equation $\frac{dy}{dx} - y = 1$, given that $y(0) = 1$, is : (A) 0 (B) 1 (C) 2 (D) infinitely many	
Sol.	(B)1	1
2.	For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ? (A) $\vec{a} \cdot \vec{b} \geq \lvert\vec{a}\rvert \lvert\vec{b}\rvert$ (B) $\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert \lvert\vec{b}\rvert$ (C) $\vec{a} \cdot \vec{b} \leq \lvert\vec{a}\rvert \lvert\vec{b}\rvert$ (D) $\vec{a} \cdot \vec{b} < \lvert\vec{a}\rvert \lvert\vec{b}\rvert$	
Sol.	(C) $\vec{a} \cdot \vec{b} \leq \lvert\vec{a}\rvert \lvert\vec{b}\rvert$	1
3.	The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by : (A) (1, 0, 0) (B) (2, 0, 0) (C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)	
Sol.	(D)(0, 0, 0)	1
4.	The common region determined by all the constraints of a linear programming problem is called : (A) an unbounded region (B) an optimal region (C) a bounded region (D) a feasible region	
Sol.	(D)a feasible region	1
5.	Let E be an event of a sample space S of an experiment, then $P(S\mid E) =$ (A) $P(S \cap E)$ (B) $P(E)$ (C) 1 (D) 0	
Sol.	(C)1	1

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6.	The number of all scalar matrices of order 3, with each entry – 1, 0 or 1, is : (A) 1 (B) 3 (C) 2 (D) 3^9	
Sol.	(B)3	1
7.	$\frac{d}{dx}[\cos(\log x + e^x)]$ at $x = 1$ is : (A) $-\sin e$ (B) $\sin e$ (C) $-(1+e)\sin e$ (D) $(1+e)\sin e$	
Sol.	(C) $-(1+e)\sin e$	1
8.	The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')$ is : (A) 1 (B) 2 (C) 3 (D) not defined	
Sol.	(D)not defined	1
9.	The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is : (A) $2\hat{j}$ (B) $\hat{j}$ (C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$	
Sol.	(B) $\hat{j}$	1
10.	Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are : (A) 2, – 1, 6 (B) 2, 1, 6 (C) 2, 1, 3 (D) 2, – 1, 3	
Sol.	(D)2, – 1, 3	1
11.	If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is : (A) 1 (B) 2 (C) 0 (D) – 2	
Sol.	(B)2	1

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12.	If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is : (A) 90° (B) 120° (C) 60° (D) 0°	
Sol.	(A) 90°	1
13.	If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is : (A) 0 (B) 9 (C) 27 (D) 729	
Sol.	(A) 0	1
14.	Which of the following statements is ***not*** true about equivalence classes A_i (i = 1, 2, ... n) formed by an equivalence relation R defined on a set A ? (A) $\bigcup_{i=1}^{n} A_i = A$ (B) $A_i \cap A_j \neq \phi, i \neq j$ (C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$ (D) All elements of A_i are related to each other, for all i	
Sol.	(B) $A_i \cap A_j \neq \emptyset, i \neq j$	1
15.	If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is : (A) 0 (B) 1 (C) 2 (D) 4	
Sol.	(D)4	1

16.	The number of points of discontinuity of $f(x) = \begin{cases} \lvert x\rvert+3, & \text{if } x \le -3 \\ -2x, & \text{if } -3<x<3 \\ 6x+2, & \text{if } x \ge 3 \end{cases}$ is : (A) 0 (B) 1 (C) 2 (D) infinite	
Sol.	(B)1	1
17.	The function $f(x) = x^3 - 3x^2 + 12x - 18$ is : (A) strictly decreasing on R (B) strictly increasing on R (C) neither strictly increasing nor strictly decreasing on R (D) strictly decreasing on $(-\infty, 0)$	
Sol.	(B)strictly increasing on R	1
18.	If $\int_0^2 2\,e^{2x}\,dx = \int_0^a e^x\,dx$, the value of 'a' is : (A) 1 (B) 2 (C) 4 (D) $\frac{1}{2}$	
Sol.	(C)4	1
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	

19.	*Assertion (A) :* For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$. *Reason (R) :* For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.	
Sol.	(C)Assertion (A) is true, but Reason (R) is false.	1
20.	*Assertion (A) :* For any symmetric matrix A, B′AB is a skew-symmetric matrix. *Reason (R) :* A square matrix P is skew-symmetric if P′ = – P.	
Sol.	(D)Assertion (A) is false, but Reason (R) is true.	1
	SECTION B	
21	If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of (M – m).	
Sol.	$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2} = \frac{(x+1)(x-1)}{x^2}$ $f'(x) = 0 \Rightarrow x = -1,\ 1$ $f''(x) = \frac{2}{x^3} \Rightarrow f''(-1) = -2 < 0$ $\therefore$ -1 is a point of local maximum The local maximum value $= f(-1) = -2 = M$ $f''(1) = 2 > 0$ $\therefore$ 1 is point of local minimum The local minimum value $= f(1) = 2 = m$ $M - m = -4$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
22.	**Evaluate :** $$\int_0^{a^3} \frac{x^2}{x^6 + a^6}\, dx$$	
Sol.	Put $x^3 = t \Rightarrow x^2 dx = \frac{dt}{3}$ Given integral $= \frac{1}{3}\int_0^{a^9} \frac{dt}{t^2 + (a^3)^2}$	$\frac{1}{2}$ $\frac{1}{2}$

	$= \frac{1}{3a^3}\tan^{-1}\frac{t}{a^3}\Big\|_0^{a^9}$ $= \frac{1}{3a^3}tan^{-1}a^6$	$\frac{1}{2}$ $\frac{1}{2}$
23.	Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.	
Sol.	$f'(x) = e^x + e^{-x} + 1 - \frac{1}{1+x^2}$ $= e^x + \frac{1}{e^x} + \frac{x^2}{1+x^2} > 0\ for\ all\ x \in R$ ∴ f is strictly increasing over its domain R	1 1
24.	(a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.	
Sol.	The given expression = $\frac{-\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$ $= \frac{-\pi}{12}$	$1\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.	
Sol.	$-1 \leqslant x^2 - 4 \leqslant 1$ $\Rightarrow 3 \leqslant x^2 \leqslant 5$ Domain = $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$\frac{1}{2}$ $\frac{1}{2}$ 1
25.	(a) Check the differentiability of $f(x) = \|\cos x\|$ at $x = \frac{\pi}{2}$.	
Sol.	$f(x) = \|\cos x\| = \begin{cases} cosx & 0 \le x \le \frac{\pi}{2} \\ -cosx & \frac{\pi}{2} \le x \le \pi \end{cases}$ LHD at $\frac{\pi}{2} = \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}-h\right)-f\left(\frac{\pi}{2}\right)}{-h} = \lim_{h\to 0}\frac{\cos\left(\frac{\pi}{2}-h\right)-0}{-h} = \lim_{h\to 0}\frac{sinh}{-h} = -1$	1

	RHD at $\frac{\pi}{2} = \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}+h\right)-f\left(\frac{\pi}{2}\right)}{h} = \lim_{h\to 0}\frac{-\cos\left(\frac{\pi}{2}+h\right)-0}{h} = \lim_{h\to 0}\frac{\sinh}{h} = 1$ LHD $\neq$ RHD $\therefore$ f is not differentiable at x = $\frac{\pi}{2}$	$\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(b) If y = A sin 2x + B cos 2x and $\frac{d^2y}{dx^2} - ky = 0$, find the value of k.	
Sol.	$\frac{dy}{dx} = 2A\cos 2x - 2B\sin 2x$ $\Rightarrow \frac{d^2y}{dx^2} = -4A\sin 2x - 4B\cos 2x = -4y$ $\Rightarrow \frac{d^2y}{dx^2} + 4y = 0$ $\Rightarrow k = -4$	1 $\frac{1}{2}$ $\frac{1}{2}$
	Section C	
26.	(a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; y = 2, when x = 1.	
Sol.	Given differential equation can be written as $\frac{dy}{dx} = \frac{2xy+y^2}{2x^2} = \frac{y}{x} + \frac{y^2}{2x^2}$ Let $y = vx \Rightarrow v + x\frac{dv}{dx}$ The equation becomes $x\frac{dv}{dx} = \frac{1}{2}v^2$ $\Rightarrow \frac{dv}{v^2} = \frac{1}{2}\times\frac{dx}{x}$ Integrating both sides, we get $\frac{-1}{v} = \frac{1}{2}\log\lvert x\rvert + C$ $\Rightarrow -\frac{x}{y} = \frac{1}{2}\log\lvert x\rvert + C$ $x = 1, y = 2$ gives $C = -\frac{1}{2}$ The particular solution is $-\frac{x}{y} = \frac{1}{2}\log\lvert x\rvert - \frac{1}{2}$ $or, y = \frac{2x}{1-\log\lvert x\rvert}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$

<table>
<tr><td></td><td>OR</td><td></td></tr>
<tr><td></td><td>(b) Find the general solution of the differential equation :
$y\,dx = (x + 2y^2)\,dy$</td><td></td></tr>
<tr><td>Sol.</td><td>Given differential equation can be written as
$\frac{dx}{dy} - \frac{x}{y} = 2y$
Integrating Factor = $e^{\int \frac{-1}{y} dy} = \frac{1}{y}$
Solution is $x\frac{1}{y} = \int 2dy$
$\Rightarrow \frac{x}{y} = 2y + C$
$\Rightarrow x = 2y^2 + Cy$</td><td>1
1
$\frac{1}{2}$
$\frac{1}{2}$</td></tr>
<tr><td>27.</td><td>If vectors $\vec{a}$, $\vec{b}$ and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$.</td><td></td></tr>
<tr><td>Sol.</td><td>$|\vec{a}| = |\vec{b}| = |2\vec{a} + 3\vec{b}| = 1$
$(2\vec{a} + 3\vec{b})^2 = |2\vec{a} + 3\vec{b}|^2$
$\Rightarrow 4|\vec{a}|^2 + 12\vec{a}.\vec{b} + 9|\vec{b}|^2 = 1$
$\Rightarrow cos\theta = -1$, where θ is angle between $\vec{a}$ and $\vec{b}$
Hence, $\theta = \pi$</td><td>$\frac{1}{2}$
$\frac{1}{2}$
1
$\frac{1}{2}$
$\frac{1}{2}$</td></tr>
<tr><td>28.</td><td>A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.</td><td></td></tr>
<tr><td>Sol.</td><td><table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>P(X)</td><td>$\frac{6}{36} = \frac{1}{6}$</td><td>$\frac{10}{36} = \frac{5}{18}$</td><td>$\frac{8}{36} = \frac{2}{9}$</td><td>$\frac{6}{36} = \frac{1}{6}$</td><td>$\frac{4}{36} = \frac{1}{9}$</td><td>$\frac{2}{36} = \frac{1}{18}$</td></tr></table></td><td>$\frac{1}{2} \times 6 = 3$</td></tr>
<tr><td>29</td><td>Find :
$\int x^2 . \sin^{-1}(x^{3/2})\,dx$</td><td></td></tr>
</table>

*These answers are meant to be used by evaluators.

Sol.	Let $x^{\frac{3}{2}} = t$ $\Rightarrow \frac{3}{2} x^{\frac{1}{2}} dx = dt$ The given integral becomes $\frac{2}{3}\int t\, sin^{-1}t dt$ $= \frac{2}{3}\left[sin^{-1}t \times \frac{t^2}{2} - \int \frac{1}{\sqrt{1-t^2}} \times \frac{t^2}{2} dt\right]$ $= \frac{1}{3}\left[sin^{-1}t \times t^2 + \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt\right]$ $= \frac{1}{3}\left[sin^{-1}t \times t^2 + \int \sqrt{1-t^2}\, dt - \int \frac{1}{\sqrt{1-t^2}} dt\right]$ $= \frac{1}{3}\left[t^2 sin^{-1}t + \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2} sin^{-1}t - sin^{-1}t\right]$+C $= \frac{1}{3}\left[t^2 sin^{-1}t + \frac{t}{2}\sqrt{1-t^2} - \frac{1}{2} sin^{-1}t\right] + C$ $= \frac{1}{3}\left[x^3 sin^{-1}\left(x^{\frac{3}{2}}\right) + \frac{x^{\frac{3}{2}}}{2}\sqrt{1-x^3} - \frac{1}{2} sin^{-1}\left(x^{\frac{3}{2}}\right)\right] + C$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
30.	(a) If $x^{30} y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.	
Sol.	(a)Taking log of both sides, we get $30 \log x + 20 \log y = 50 \log (x + y)$ Differentiating both sides w.r.t. x, we get $\frac{30}{x} + \frac{20}{y}\frac{dy}{dx} = \frac{50}{x+y}(1 + \frac{dy}{dx})$ $\Rightarrow \frac{dy}{dx}\left(\frac{20x - 30y}{y(x+y)}\right) = \frac{20x - 30y}{x(x+y)}$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x}$	1 1 1
	OR	
	(b) Find $\frac{dy}{dx}$, if $5^x + 5^y = 5^{x+y}$.	
Sol.	Differentiating both sides w.r.t. x, we get $5^x log5 + 5^y log5 \frac{dy}{dx} = 5^{x+y} log5(1 + \frac{dy}{dx})$ $\Rightarrow 5^x + 5^y \frac{dy}{dx} = (5^x + 5^y)(1 + \frac{dy}{dx})$ $\Rightarrow 5^x + 5^y \frac{dy}{dx} = 5^x + 5^x \frac{dy}{dx} + 5^y + 5^y \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = -\frac{5^y}{5^x} = -5^{y-x}$	2 1

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31.	(a) **Evaluate :** $$\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$$	
Sol.	(a) $\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}} dx$ $= \int_{-2}^{2} \frac{2-x}{\sqrt{4-x^2}} dx$ $= \int_{-2}^{2} \frac{2}{\sqrt{4-x^2}} dx - \int_{-2}^{2} \frac{x}{\sqrt{4-x^2}} dx$ $= 2\int_{0}^{2} \frac{2}{\sqrt{4-x^2}} dx - 0$ [$\frac{2}{\sqrt{4-x^2}}$is even, $\frac{x}{\sqrt{4-x^2}}$ is odd] $= 4\int_{0}^{2} \frac{1}{\sqrt{4-x^2}} dx$ $= 4 \sin^{-1} \frac{x}{2}\Big\vert_{0}^{2}$ $= 2\pi$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(b) **Find :** $$\int \frac{1}{x\,[(\log x)^2 - 3\log x - 4]}\, dx$$	
Sol.	Let $logx = t \Longrightarrow \frac{1}{x} dx = dt$ The given integral becomes $= \int \frac{1}{t^2-3t-4} dt$ $= \int \frac{1}{(t-\frac{3}{2})^2 - (\frac{5}{2})^2} dx$ $= \frac{1}{5} log \left\vert\frac{t-4}{t+1}\right\vert + C$ $= \frac{1}{5} log \left\vert\frac{logx-4}{logx+1}\right\vert + C$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$
	Section D	
32.	Find the value of p for which the lines $\vec{r} = \lambda\hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k}$ and $\vec{r} = \hat{i} - 3\mu\hat{j} + (p\mu + 7)\hat{k}$ are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.	

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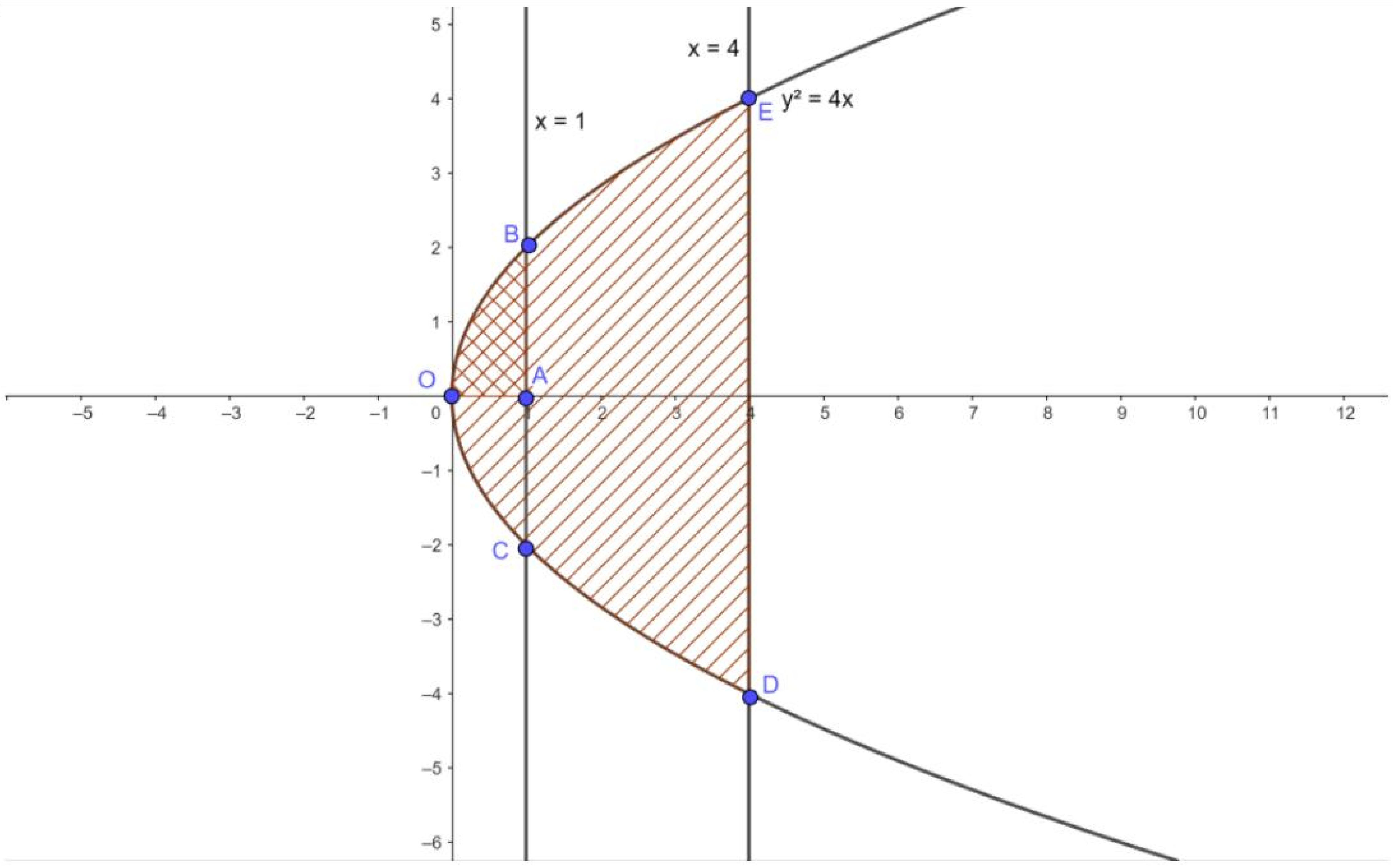
Sol.	Given lines are $\vec{r} = \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{r} = \hat{i} + 7\hat{k} + \mu(-3\hat{j} + p\hat{k})$ The lines are perpendicular $(\hat{i} + 2\hat{j} + 3\hat{k}).(-3\hat{j} + p\hat{k}) = 0 \Rightarrow -6 + 3p = 0 \Rightarrow p = 2$ The coordinates of any point on the two lines are $(\lambda, 2\lambda + 1, 3\lambda + 2)$ and $(1, -3\mu, 2\mu + 7)$ For the point of intersection, we must have $\lambda = 1, 2\lambda + 1 = -3\mu, 3\lambda + 2 = 2\mu + 7$ for some λ and μ. Solving first two equations, we get, $\lambda = 1, \mu = -1$, which satisfies the third equation as $5 = 5$ is true. Hence, the lines intersect each other. The Point of intersection is (1, 3, 5)	1 1 1 1 1
33.	(a) If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find $(AB)^{-1}$. Also, find $\lvert (AB)^{-1} \rvert$.	
Sol	We know that $(AB)^{-1} = B^{-1}A^{-1}$ $\lvert A \rvert = 5(-1) + 4(1) = -1 \neq 0$. Hence, A^{-1} exists. Cofactors of the elements of A are: $A_{11} = -1, A_{12} = 0, A_{13} = 1$ $A_{21} = 8, A_{22} = 1, A_{23} = -10$ $A_{31} = -12, A_{32} = -2, A_{33} = 15$ adj $A = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$ $A^{-1} = \frac{adj\ A}{\lvert A \rvert} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$ $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix} =$ $\begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$ $\lvert (AB)^{-1} \rvert = \lvert B^{-1}A^{-1} \rvert = \lvert B^{-1} \rvert \lvert A^{-1} \rvert$ $= 1 \times \frac{1}{-1} = -1$	1 2 $\frac{1}{2}$ 1 $\frac{1}{2}$

These answers are meant to be used by evaluators.

	OR	
	(b) Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, find A^{-1}. Use it to solve the following system of equations : $x + y + z = 1$ $2x + 3y + 2z = 2$ $x + y + 2z = 4$	
Sol.	$\lvert A\rvert = 1(4) - 1(2) + 1(-1) = 1$ Cofactors of the elements of A are: $A_{11} = 4, A_{12} = -2, A_{13} = -1$ $A_{21} = -1, A_{22} = 1, A_{23} = 0$ $A_{31} = -1, A_{32} = 0, A_{33} = 1$ $\therefore \text{adj } A = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ $A^{-1} = \frac{adj\ A}{\lvert A\rvert} = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ Given system of equations can be written as AX = B, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ $X = A^{-1}B = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$ $x = -2, y = 0, z = 3$	1 2 $\frac{1}{2}$ 1 $\frac{1}{2}$
34.	If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.	

Sol.	 $A_1 =$ Area (region OABO) $= \int_0^1 2\sqrt{x}\, dx = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{4}{3}$ $A_2 =$ Area (region ODEO) $= 2\int_0^4 2\sqrt{x}\, dx = 4 \times \frac{2}{3}[2^3] = \frac{64}{3}$ $A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1:16$	2 1 1 1
35.	(a) Show that a function $f : R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.	
Sol.	Let $f(x_1) = f(x_2)$ *for some* $x_1, x_2 \in R$ Then $\frac{2x_1}{1+{x_1}^2} = \frac{2x_2}{1+{x_2}^2}$ $\Rightarrow x_1 + x_1{x_2}^2 = x_2 + {x_1}^2 x_2$ $\Rightarrow (x_1 - x_2) - x_1 x_2 (x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)(1 - x_1 x_2) = 0$ $\Rightarrow x_1 - x_2 = 0$ or $1 - x_1 x_2 = 0$ $\Rightarrow x_1 = x_2$ or $x_1 x_2 = 1$ so if $x_1 x_2 = 1, x_1 \neq x_2$ Hence f is not one -one Let y = f(x) where $x \in R$ Then $y = \frac{2x}{1+x^2}$. Here, for x = 0, y = 0 If $y \neq 0$, then $y = \frac{2x}{1+x^2}$ $\Rightarrow yx^2 - 2x + y = 0$	2

These answers are meant to be used by evaluators.

	$\Rightarrow x = \frac{2 \pm \sqrt{4-4y^2}}{2y}$ For x to be real, $4 - 4y^2 \geq 0$ $\Rightarrow y^2 \leq 1$ $\Rightarrow -1 \leq y \leq 1$ Hence, range = $[-1,1] \neq codomain$ Hence, f is not onto. For the given function to become onto, $A = [-1,1]$	2 1
	OR	
	(b) A relation R is defined on N × N (where N is the set of natural numbers) as : (a, b) R (c, d) ⇔ a – c = b – d Show that R is an equivalence relation.	
Sol.	Let $(a,b) \in N \times N$ We have $a - a = b - b$ This implies that (a, b) R (a, b) $\forall (a,b) \in N \times N$ Hence R is reflexive Let (a, b) R (c, d) for some $(a,b),(c,d) \in N \times N$ Then $a - c = b - d$ $\Rightarrow c - a = d - b$ ⟹(c, d) R (a, b) Hence, R is symmetric. Let (a, b) R (c, d), (c, d) R (e, f) for some $(a,b),(c,d),(e,f) \in N \times N$ Then $a - c = b - d, c - e = d - f$ $\Rightarrow a - c + c - e = b - d + d - f$ $\Rightarrow a - e = b - f$ ⟹(a, b) R (e, f) Hence, R is transitive Thus, R is an equivalence relation.	$1\frac{1}{2}$ $1\frac{1}{2}$ 2

These answers are meant to be used by evaluators.

Section E

36.

The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

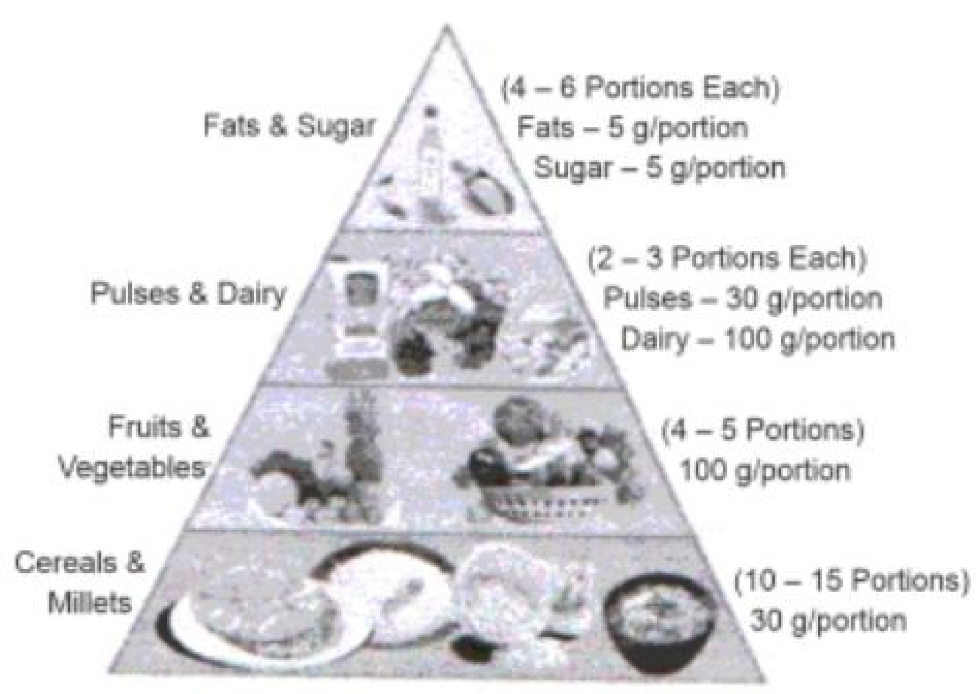


Figure-1

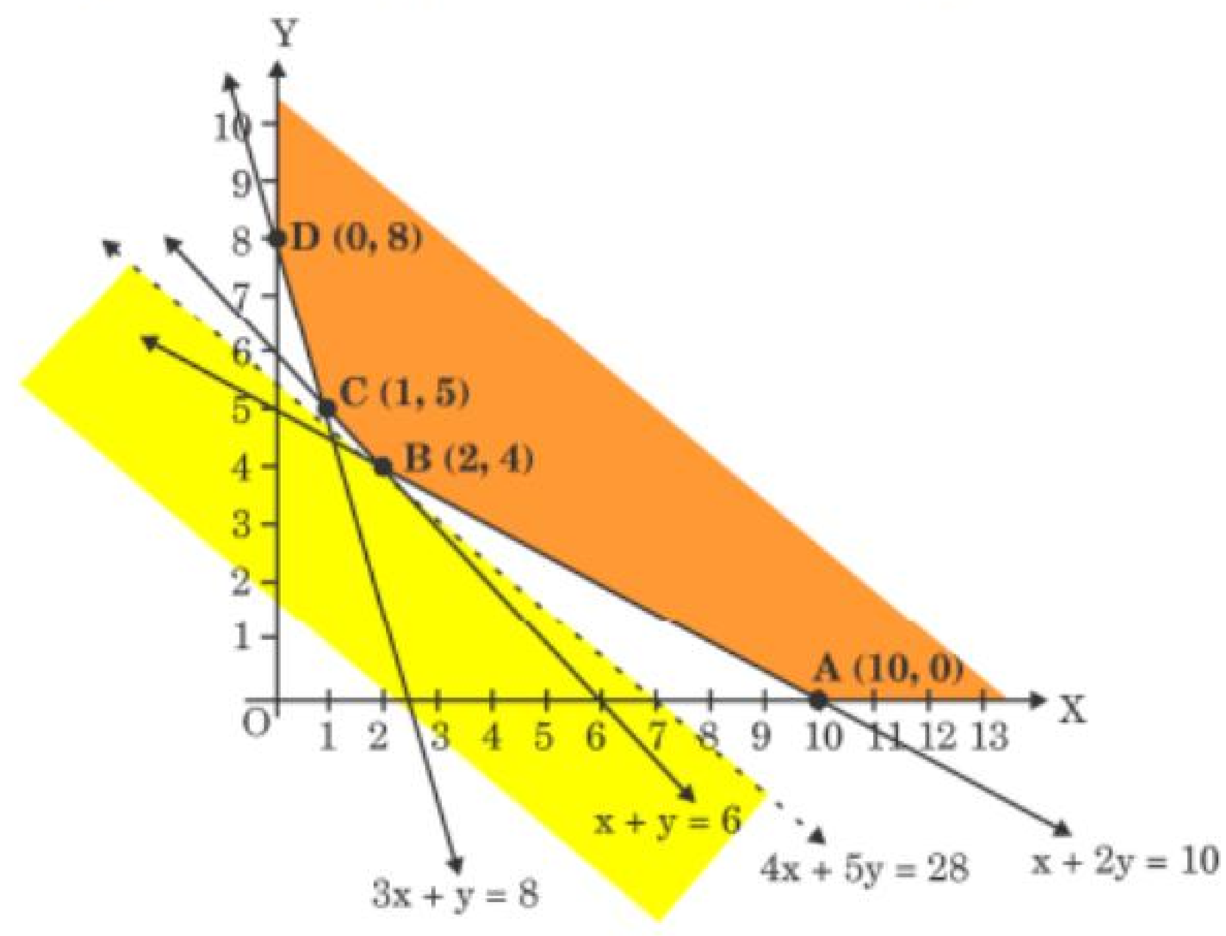


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions :

(i) Identify and write all the constraints which determine the given feasible region in Figure-2.

(ii) If the objective is to minimize cost Z = 16x + 20y, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.

Sol.

(i)Constraints are $x + 2y \geq 10$

$x + y \geq 6$ $\quad 1\frac{1}{2}$

$3x + y \geq 8$

$x \geq 0$ $\quad \frac{1}{2}$

$y \geq 0$

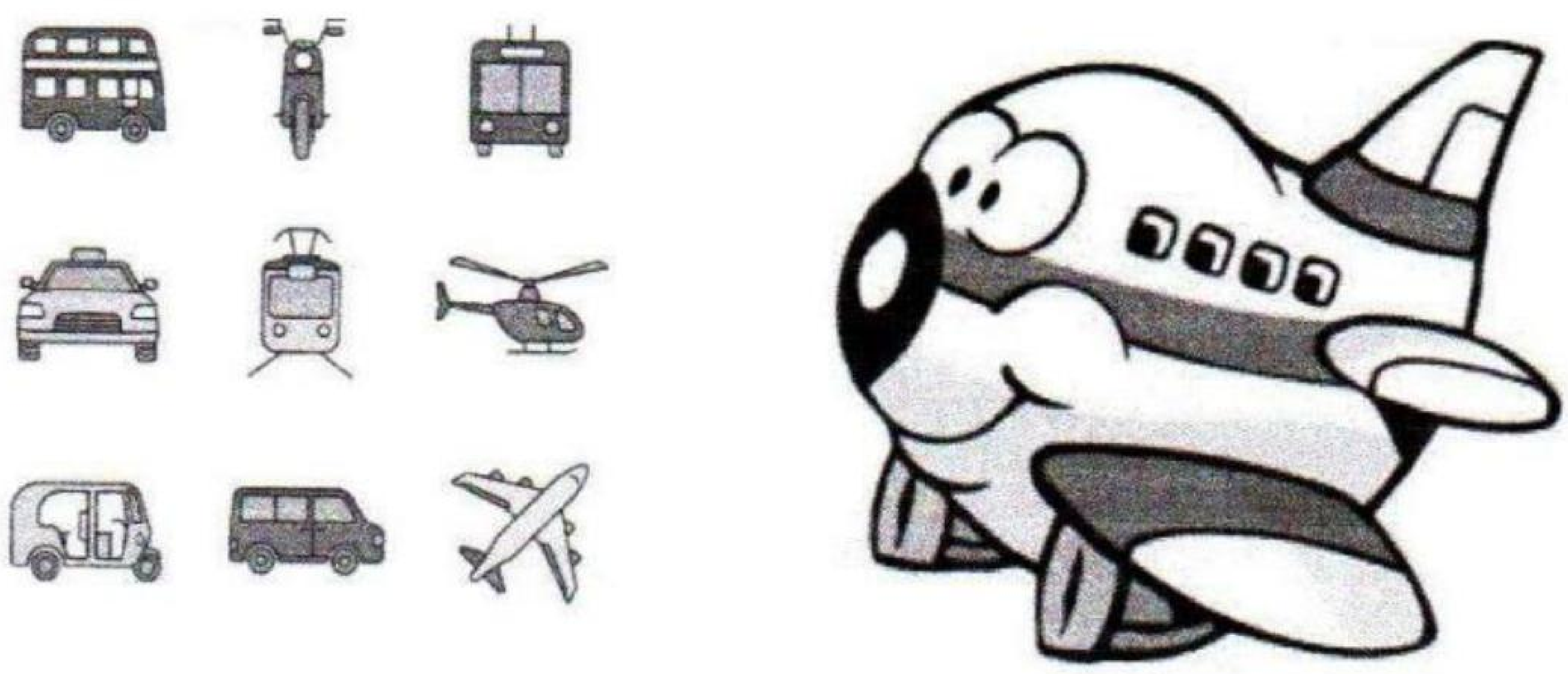
(ii)

Corner points	Value of Z = 16x + 20y
A (10, 0)	160
B (2, 4)	112
C (1, 5)	116
D (0, 8)	160

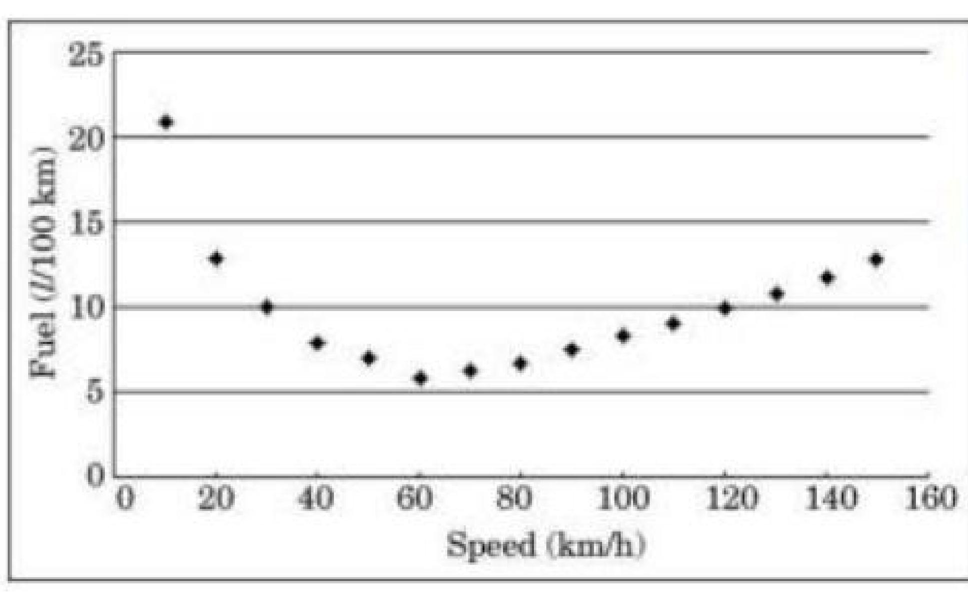
$1\frac{1}{2}$

The minimum cost is ₹112 $\quad \frac{1}{2}$

*These answers are meant to be used by evaluators.

37.	Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.  Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive. Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey. On the basis of the above information, answer the following questions : (i) Find the probability that the airplane will not crash. (ii) Find $P(A \mid E_1) + P(A \mid E_2)$. (iii) (a) Find P(A). **OR** (iii) (b) Find $P(E_2 \mid A)$.	
Sol.	(i) $P(E_2) = 1 - 0.0000001$ $= 0.9999999$ (ii) $P(A/E_1) + P(A/E_2) = \frac{95}{100} + 1 = \frac{195}{100}$	 1 1
	(iii)(a) $P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)$ $= \frac{1}{10000000} \times \frac{95}{100} + \frac{9999999}{10000000} \times 1$ $= \frac{95+999999900}{1000000000} = \frac{999999995}{1000000000}$	1 1

*These answers are meant to be used by evaluators.

	OR	
	(iii)(b) $P(E_2/A) = \frac{P(E_2)\times P(A/E_2)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)}$ $= \frac{\frac{9999999}{10000000}}{\frac{999999995}{1000000000}} = \frac{999999900}{999999995}$	1 1
38.	Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.  The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$. On the basis of the above information, answer the following questions : (i) Find F, when V = 40 km/h. (ii) Find $\frac{dF}{dV}$. (iii) (a) Find the speed V for which fuel consumption F is minimum. **OR** (iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0{\cdot}01$.	
Sol.	(i) When V = 40 km/h, F = 36/5 ℓ/100km (ii) $\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}$ (iii)(a) $\frac{dF}{dV} = 0$ $\Rightarrow$V = 62.5 km/h	1 1 1

These answers are meant to be used by evaluators.

	$\frac{d^2F}{dV^2} = \frac{1}{250} > 0$ at V = 62.5 km/h Hence, F is minimum when V = 62.5 km/h	$\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(iii) (b) $\frac{dF}{dV} = -0.01$ $\Rightarrow \frac{V}{250} - \frac{1}{4} = \frac{-1}{100}$ $\Rightarrow V = 60$ km/h $F = \frac{60^2}{500} - \frac{60}{4} + 14 = 6.2\,\ell/100km$ Quantity of fuel required for 600 km $= 6.2 \times 6\,\ell = 37.2\,\ell$	1 $\frac{1}{2}$ $\frac{1}{2}$