

**Marking Scheme**  
**Strictly Confidential**  
 (For Internal and Restricted use only)  
 Senior School Certificate Examination, 2024  
**MATHEMATICS PAPER CODE 65/4/2**

**General Instructions:**

<b>1</b>	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
<b>2</b>	<b>“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”</b>
<b>3</b>	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. <b>However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.</b>
<b>4</b>	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
<b>5</b>	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
<b>6</b>	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. <b>This is most common mistake which evaluators are committing.</b>
<b>7</b>	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
<b>8</b>	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
<b>9</b>	<b><u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u></b>

10	<b><u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u></b>
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- <ul style="list-style-type: none"> <li>● Leaving answer or part thereof unassessed in an answer book.</li> <li>● Giving more marks for an answer than assigned to it.</li> <li>● Wrong totaling of marks awarded on an answer.</li> <li>● Wrong transfer of marks from the inside pages of the answer book to the title page.</li> <li>● Wrong question wise totaling on the title page.</li> <li>● Wrong totaling of marks of the two columns on the title page.</li> <li>● Wrong grand total.</li> <li>● Marks in words and figures not tallying/not same.</li> <li>● Wrong transfer of marks from the answer book to online award list.</li> <li>● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)</li> <li>● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.</li> </ul>
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the “ <b>Guidelines for spot Evaluation</b> ” before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.



5.	If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$ , then value of $k$ is : (A) 3                      (B) 5                      (C) 7                      (D) 9	
<b>Ans:</b>	<b>(B) 5</b>	<b>1</b>
6.	If $e^{x^2y} = c$ , then $\frac{dy}{dx}$ is : (A) $\frac{xe^{x^2y}}{2y}$ (B) $\frac{-2y}{x}$ (C) $\frac{2y}{x}$ (D) $\frac{x}{2y}$	
<b>Ans:</b>	<b>(B) <math>\frac{-2y}{x}</math></b>	<b>1</b>
7.	The value of constant $c$ that makes the function $f$ defined by $f(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4 \end{cases}$ continuous for all real numbers is (A) -2                      (B) -1                      (C) 0                      (D) 2	
<b>Ans:</b>	<b>(A) -2</b>	<b>1</b>
8.	The value of $\int_{-1}^1  x  dx$ is: (A) -2                      (B) -1                      (C) 1                      (D) 2	
<b>Ans:</b>	<b>(C) 1</b>	<b>1</b>
9.	The number of arbitrary constants in the particular solution of the differential equation $\log \left( \frac{dy}{dx} \right) = 3x + 4y; y(0) = 0$ is/are (A) 2    (B) 1 (C) 0    (D) 3	
<b>Ans:</b>	<b>(C) 0</b>	<b>1</b>
10.	If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is (A) 0                      (B) 5                      (C) 10                      (D) 25	
<b>Ans:</b>	<b>(D) 25</b>	<b>1</b>

<b>11.</b>	If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ , then the value of $I - A + A^2 - A^3 + \dots$ is  (A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	
<b>Ans:</b>	(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$	<b>1</b>
<b>12.</b>	Given that $A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ , matrix A is:  (A) $7 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (C) $\frac{1}{7} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (D) $\frac{1}{49} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$	
<b>Ans:</b>	(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$	<b>1</b>
<b>13.</b>	The integrating factor of the differential equation $(x + 2y^2) \frac{dy}{dx} = y$ ( $y > 0$ ) is :  (A) $\frac{1}{x}$ (B) $x$ (C) $y$ (D) $\frac{1}{y}$	
<b>Ans:</b>	(D) $\frac{1}{y}$	<b>1</b>
<b>14.</b>	A vector perpendicular to the line $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ is :  (A) $5\hat{i} + \hat{j} + 6\hat{k}$ (B) $\hat{i} + 3\hat{j} + 5\hat{k}$ (C) $2\hat{i} - 2\hat{j}$ (D) $9\hat{i} - 3\hat{j}$	
<b>Ans:</b>	(B) $\hat{i} + 3\hat{j} + 5\hat{k}$	<b>1</b>
<b>15.</b>	The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ , $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of  (A) an equilateral triangle      (B) an obtuse-angled triangle (C) an isosceles triangle      (D) a right-angled triangle	
<b>Ans:</b>	(D) a right-angled triangle	<b>1</b>
<b>16.</b>	Let $\vec{a}$ be any vector such that $ \vec{a}  = a$ . The value of $ \vec{a} \times \hat{i} ^2 +  \vec{a} \times \hat{j} ^2 +  \vec{a} \times \hat{k} ^2$ is:  (A) $a^2$ (B) $2a^2$ (C) $3a^2$ (D) $0$	
<b>Ans:</b>	(B) $2a^2$	<b>1</b>

17.	If $\vec{a}$ and $\vec{b}$ are two vectors such that $ \vec{a} =1,  \vec{b} =2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$ , then the angle between $2\vec{a}$ and $-\vec{b}$ is:  (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$	
<b>Ans:</b>	<b>(C) <math>\frac{5\pi}{6}</math></b>	<b>1</b>
18.	The function $f(x) = kx - \sin x$ is strictly increasing for  (A) $k > 1$ (B) $k < 1$ (C) $k > -1$ (D) $k < -1$	
<b>Ans:</b>	<b>(A) <math>k &gt; 1</math></b>	<b>1</b>
<p><b>ASSERTION-REASON BASED QUESTIONS</b></p> <p>Questions No. 19 &amp; 20, are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A). (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.</p>		
19.	<p><b>Assertion (A) :</b> The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of <math>Z = x + 2y</math> occurs at infinite points.</p> <p><b>Reason (R) :</b> The optimal solution of a LPP having bounded feasible region must occur at corner points.</p>	
<b>Ans:</b>	<b>(B) Both A and R are true but R is not the correct explanation of A.</b>	<b>1</b>
20.	<p><b>Assertion (A) :</b> The relation <math>R = \{(x, y) : (x + y) \text{ is a prime number and } x, y \in \mathbb{N}\}</math> is not a reflexive relation.</p> <p><b>Reason (R) :</b> The number '2n' is composite for all natural numbers n.</p>	
<b>Ans:</b>	<b>(C) Assertion (A) is true, but Reason (R) is false.</b>	<b>1</b>

<b>SECTION B</b>		
<b>In this section there are 5 very short answer type questions of 2 marks each.</b>		
<b>21.</b>	The volume of a cube is increasing at the rate of 6 cm <sup>3</sup> /s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?	
<b>Sol.</b>	<p>Given, <math>\frac{dV}{dt} = 6 \text{ cm}^3 / \text{sec}</math>. Since, <math>V = x^3</math></p> $\frac{dV}{dt} = 3x^2 \frac{dx}{dt} \Rightarrow 6 = 3x^2 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{2}{x^2} \text{ cm / sec}$ <p>Now, Surface Area = <math>S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \frac{dx}{dt} = 3 \text{ cm}^2 / \text{sec}</math></p>	1 1
<b>22(a).</b>	Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$ , where $-\frac{\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.	
<b>Sol.</b>	$y = \tan^{-1}\left[\frac{\cos x}{1 - \sin x}\right] = \tan^{-1}\left[\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{(\cos \frac{x}{2} - \sin \frac{x}{2})^2}\right]$ $y = \tan^{-1}\left[\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] = \left(\frac{\pi}{4} + \frac{x}{2}\right)$	½ 1 ½
<b>OR</b>		
<b>22(b).</b>	Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ .	
<b>Sol.</b>	$\tan^{-1}(1) + \pi - \cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \frac{\pi}{4}$ $= \frac{2\pi}{3}$	1 ½ ½
<b>23.</b>	Show that $f(x) = \frac{4 \sin x}{2 + \cos x} - x$ is an increasing function of $x$ in $\left[0, \frac{\pi}{2}\right]$ .	
<b>Sol.</b>	$f(x) = \frac{4 \sin x}{2 + \cos x} - x \Rightarrow f'(x) = \frac{\cos x(4 - \cos x)}{(2 + \cos x)^2}$ <p>when <math>x \in \left[0, \frac{\pi}{2}\right]</math>, <math>\cos x \geq 0 \Rightarrow \frac{\cos x(4 - \cos x)}{(2 + \cos x)^2} \geq 0</math></p> <p>Thus, <math>f</math> is increasing on <math>\left[0, \frac{\pi}{2}\right]</math>.</p>	1 1

<b>24 (a).</b>	If $y = \cos^3(\sec^2 2t)$ , find $\frac{dy}{dt}$ .	
<b>Sol.</b>	$y = \cos^3(\sec^2 2t)$ $\Rightarrow \frac{dy}{dt} = 3 \cos^2(\sec^2 2t)[- \sin(\sec^2 2t)] \times \frac{d(\sec^2 2t)}{dt}$ $\Rightarrow \frac{dy}{dt} = -3 \cos^2(\sec^2 2t) \cdot \sin(\sec^2 2t) \times 2 \sec 2t \cdot \sec 2t \tan 2t \cdot 2$ $\therefore \frac{dy}{dt} = -12 \cos^2(\sec^2 2t) \times \sin(\sec^2 2t) \times \sec^2 2t \times \tan 2t.$	 $\frac{1}{2}$  <b>1</b>  $\frac{1}{2}$
	<b>OR</b>	
<b>24 (b).</b>	If $x^y = e^{x-y}$ , prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$ .	
<b>Sol.</b>	<p>As, <math>x^y = e^{x-y} \Rightarrow \log(x^y) = \log(e^{x-y})</math></p> $\Rightarrow y \log x = (x - y) \Rightarrow y = \frac{x}{1 + \log x}$ <p>Now, Differentiating both the sides wrt <math>x</math></p> $\frac{dy}{dx} = \frac{(\log x + 1) \cdot 1 - x \left(\frac{1}{x}\right)}{(\log x + 1)^2} = \frac{\log x}{(1 + \log x)^2}$	   <b>1</b>   <b>1</b>
<b>25.</b>	Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \cdot \log\left(\frac{1-x}{1+x}\right) dx$	
<b>Sol.</b>	<p>Let <math>f(x) = \cos x \cdot \log\left(\frac{1-x}{1+x}\right)</math></p> <p>So, <math>f(-x) = \cos(x) \cdot \log\left(\frac{1+x}{1-x}\right) = -f(x)</math> [Odd function]</p> <p>Thus, <math>I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \cdot \log\left(\frac{1-x}{1+x}\right) dx = 0</math></p>	   <b>1</b>   <b>1</b>
	<b>SECTION C</b>	
	<b>In this section there are 6 short answer type questions of 3 marks each.</b>	
<b>26.</b>	Given that $x^y + y^x = a^b$ , where $a$ and $b$ are positive constants, find $\frac{dy}{dx}$ .	



<b>Sol.</b>	$\text{Let } u = x^y \Rightarrow \frac{du}{dx} = x^y \left( \frac{y}{x} + \log x \cdot \frac{dy}{dx} \right), v = y^x \Rightarrow \frac{dv}{dx} = y^x \left( \frac{x}{y} \frac{dy}{dx} + \log y \right)$ $\text{Since, } u + v = a^b \Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$ $\therefore \frac{dy}{dx} = \frac{-(x^{y-1}y + y^x \log y)}{(x^y \log x + y^{x-1}x)}$	<p><b>1 + 1</b></p> <p><b>1</b></p>
<b>27(a).</b>	<p>Find the particular solution of the differential equation <math>\frac{dy}{dx} = y \cot 2x</math>,</p> <p>given that <math>y\left(\frac{\pi}{4}\right) = 2</math>.</p>	
<b>Sol.</b>	$\frac{dy}{dx} = y \cot 2x \Rightarrow \int \frac{dy}{y} = \int \cot 2x dx$ $\Rightarrow \log  y  = \frac{1}{2} \log  \sin 2x  + \log c$ <p>Thus, <math>y = c \cdot \sqrt{\sin 2x}</math></p> <p>when <math>y\left(\frac{\pi}{4}\right) = 2</math>, gives <math>c = 2</math></p> <p><math>\therefore y = 2\sqrt{\sin 2x}</math> is the required Particular solution of given D.E.</p>	<p><b>1</b></p> <p><b>1</b></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
<b>OR</b>		
<b>27(b).</b>	<p>Find the particular solution of the differential equation</p> $(x e^{\frac{y}{x}} + y) dx = x dy, \text{ given that } y = 1 \text{ when } x = 1.$	
<b>Sol.</b>	$\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x} = f\left(\frac{y}{x}\right) \text{ so, its a homogeneous differential equation}$ <p>Let <math>y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}</math></p> <p>Now, <math>v + x \frac{dv}{dx} = e^v + v</math></p> $\Rightarrow \int e^{-v} dv = \int \frac{1}{x} dx$ $\Rightarrow -e^{-v} = \log  x  + c \Rightarrow -e^{-\frac{y}{x}} = \log  x  + c \dots (1)$ <p>Now, <math>x = 1, y = 1</math>, gives <math>c = -e^{-1}</math></p> <p>Thus, <math>\boxed{\log  x  + e^{-\frac{y}{x}} = e^{-1}}</math></p>	<p><b>1</b></p> <p><math>\frac{1}{2}</math></p> <p><b>1</b></p> <p><math>\frac{1}{2}</math></p>
<b>28.</b>	<p>Find: <math>\int \frac{2x+3}{x^2(x+3)} dx</math></p>	

<b>Sol.</b>	$I = \int \frac{2x+3}{x^2(x+3)} dx = \int \frac{x+3}{x^2(x+3)} dx + \int \frac{x}{x^2(x+3)} dx$ $I = \int \frac{1}{x^2} dx + \frac{1}{3} \int \frac{x+3-x}{x(x+3)} dx = \int \frac{1}{x^2} dx + \frac{1}{3} \int \frac{1}{x} dx - \frac{1}{3} \int \frac{1}{x+3} dx$ $I = \frac{-1}{x} + \frac{1}{3} \log x  - \frac{1}{3} \log x+3  + c$	   <b>1 ½</b>   <b>1 ½</b>
<b>29(a).</b>	<p>A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.</p>	
<b>Sol.</b>	<p>Let <math>E_1</math> be the event of lost card is King,  <math>E_2</math> be the event of lost card not a King and  <math>A</math> be the event of drawing a King from remaining 51 cards. }  so, <math>P(E_1) = \frac{1}{13}</math>, <math>P(E_2) = \frac{12}{13}</math>, <math>P(A E_1) = \frac{3}{51}</math>, <math>P(A E_2) = \frac{4}{51}</math> }  Now, Required probability is <math>P(E_1   A)</math>,</p> $P(E_1   A) = \frac{P(A E_1) \times P(E_1)}{P(A E_1) \times P(E_1) + P(A E_2) \times P(E_2)} = \frac{\frac{1}{13} \times \frac{3}{51}}{\frac{1}{13} \times \frac{3}{51} + \frac{12}{13} \times \frac{4}{51}} = \frac{1}{17}$	   <b>½</b>   <b>1 ½</b>   <b>1</b>
<b>OR</b>		
<b>29(b).</b>	<p>A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.</p>	
<b>Sol.</b>	<p>Let <math>P(1)=P(3)=P(5) = p</math>, so <math>P(2)=P(4)=P(6) = 2p</math>  As, <math>P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1 \Rightarrow 9p = 1 \Rightarrow p = \frac{1}{9}</math>  <math>P(\text{Getting } 6) = \frac{2}{9}</math>, <math>P(\text{Not getting six}) = \frac{7}{9}</math>  Let <math>X</math> represents the Number of sixes  Possible values of <math>X</math> are 0, 1 or 2  Now, <math>P(X=0) = \frac{7}{9} \times \frac{7}{9} = \frac{49}{81}</math>, <math>P(X=1) = 2 \times \frac{7}{9} \times \frac{2}{9} = \frac{28}{81}</math>, <math>P(X=2) = \frac{2}{9} \times \frac{2}{9} = \frac{4}{81}</math>  Required probability distribution of number of sixes is</p>	   <b>½</b>      <b>½</b>

$X$	0	1	2
$P(X)$	$\frac{49}{81}$	$\frac{28}{81}$	$\frac{4}{81}$

$$\text{Mean of } X = \sum_{i=1}^3 X_i P(X_i) = 0 + \frac{28}{81} + \frac{8}{81} = \frac{36}{81} = \frac{4}{9}$$

1½

½

30.

Solve the following linear programming problem graphically :

$$\text{Maximise } Z = 2x + 3y$$

subject to the constraints :

$$x + y \leq 6$$

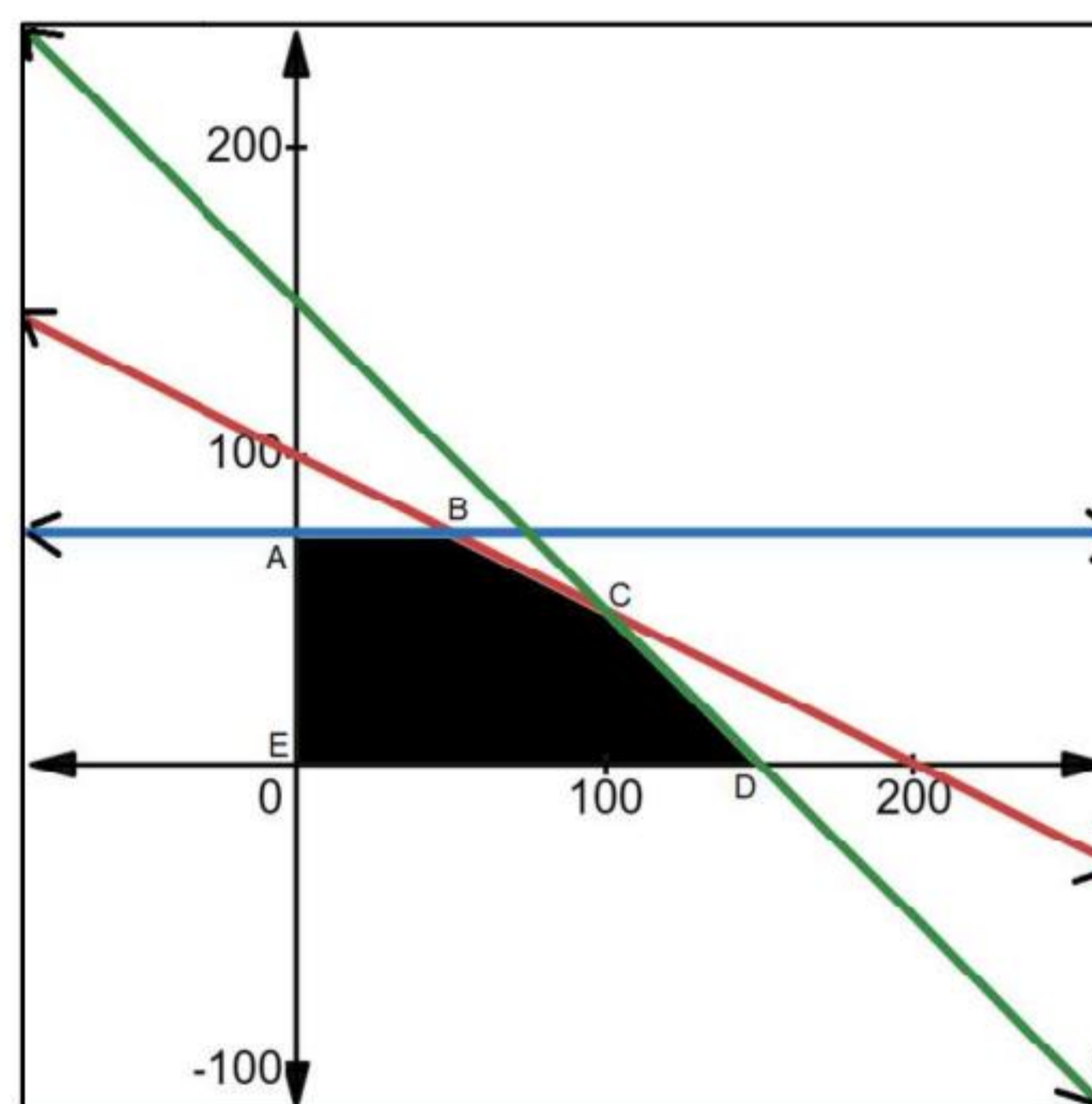
$$x \geq 2$$

$$y \leq 3$$

$$x, y \geq 0$$

Sol.

On plotting the graph of  $x + 2y \leq 200, x + y \leq 150, y \leq 75, \&x \geq 0, y \geq 0$  we get the following graph and common shaded region is the region ABCDE.



Now, Corner points of the common shaded region are  $A(0, 75), B(50, 75), C(100, 50), D(150, 0) \& E(0, 0)$ . Thus,

Corner points	Value of $Z = x + 3y$
$A(0, 75)$	225
$B(50, 75)$	275
$C(100, 50)$	250
$D(150, 0)$	150
$E(0, 0)$	0

So, Maximum Value of  $Z$  is 275 at  $x = 50 \& y = 75$ .

For  
correct  
Graph  
1½

For  
correct  
Table  
1

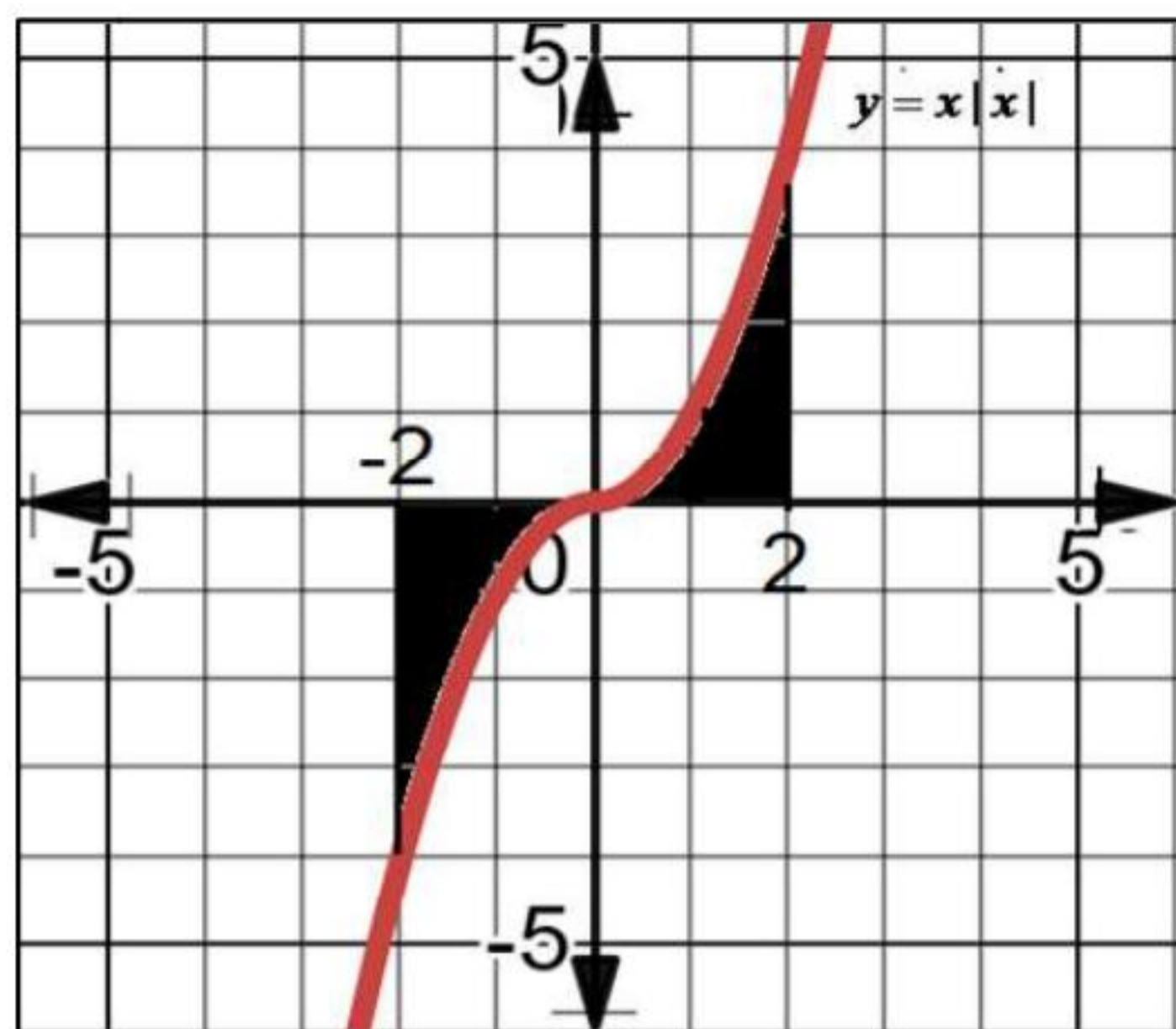
½

<b>31(a).</b>	Evaluate: $\int_0^{\frac{\pi}{4}} \frac{x}{1 + \cos 2x + \sin 2x} dx$	
<b>Sol.</b>	$I = \int_0^{\frac{\pi}{4}} \frac{x}{1 + \cos 2x + \sin 2x} dx \dots(1)$ <p>On applying <math>\int_0^a f(x) dx = \int_0^a f(a-x) dx</math>,</p> $\text{we get } I = \int_0^{\frac{\pi}{4}} \frac{\frac{\pi}{4} - x}{1 + \cos 2x + \sin 2x} dx \dots(2)$ <p>On adding Eq. (1) and (2), we get <math>2I = \frac{\pi}{4} \int_0^{\frac{\pi}{4}} \frac{1}{1 + \cos 2x + \sin 2x} dx</math></p> $I = \frac{\pi}{16} \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x + \sin x \cos x} dx = \frac{\pi}{16} \int_0^{\frac{\pi}{4}} \frac{\sec^2 x dx}{1 + \tan x}$ $I = \frac{\pi}{16} (\log  1 + \tan x )_0^{\frac{\pi}{4}}$ $I = \frac{\pi}{16} \log 2$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><b>1</b></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
<b>OR</b>		
<b>31(b).</b>	Find: $\int e^x \left[ \frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}} \right] dx$	
<b>Sol.</b>	$I = \int e^x \left( \frac{x}{\sqrt{1+x^2}} + \frac{1}{(1+x^2)^{\frac{3}{2}}} \right) dx$ <p>Let <math>f(x) = \frac{x}{\sqrt{1+x^2}}</math>,</p> $f'(x) = \frac{\sqrt{1+x^2} - x \frac{x}{\sqrt{1+x^2}}}{1+x^2} = \frac{1+x^2 - x^2}{(1+x^2)\sqrt{1+x^2}} = \frac{1}{(1+x^2)^{\frac{3}{2}}}$ <p>On applying <math>\int e^x (f(x) + f'(x)) dx = e^x f(x) + c</math>, <math>I = e^x \frac{x}{\sqrt{1+x^2}} + c</math></p>	<p><math>\frac{1}{2}</math></p> <p><b>1</b> <math>\frac{1}{2}</math></p> <p><b>1</b></p>
<b>SECTION D</b>		
<b>In this section there are 4 long answer type questions of 5 marks each.</b>		
<b>32(a).</b>	Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{1\}$ . Consider the function $f : A \rightarrow B$ , defined by $f(x) = \frac{x-3}{x-5}$ . Show that $f$ is one-one and onto.	

<b>Sol.</b>	$\left. \begin{aligned} & \text{Let } f(x_1) = f(x_2), \text{ for some } x_1, x_2 \in A \\ & \Rightarrow \frac{x_1 - 3}{x_1 - 5} = \frac{x_2 - 3}{x_2 - 5} \\ & \Rightarrow (x_1 - 3)(x_2 - 5) = (x_2 - 3)(x_1 - 5) \\ & \Rightarrow x_1 = x_2, \text{ So } \underline{f \text{ is one-one Function.}} \end{aligned} \right\}$ $\left. \begin{aligned} & \text{Let } y = f(x) = \frac{x-3}{x-5} \Rightarrow y(x-5) = x-3 \\ & \Rightarrow yx - 5y = x - 3 \\ & \Rightarrow x = \frac{5y-3}{y-1}, \text{ We observe that } x \text{ is defined for all values of } y \text{ except } y = 1, \\ & \text{So, Range} = R - \{1\} \text{ and Co-domain is Given } R - \{1\} \quad [As, f : A \rightarrow B] \\ & \text{Since, Range} = \text{Co-domain, } f \text{ is onto Function.} \\ & \text{Thus, } f \text{ is one-one \& onto function.} \end{aligned} \right\}$	$2 \frac{1}{2}$
	<b>OR</b>	
<b>32(b).</b>	Check whether the relation S in the set of real numbers R defined by $S = \{(a, b) : \text{where } a - b + \sqrt{2} \text{ is an irrational number}\}$ is reflexive, symmetric or transitive.	
<b>Sol.</b>	$\left. \begin{aligned} & \underline{\text{Reflexive:}} \text{ For } a \in S \\ & \Rightarrow a - a + \sqrt{2} \text{ is irrational number} \\ & \Rightarrow \sqrt{2} \text{ is irrational number} \\ & \Rightarrow (a, a) \in S \\ & \text{Thus, S is } \underline{\text{Reflexive Relation.}} \end{aligned} \right\}$ $\left. \begin{aligned} & \underline{\text{Symmetric:}} \text{ Let } (a, b) \in S \Rightarrow a - b + \sqrt{2} \text{ is irrational number} \\ & \text{but } b - a + \sqrt{2} \text{ may not be irrational number} \\ & \text{For example, } (\sqrt{2}, 1) \in S \Rightarrow \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1 \text{ is irrational number} \\ & (1, \sqrt{2}) \notin S \text{ as } 1 - \sqrt{2} + \sqrt{2} = 1 \text{ is not irrational number} \\ & \therefore (b, a) \notin S, \text{ So S is } \underline{\text{NOT Symmetric Relation.}} \end{aligned} \right\}$ $\left. \begin{aligned} & \underline{\text{Transitive:}} \text{ Let } (a, b) \in S \Rightarrow a - b + \sqrt{2} \text{ is irrational number} \\ & \text{\& } (b, c) \in S \Rightarrow b - c + \sqrt{2} \text{ is irrational number} \\ & \text{but } a - c + \sqrt{2} \text{ may not be irrational number} \\ & \text{For example, } (1, \sqrt{3}) \in S \Rightarrow 1 - \sqrt{3} + \sqrt{2} \text{ is irrational number} \\ & (\sqrt{3}, \sqrt{2}) \in S \Rightarrow \sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3} \text{ is irrational number} \\ & \text{But } (1, \sqrt{2}) \notin S \text{ as } 1 - \sqrt{2} + \sqrt{2} = 1 \text{ is not irrational number} \\ & \therefore (a, c) \notin S, \text{ So S is } \underline{\text{NOT Transitive Relation.}} \\ & \text{Thus, S is Reflexive But Neither Symmetric nor Transitive Relation.} \end{aligned} \right\}$	$1 \frac{1}{2}$  $1 \frac{1}{2}$  $2$

33(a).	Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, -5).	
Sol.	<p>Equation of the given line in standard form is</p> $L_1 : \frac{x}{2} = \frac{y-3}{2} = \frac{z-1}{1}$ <p>Equation of the line parallel to <math>L_1</math> &amp; passing through (4, 0, -5) is</p> $L_2 : \frac{x-4}{2} = \frac{y}{2} = \frac{z+5}{1}$ <p>Vector Equation of Lines are <math>L_1 : \vec{r} = (0\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})</math></p> $L_2 : \vec{r} = (4\hat{i} + 0\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k})$ <p>Now, <math>\vec{a}_2 - \vec{a}_1 = (4\hat{i} + 0\hat{j} - 5\hat{k}) - (0\hat{i} + 3\hat{j} + \hat{k}) = (4\hat{i} - 3\hat{j} - 6\hat{k})</math></p> $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$ $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & -6 \\ 2 & 2 & 1 \end{vmatrix} = 9\hat{i} - 16\hat{j} + 14\hat{k}$ $ \vec{b}  = \sqrt{4+4+1} = 3$ <p>Thus, Distance between the lines is</p> $\text{S.D.} = \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} } = \frac{\sqrt{81+256+196}}{3} = \frac{\sqrt{533}}{3} \text{ units}$	<p><math>\frac{1}{2}</math></p> <p><b>1</b></p> <p><math>\frac{1}{2}</math></p> <p><b>1</b></p> <p><math>\frac{1}{2}</math></p> <p><b>1 <math>\frac{1}{2}</math></b></p>
<b>OR</b>		
33(b).	If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, -4, 7).	
	$L_1 : \frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2} \Rightarrow \text{direction ratio's of } L_1 = \langle -3, 2k, 2 \rangle$ $L_2 : \frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7} \Rightarrow \text{direction ratio's of } L_2 = \langle 3k, 1, -7 \rangle$ <p>Since <math>L_1 \perp L_2</math>,</p> $-9k + 2k - 14 = 0 \Rightarrow k = -2$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><b>1</b></p>

	<p>Thus, d.r.'s of <math>L_1 = \langle -3, -4, 2 \rangle</math>, d.r.'s of <math>L_2 = \langle -6, 1, -7 \rangle</math></p> <p>Now the vector perpendicular to both <math>L_1</math> &amp; <math>L_2</math> is given by</p> $\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 2 \\ -6 & 1 & -7 \end{vmatrix} = 26\hat{i} - 33\hat{j} - 27\hat{k}$ <p>Thus, Equation of the required line is</p> $\vec{r} = (3\hat{i} - 4\hat{j} + 7\hat{k}) + \lambda(26\hat{i} - 33\hat{j} - 27\hat{k})$	2 1
34.	<p>Use the product of matrices <math>\begin{pmatrix} 1 &amp; 2 &amp; -3 \\ 3 &amp; 2 &amp; -2 \\ 2 &amp; -1 &amp; 1 \end{pmatrix} \begin{pmatrix} 0 &amp; 1 &amp; 2 \\ -7 &amp; 7 &amp; -7 \\ -7 &amp; 5 &amp; -4 \end{pmatrix}</math> to solve the following system of equations :</p> $x + 2y - 3z = 6$ $3x + 2y - 2z = 3$ $2x - y + z = 2$	
Sol.	$AB = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix} = 7I$ $\text{Thus, } A^{-1} = \frac{1}{7}B = \frac{1}{7} \begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}$ <p>so, Given equation can be written into a matrix equation as</p> $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \Rightarrow X = A^{-1}.C$ $A \quad X = C$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 7 \\ -35 \\ -35 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ -5 \end{pmatrix}$ <p><math>\therefore x = 1, y = -5, z = -5</math></p>	2 1 $\frac{1}{2}$ 1 $\frac{1}{2}$
35(a).	<p>Sketch the graph of <math>y = x x </math> and hence find the area bounded by this curve, X-axis and the ordinates <math>x = -2</math> and <math>x = 2</math>, using integration.</p>	
Sol.		



$$\text{As, } y = x|x| = \begin{cases} -x^2, & x \leq 0 \\ x^2, & x \geq 0 \end{cases}$$

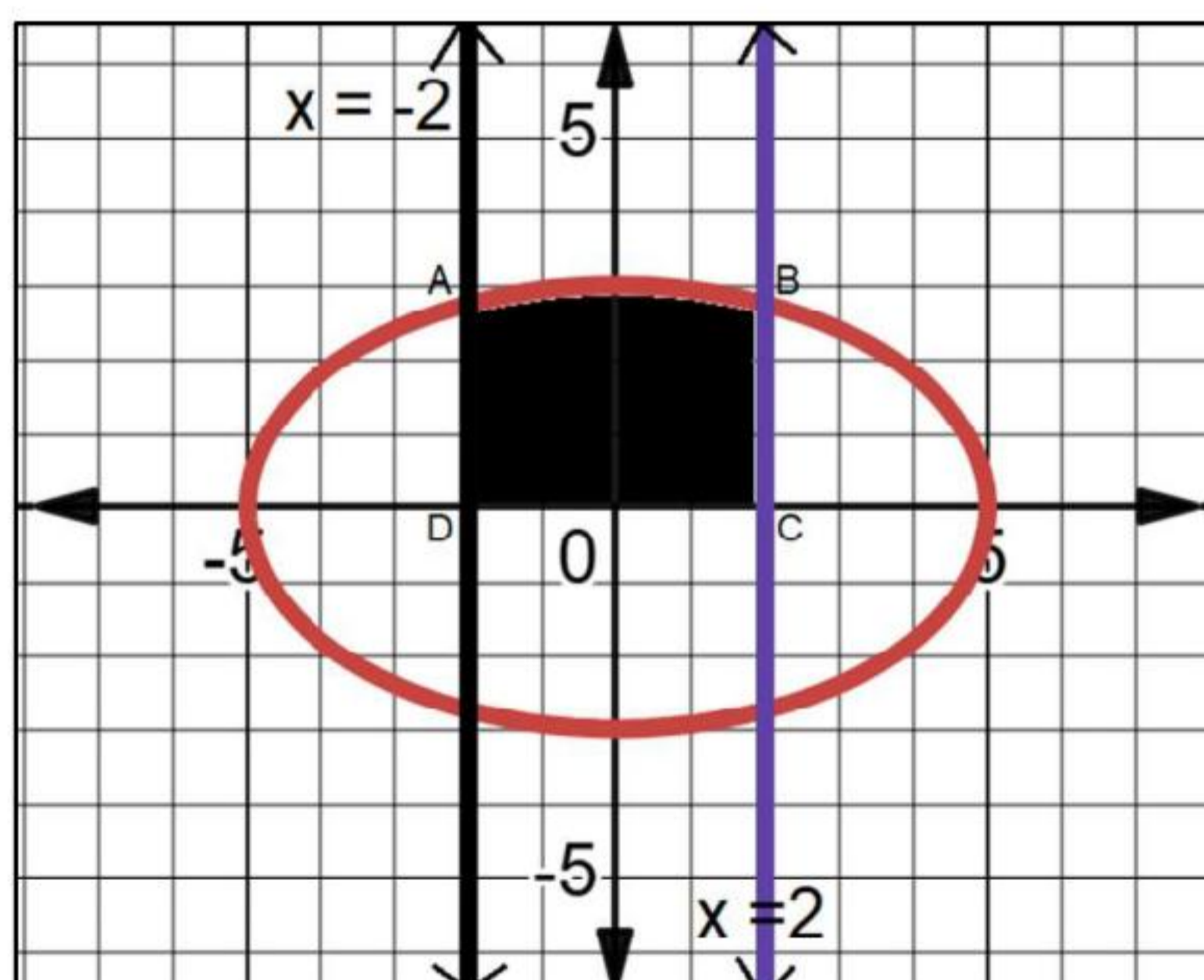
$$\begin{aligned} \text{Area of the shaded region} &= \int_{-2}^2 y \, dx = 2 \int_0^2 y \, dx = 2 \int_0^2 x^2 \, dx \\ &= 2 \left( \frac{x^3}{3} \right)_0^2 \\ &= 2 \left( \frac{8}{3} \right) = \frac{16}{3} \end{aligned}$$

OR

35(b).

Using integration, find the area bounded by the ellipse  $9x^2 + 25y^2 = 225$ , the lines  $x = -2$ ,  $x = 2$ , and the X-axis.

Sol.



For  
correct  
figure  
 $1 \frac{1}{2}$

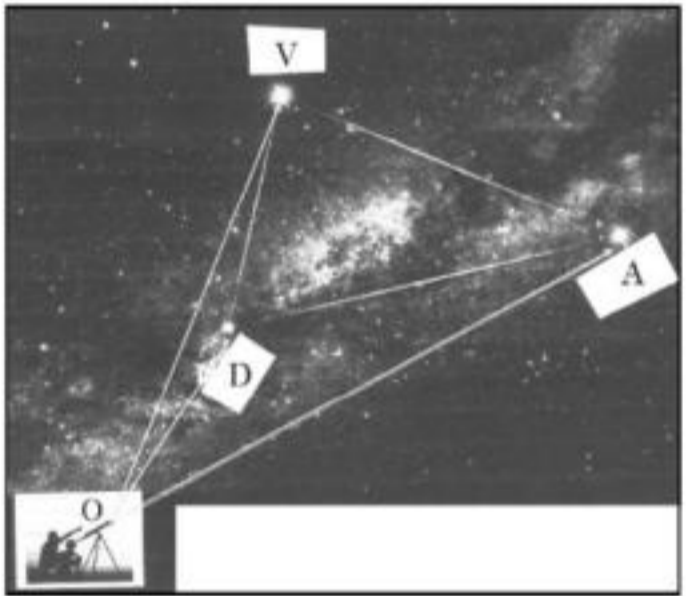
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correct  
figure  
 $1 \frac{1}{2}$



	<p>As, <math>9x^2 + 25y^2 = 225 \Rightarrow y = \pm \frac{3}{5} \sqrt{5^2 - x^2}</math></p> <p>Required Area = <math>\int_{-2}^2 \frac{3}{5} \sqrt{5^2 - x^2} dx = \frac{6}{5} \int_0^2 \sqrt{5^2 - x^2} dx</math></p> $= \frac{6}{5} \left( \frac{x\sqrt{5^2 - x^2}}{2} + \frac{25}{2} \sin^{-1} \left( \frac{x}{5} \right) \right)_0^2$ $= \frac{6}{5} \left( \frac{2\sqrt{21}}{2} + \frac{25}{2} \sin^{-1} \left( \frac{2}{5} \right) \right)$ $= \left( \frac{6\sqrt{21}}{5} + 15 \sin^{-1} \left( \frac{2}{5} \right) \right)$	<p>1 ½</p> <p>1</p> <p>1</p>
<p><b>SECTION E</b></p> <p><b>In this section there are 3 case-study based questions of 4 marks each.</b></p>		
<p>36.</p>	<p>An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0,0,0) and the three stars have their locations at the points D, A and V having position vectors <math>2\hat{i} + 3\hat{j} + 4\hat{k}</math>, <math>7\hat{i} + 5\hat{j} + 8\hat{k}</math> and <math>-3\hat{i} + 7\hat{j} + 11\hat{k}</math> respectively.</p>  <p>Based on the above information, answer the following questions:</p> <p>(i) How far is the star V from star A?</p> <p>(ii) Find a unit vector in the direction of <math>\overrightarrow{DA}</math>.</p> <p>(iii) Find the measure of <math>\angle VDA</math>.</p> <p style="text-align: center;">OR</p> <p>What is the projection of vector <math>\overrightarrow{DV}</math> on vector <math>\overrightarrow{DA}</math>?</p>	<p>1</p> <p>1</p> <p>2</p> <p>2</p>
<p><b>Sol.</b></p>	<p>(i) <math>\overrightarrow{AV} = \text{Position Vector of V} - \text{Position Vector of A} = -10\hat{i} + 2\hat{j} + 3\hat{k}</math></p> <p>Thus, <math> \overrightarrow{AV}  = \sqrt{100 + 4 + 9} = \sqrt{113}</math> units</p> <p>(ii) <math>\overrightarrow{DA} = \text{Position Vector of A} - \text{Position Vector of D} = 5\hat{i} + 2\hat{j} + 4\hat{k}</math></p> <p>Unit vector in the direction of <math>\overrightarrow{DA} = \frac{5\hat{i} + 2\hat{j} + 4\hat{k}}{3\sqrt{5}}</math></p>	<p>½</p> <p>½</p> <p>½</p> <p>½</p>

	<p>(iii) <math>\overrightarrow{DV} = -5\hat{i} + 4\hat{j} + 7\hat{k}</math></p> $\angle VDA = \cos^{-1} \left( \frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{ \overrightarrow{DV}   \overrightarrow{DA} } \right) = \cos^{-1} \left( \frac{11\sqrt{2}}{90} \right)$ <p style="text-align: center;">OR</p> <p>(iii) <math>\overrightarrow{DV} = -5\hat{i} + 4\hat{j} + 7\hat{k}</math></p> $\text{Projection of } \overrightarrow{DV} \text{ on } \overrightarrow{DA} = \left( \frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{ \overrightarrow{DA} } \right) = \frac{11\sqrt{5}}{15}$	<p><math>\frac{1}{2}</math></p> <p><math>1 \frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>1 \frac{1}{2}</math></p>
37.	<p>Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is <math>\frac{1}{5}</math>, Jaspreet's selection is <math>\frac{1}{3}</math> and Alia's selection is <math>\frac{1}{4}</math>. The event of selection is independent of each other.</p> <div style="text-align: center;">  </div> <p>Based on the above information, answer the following questions:</p> <p>(i) What is the probability that at least one of them is selected ?</p> <p>(ii) Find <math>P(G \overline{H})</math> where G is the event of Jaspreet's selection and <math>\overline{H}</math> denotes the event that Rohit is not selected.</p> <p>(iii) Find the probability that exactly one of them is selected.</p> <p style="text-align: center;">OR</p> <p>(iii) Find the probability that exactly two of them are selected.</p>	<p>1</p> <p>1</p> <p>2</p> <p>2</p>
Sol.	<p>Given <math>P(\text{Rohit}) = \frac{1}{5}</math>, <math>P(\text{Jaspreet}) = \frac{1}{3}</math>, <math>P(\text{Alia}) = \frac{1}{4}</math></p> <p>(i) <math>P(\text{at least one of them is selected}) = 1 - P(\text{no one is selected})</math></p> $= 1 - \left( \frac{4}{5} \times \frac{2}{3} \times \frac{3}{4} \right) = \frac{3}{5}$ <p>(ii) <math>P(G \overline{H}) = \frac{P(G \cap \overline{H})}{P(\overline{H})} = \frac{1}{3}</math></p> <p>(iii) <math>P(\text{exactly one of them selected})</math></p> $= P(R) \times P(\overline{J}) \times P(\overline{A}) + P(\overline{R}) \times P(J) \times P(\overline{A}) + P(\overline{R}) \times P(\overline{J}) \times P(A)$ $= \frac{6+12+8}{60} = \frac{13}{30}$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>

	OR	
	<p>(iii) P(exactly two of them selected)</p> $= P(R) \times P(J) \times P(\bar{A}) + P(R) \times P(\bar{J}) \times P(A) + P(\bar{R}) \times P(J) \times P(A)$ $= \frac{3+2+4}{60} = \frac{3}{20}$	1 1
38.	<p>A store has been selling calculators at Rs. 350 each. A market survey indicates that a reduction in price (<math>p</math>) of calculator increases the number of units (<math>x</math>) sold. The relation between the price and quantity sold is given by demand function</p> $p = 450 - \frac{x}{2}$ <div style="text-align: center;">  </div> <p>Based on the above information, answer the following questions:</p> <p>(i) Determine the number of units (<math>x</math>) that should be sold to maximise the revenue <math>R(x) = xp(x)</math>. Also verify the result.</p> <p>(ii) What rebate in price of calculator should the store give to maximise the revenue?</p>	
Sol.	<p>(i) Revenue by selling <math>x</math> items <math>= R(x) = x.p(x) = 450x - \frac{x^2}{2}</math></p> $\frac{dR}{dx} = 450 - x$ <p>For Maxima or Minima, <math>\frac{dR}{dx} = 0 \Rightarrow x = 450</math></p> $\frac{d^2R}{dx^2} = -1 < 0$ <p>(Revenue is maximum when <math>x = 450</math> units are sold)</p> <p>(ii) At <math>x = 450</math>, <math>p = 450 - \frac{450}{2} = 225</math></p> <p>So, Rebate <math>= 350 - 225 = \text{Rs.}125</math> per calculator</p>	1/2  1  1/2 1 1