Marking Scheme

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Senior School Certificate Examination, 2024

MATHEMATICS PAPER CODE 65/3/3

General Instructions:

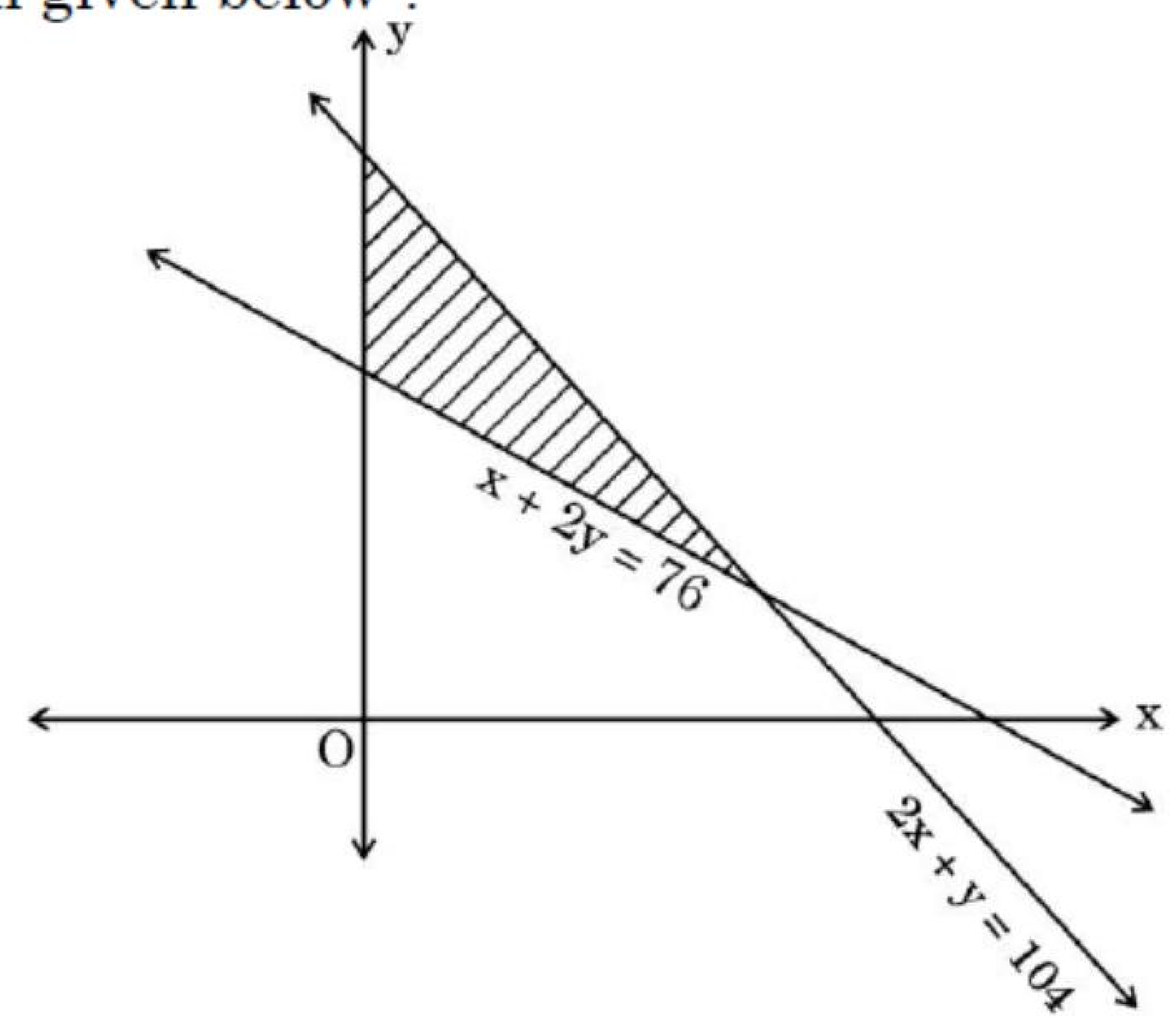
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after delibration and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**

**These answers are meant to be used by evaluators.*

10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. NO.	EXPECTED ANSWER / VALUE POINT	MARKS
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) of 1 mark each.	
Q1	The value of $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$ is : (A) 0 (B) 2 (C) 7 (D) – 2	
Ans	(A) 0	1
Q2	If $y = \sin^{-1} x$, then $\frac{d^2y}{dx^2}$ is : (A) sec y (B) sec y tan y (C) $\sec^2 y \tan y$ (D) $\tan^2 y \sec y$	
Ans	(C) $\sec^2 y \tan y$	1
Q3	If $\lvert \vec{a} \rvert = 2$ and $-3 \le k \le 2$, then $\lvert k\vec{a} \rvert \in$: (A) [– 6, 4] (B) [0, 4] (C) [4, 6] (D) [0, 6]	
Ans	(D) [0, 6]	1
Q4	If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is : (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π	
Ans	(C) $\frac{\pi}{2}$	1

These answers are meant to be used by evaluators.

Q5	Of the following, which group of constraints represents the feasible region given below ?  (A) $x + 2y \le 76, 2x + y \ge 104, x, y \ge 0$ (B) $x + 2y \le 76, 2x + y \le 104, x, y \ge 0$ (C) $x + 2y \ge 76, 2x + y \le 104, x, y \ge 0$ (D) $x + 2y \ge 76, 2x + y \ge 104, x, y \ge 0$	
Ans	(C) $x + 2y \ge 76, 2x + y \le 104, x, y \ge 0$	1
Q6	If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is : (A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (B) $30\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (C) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (D) $\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$	
Ans	(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$	1

*These answers are meant to be used by evaluators.

Q7	If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ? (A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\, i, j$ (C) $a_{ij} = 0, \forall\, i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$	
Ans	(D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$	1
Q8	Let Z denote the set of integers, then function $f : Z \to Z$ defined as $f(x) = x^3 - 1$ is : (A) both one-one and onto (B) one-one but not onto (C) onto but not one-one (D) neither one-one nor onto	
Ans	(B) one-one but not onto	1
Q9	Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, $(a + b + c + d)$ is equal to : (A) 2a (B) 2b (C) 2c (D) 0	
Ans	(A) 2a	1
Q10	A function $f(x) = \lvert 1 - x + \lvert x \rvert \rvert$ is : (A) discontinuous at $x = 1$ only (B) discontinuous at $x = 0$ only (C) discontinuous at $x = 0, 1$ (D) continuous everywhere	
Ans	(D) continuous everywhere	1
Q11	The rate of change of surface area of a sphere with respect to its radius 'r', when r = 4 cm, is : (A) 64π cm^2/cm (B) 48π cm^2/cm (C) 32π cm^2/cm (D) 16π cm^2/cm	
Ans	(C) 32π cm^2/cm	1

These answers are meant to be used by evaluators.

Q12	$\int_{-a}^{a} f(x)\,dx = 0$, if : (A) $f(-x) = f(x)$ (B) $f(-x) = -f(x)$ (C) $f(a-x) = f(x)$ (D) $f(a-x) = -f(x)$	
Ans	(B) $f(-x) = -f(x)$	1
Q13	$x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a : (A) variable separable differential equation. (B) homogeneous differential equation. (C) first order linear differential equation. (D) differential equation whose degree is not defined.	
Ans	(C) first order linear differential equation.	1
Q14	If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are : (A) collinear vectors which are not parallel (B) parallel vectors (C) perpendicular vectors (D) unit vectors	
Ans	(C) perpendicular vectors	1
Q15	If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ? (A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ (B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$ (C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ (D) $\cos \alpha + \cos \beta + \cos \gamma = 1$	
Ans	(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$	1
Q16	The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called : (A) feasible solutions (B) constraints (C) optimal solutions (D) infeasible solutions	
Ans	(B) constraints	1

Q17	Let E and F be two events such that P(E) = 0·1, P(F) = 0·3, $P(E \cup F)$ = 0·4, then P(F | E) is : (A) 0·6 (B) 0·4 (C) 0·5 (D) 0	
Ans	(D) 0	1
Q18	If A and B are two skew symmetric matrices, then (AB + BA) is : (A) a skew symmetric matrix (B) a symmetric matrix (C) a null matrix (D) an identity matrix	
Ans	(B) a symmetric matrix	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19	*Assertion (A) :* For any non-zero unit vector $\vec{a}$, $\vec{a} \cdot (-\vec{a}) = (-\vec{a}) \cdot \vec{a} = -1$. *Reason (R) :* Angle between $\vec{a}$ and $(-\vec{a})$ is $\frac{\pi}{2}$.	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1
Q20	*Assertion (A) :* Every scalar matrix is a diagonal matrix. *Reason (R) :* In a diagonal matrix, all the diagonal elements are 0.	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1

SECTION B

Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

Q21	$\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors. If θ is the angle between $\vec{a}$ and $(2\vec{a} + 3\vec{b} + 6\vec{c})$, find the value of cos θ.

Ans	Given $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert=1$ and $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$ Now, $\lvert 2\vec{a}+3\vec{b}+6\vec{c}\rvert^2=4\lvert\vec{a}\rvert^2+9\lvert\vec{b}\rvert^2+36\lvert\vec{c}\rvert^2=49$ $\Rightarrow\lvert 2\vec{a}+3\vec{b}+6\vec{c}\rvert=7$ $\cos\theta=\frac{\vec{a}.(2\vec{a}+3\vec{b}+6\vec{c})}{\lvert\vec{a}\rvert\lvert 2\vec{a}+3\vec{b}+6\vec{c}\rvert}=\frac{2\lvert\vec{a}\rvert^2}{\lvert\vec{a}\rvert\lvert 2\vec{a}+3\vec{b}+6\vec{c}\rvert}$ $\therefore\cos\theta=\frac{2}{7}$	½ ½ ½ ½
Q22	Evaluate : $\cot^2\left\{\text{cosec}^{-1}3\right\}+\sin^2\left\{\cos^{-1}\left(\frac{1}{3}\right)\right\}$	
Ans	$\left[\text{cosec}^2\left(\text{cosec}^{-1}3\right)-1\right]+\left[1-\cos^2\left(\cos^{-1}\frac{1}{3}\right)\right]$ $=(9-1)+\left(1-\frac{1}{9}\right)$ $=\frac{80}{9}$	1 ½ ½
Q23(a)	If $x=e^{x/y}$, prove that $\frac{dy}{dx}=\frac{\log x-1}{(\log x)^2}$	
Ans	$x=e^{\frac{x}{y}}\Rightarrow\log x=\frac{x}{y}\Rightarrow y=\frac{x}{\log x}$ $\Rightarrow\frac{dy}{dx}=\frac{(\log x)(1)-x\left(\frac{1}{x}\right)}{(\log x)^2}=\frac{\log x-1}{(\log x)^2}$	1 1
Q23(b)	Check the differentiability of $f(x)=\begin{cases}x^2+1, & 0\le x<1\\ 3-x, & 1\le x\le 2\end{cases}$ at $x=1$.	
Ans	LHD at $x=1$ $=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to 0}\frac{\left[(1-h)^2+1\right]-2}{-h}=2$ RHD at $x=1$ $=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{\left[3-(1+h)\right]-2}{h}=-1$ as LHD $\neq$ RHD, so $f(x)$ is not differentiable at $x=1$	1 ½ ½

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Q24(a)	Evaluate : $\int_0^{\pi/2} \sin 2x \cos 3x \, dx$	
Ans	$I = \int_0^{\frac{\pi}{2}} \sin 2x \cos 3x \, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 5x - \sin x) dx$ $= \frac{1}{2}\left[-\frac{1}{5}\cos 5x + \cos x\right]_0^{\frac{\pi}{2}}$ $= -\frac{2}{5}$	 1 ½ ½
Q24(b)	Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and F(1) = 0, find F(x).	
Ans	$F(x) = \int \frac{1}{\sqrt{2x - x^2}} dx$ $= \int \frac{1}{\sqrt{1 - (x-1)^2}} dx$ $= \sin^{-1}(x-1) + c$ when $x = 1, y = 0$ gives $c = 0$ $\therefore F(x) = \sin^{-1}(x-1)$	 ½ ½ ½ ½
Q25	Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $\lvert \overrightarrow{AB} \rvert : \lvert \overrightarrow{BC} \rvert$.	
Ans	Position vector of $C = \vec{r} = \frac{4\vec{b} - \vec{a}}{3}$ i.e. $\vec{r} = \frac{1}{3}(-5\hat{i} + 2\hat{j} + 5\hat{k})$ Now, $\overrightarrow{AB} = -2\hat{i} - \hat{j} + 2\hat{k} \Rightarrow \lvert\overrightarrow{AB}\rvert = 3$ $\overrightarrow{BC} = -\frac{1}{3}(2\hat{i} + \hat{j} - 2\hat{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 1$ $\lvert\overrightarrow{AB}\rvert : \lvert\overrightarrow{BC}\rvert = 3:1$	 1 1

These answers are meant to be used by evaluators.

SECTION C

Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

Q26

Solve the following linear programming problem graphically :

Maximize $z = x + y$

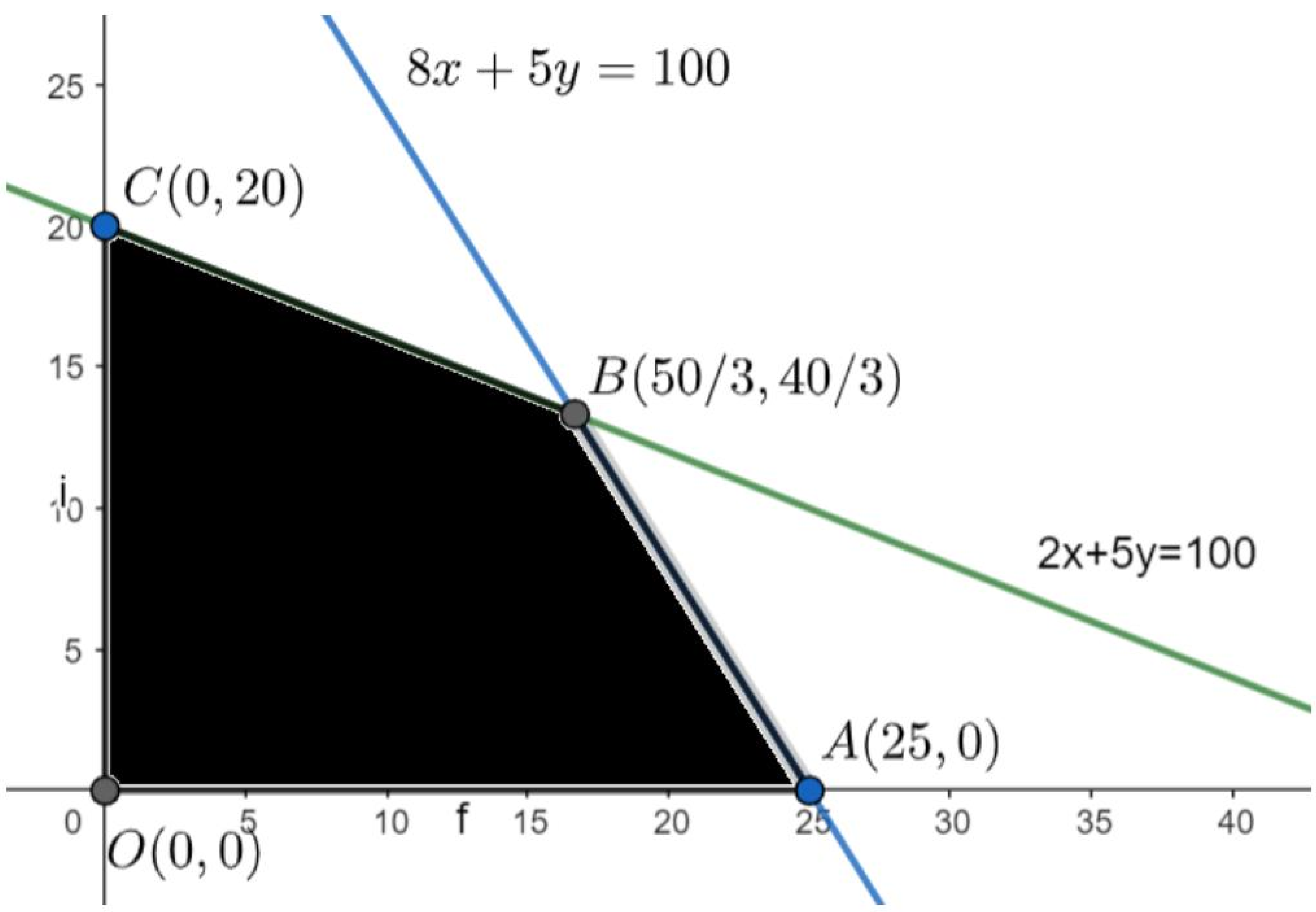
subject to constraints

$$2x + 5y \le 100$$

$$8x + 5y \le 200$$

$$x \ge 0,\ y \ge 0.$$

Ans



For correct graph 1½

Corner Point	Value of $z = 4x + 3y$
$O(0,0)$	0
$A(25,0)$	25
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	30
$C(0,20)$	20

For correct table 1

$z_{max} = 30$ when $x = \frac{50}{3},\ y = \frac{40}{3}$ ½

Q27

The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

These answers are meant to be used by evaluators.

Ans	Let $E_1 : P$ is appointed as CEO, $E_2 : Q$ is appointed as CEO, $E_3 : R$ is appointed as CEO A : company increase profits from previous year here, $P(E_1)=\frac{4}{7}, P(E_3)=\frac{1}{7}, P(E_1)=\frac{2}{7}$ $P(A\|E_1)=0.3, P(A\|E_2)=0.8, P(A\|E_3)=0.5$ $P(E_3\|A)=\frac{P(E_3)P(A\|E_3)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)+P(E_3)P(A\|E_3)}$ $=\frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5}$ $=\frac{1}{3}$	½ 1 1 ½
Q28(a)	If x cos (p + y) + cos p sin (p + y) = 0, prove that $\cos p \frac{dy}{dx} = -\cos^2(p+y)$, where p is a constant.	
Ans	$x\cos(p+y)+\cos p\sin(p+y)=0$ $\Rightarrow x=\frac{-\cos p\sin(p+y)}{\cos(p+y)} \Rightarrow x=-\cos p.\tan(p+y)$ $\Rightarrow \frac{dx}{dy}=-\cos p.\sec^2(p+y)$ $\Rightarrow \frac{dy}{dx}=\frac{-1}{\cos p.\sec^2(p+y)}$ $\Rightarrow \cos p\ \frac{dy}{dx}=-\cos^2(p+y)$	1 1 ½ ½
Q28(b)	Find the value of a and b so that function f defined as : $f(x)=\begin{cases}\frac{x-2}{\|x-2\|}+a, & \text{if } x<2\\ a+b, & \text{if } x=2\\ \frac{x-2}{\|x-2\|}+b, & \text{if } x>2\end{cases}$ is a continuous function.	

Ans	$f(x)=\begin{cases}\frac{x-2}{-(x-2)}+a & ;x<2\\ a+b & ;x=2\\ \frac{x-2}{(x-2)}+b & ;x>2\end{cases}\Rightarrow f(x)=\begin{cases}-1+a & ;x<2\\ a+b & ;x=2\\ 1+b & ;x>2\end{cases}$ $\lim_{x\to 2^-} f(x)=-1+a, \lim_{x\to 2^+} f(x)=1+b$ and $f(2)=a+b$ as f is continous at $x=2\ \therefore\ -1+a=1+b=a+b$ $\Rightarrow a=1, b=-1$	 1 1 ½+½
Q29(a)	Find the intervals in which the function f(x) = $\frac{\log x}{x}$ is strictly increasing or strictly decreasing.	
Ans	$f(x)=\frac{\log x}{x}\Rightarrow f'(x)=\frac{1-\log x}{x^2}\ ;x>0$ for strictly increasing/decreasing, put $f'(x)=0\Rightarrow x=e$ for strictly increasing, $x\in(0,e)$ and for strictly decreasing $x\in(e,\infty)$	1 1 ½+½
Q29(b)	Find the absolute maximum and absolute minimum values of the function f given by f(x) = $\frac{x}{2}+\frac{2}{x}$, on the interval [1, 2].	
Ans	$f(x)=\frac{x}{2}+\frac{2}{x};\ x\in[1,2]$ $\Rightarrow f'(x)=\frac{1}{2}-\frac{2}{x^2}$ for absolute maximum / minimum, put $f'(x)=0$ $\Rightarrow x^2=4\Rightarrow x=2$ Now, $f(1)=\frac{5}{2}$ and $f(2)=2$ $\therefore$ absolute maximum value $=\frac{5}{2}$ and absolute minimum value $=2$	 1 ½ ½+½ ½
Q30	Find : $\int \frac{\sqrt{x}}{(x+1)(x-1)}dx$	

These answers are meant to be used by evaluators.

Ans	$I=\int \frac{\sqrt{x}}{(x+1)(x-1)}dx$ $\sqrt{x}=t$ gives $I=\int \frac{2t^2}{(t^2+1)(t^2-1)}dt$ Let $\frac{2t^2}{(t^2+1)(t^2-1)}=\frac{2z}{(z+1)(z-1)}$; where $t^2=z$ we have $\frac{2z}{(z+1)(z-1)}=\frac{1}{z+1}+\frac{1}{z-1}$ $\Rightarrow I=\int \frac{1}{t^2+1}dt+\int \frac{1}{t^2-1}dt$ $=\tan^{-1}t+\frac{1}{2}\log\left\|\frac{t-1}{t+1}\right\|+c$ $=\tan^{-1}\sqrt{x}+\frac{1}{2}\log\left\|\frac{\sqrt{x}-1}{\sqrt{x}+1}\right\|+c$	½ 1 1 ½
Q31(a)	Find : $\int \frac{2+\sin 2x}{1+\cos 2x}e^x\,dx$	
Ans	$I=\int \frac{2+\sin 2x}{1+\cos 2x}e^x\,dx$ $=\int \frac{2+2\sin x\cos x}{2\cos^2 x}e^x\,dx$ $=\int (\sec^2 x+\tan x)e^x dx$ $=e^x.\tan x+c$	1 1 1
Q31(b)	Evaluate : $\int_0^{\pi/4}\frac{1}{\sin x+\cos x}dx$	

These answers are meant to be used by evaluators.

Ans	$I=\int_0^{\frac{\pi}{4}}\frac{1}{\sin x+\cos x}dx$ $=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}}\frac{1}{\cos\frac{\pi}{4}\sin x+\sin\frac{\pi}{4}\cos x}dx$ $=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}}\frac{1}{\sin\left(x+\frac{\pi}{4}\right)}dx==\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}}\cos ec\left(x+\frac{\pi}{4}\right)dx$ $=\frac{1}{\sqrt{2}}\left[\log\left\|\cos ec\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right\|\right]_0^{\frac{\pi}{4}}$ $=\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)=-\frac{1}{\sqrt{2}}\log\left(\sqrt{2}-1\right)$	 1 1 1

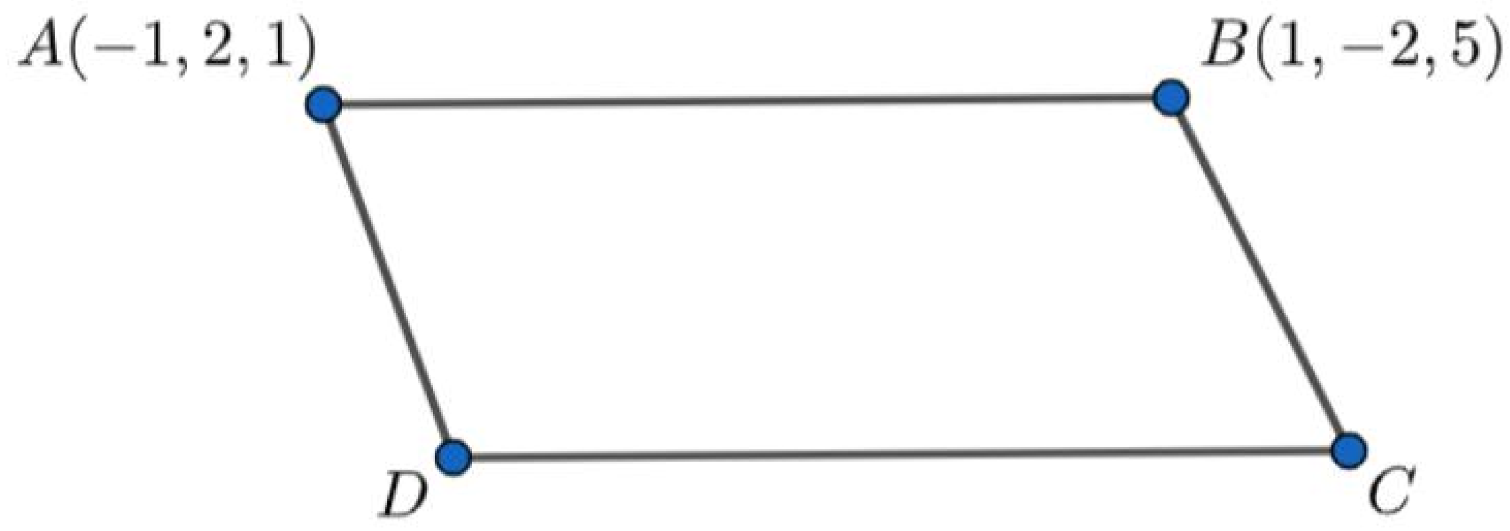
SECTION D

Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

Q32(a)	Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and $\frac{x-1}{0}=\frac{y}{-3}=\frac{z-7}{2}$ and perpendicular to these given lines.	
Ans	$l_1:\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$; $l_2:\frac{x-1}{0}=\frac{y}{-3}=\frac{z-7}{2}=\mu$ any point on l_1 is $(\lambda,2\lambda+1,3\lambda+2)$ & any point on l_2 is $(1,-3\mu,2\mu+7)$ If l_1 and l_2 intersect, $\lambda=1, 2\lambda+1=-3\mu$ and $3\lambda+2=2\mu+7\Rightarrow\lambda=1$ and $\mu=-1$ Point of intersection of l_1 and l_2 is $(1,3,5)$. Let d.r.'s of required line be $<a,b,c>$. Then, $a+2b+3c=0$ and $-3b+2c=0\Rightarrow\frac{a}{13}=\frac{b}{-2}=\frac{c}{-3}$ Required equation of line is $\frac{x-1}{13}=\frac{y-3}{-2}=\frac{z-5}{-3}$	 1 1 1 1 1

OR

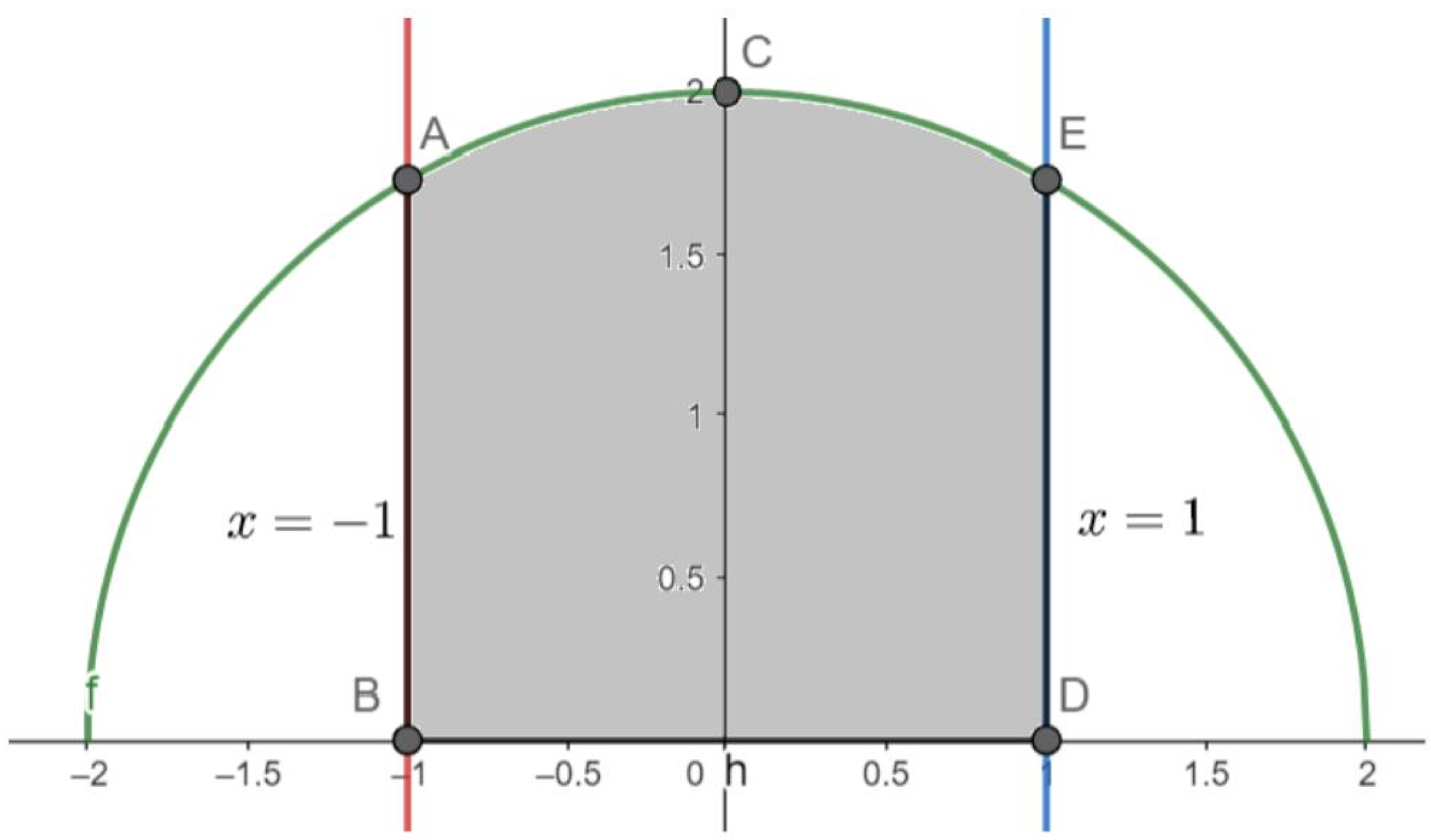
Q32(b)	Two vertices of the parallelogram ABCD are given as A(−1, 2, 1) and B(1, −2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1}=\frac{y+7}{-2}=\frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

Ans		
	d.r's of CD are $< 1, -2, 2 >$	
	$\therefore$ d.r's of AB are $< 1, -2, 2 >$	½
	$\therefore$ Equation of AB is $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{2}$	½
	$\therefore$ Equation of CD is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$	
	Let $\vec{a}_1 = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{a}_2 = 4\hat{i} - 7\hat{j} + 8\hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$	1
	Now, $\vec{a}_2 - \vec{a}_1 = 5\hat{i} - 9\hat{j} + 7\hat{k}$	
	$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -9 & 7 \\ 1 & -2 & 2 \end{vmatrix} = -4\hat{i} - 3\hat{j} - \hat{k}$	**1**
	Distance between AB and CD is given by $d = \frac{\lvert (\vec{a}_2 - \vec{a}_1) \times \vec{b} \rvert}{\lvert \vec{b} \rvert}$	
	$d = \frac{\sqrt{16+9+1}}{\sqrt{1+4+4}} = \frac{\sqrt{26}}{3}$	½
	$CD = \sqrt{2^2 + (-4)^2 + (4)^2} = 6$	½
	Area of parallelogram ABCD = $b \times h = 6 \times \frac{\sqrt{26}}{3} = 2\sqrt{26}$	1
Q33	Let $A = R - \{3\}$ and $B = R - \{a\}$. Find the value of 'a' such that the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$ is onto. Also, check whether the given function is one-one or not.	
Ans	For onto, let $f(x) = y$	
	$\frac{x-2}{x-3} = y \Rightarrow x - 2 = xy - 3y$	½

These answers are meant to be used by evaluators.

	$\Rightarrow x(1-y) = 2-3y \Rightarrow x = \dfrac{2-3y}{1-y}$ For $y=1, x \in A \therefore \text{Range} = R - \{1\} \therefore a = 1$ For one– one Let $f(x_1) = f(x_2)$ where $x_1, x_2 \in A$ $\Rightarrow \dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3}$ $\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_2 x_1 - 2x_1 - 3x_2 + 6$ $\Rightarrow x_1 = x_2$ $\therefore$ f is one – one.	1 1 **1** 1 **½**
Q34(a)	It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.	
Ans	$f(x) = x^4 - 62x^2 + ax + 9 \Rightarrow f'(x) = 4x^3 - 124x + a$ as at $x=1$, f attains local maximum value, $f'(1) = 0 \Rightarrow a = 120$ now, $f'(x) = 4x^3 - 124x + 120 = 4(x-1)(x^2+x-30) = 4(x-1)(x-5)(x+6)$ Critical points are $x = -6, 1, 5$ $f''(x) = 12x^2 - 124$ $f''(-6) > 0,\ f''(1) < 0,\ f''(5) > 0$ so f attains local maximum value at $x=1$ and local minimum value at $x = -6, 5$	**½** 1 1 1 **½** 1
Q34(b)	The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.	

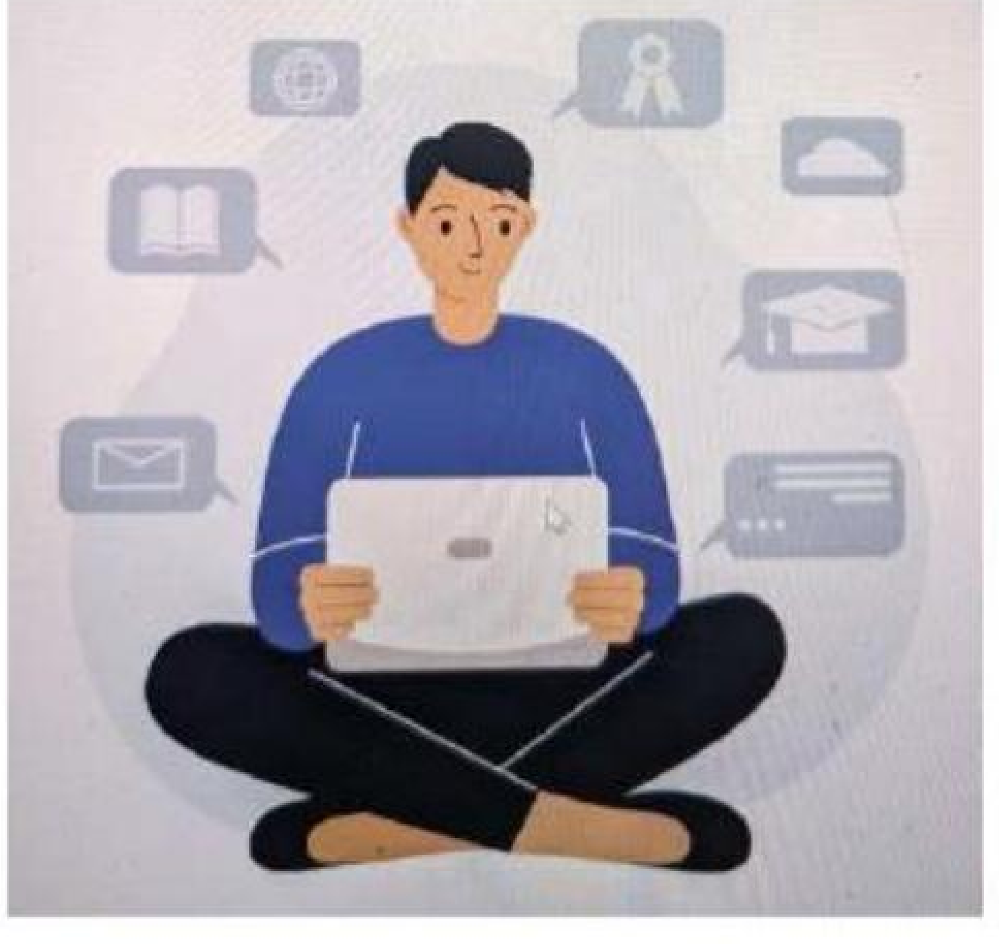
These answers are meant to be used by evaluators.

Ans	Let length of rectangle be x cm and breadth be $(150-x)$ cm. Let r be the radius of cylinder $\Rightarrow 2\pi r = x \Rightarrow r = \dfrac{x}{2\pi}$ $V = \pi r^2 h = \pi\left(\dfrac{x^2}{4\pi^2}\right)(150-x) = \dfrac{75x^2}{2\pi} - \dfrac{x^3}{4\pi}$ $\dfrac{dV}{dx} = \dfrac{150x}{2\pi} - \dfrac{3x^2}{4\pi}$ $\dfrac{dV}{dx} = 0 \Rightarrow x = 100$ cm $\left.\dfrac{d^2V}{dx^2}\right\rvert_{x=100\text{ cm}} = -\dfrac{75}{\pi} < 0 \Rightarrow V$ is maximum when $x = 100$ cm. Length of rectangle is 100 cm and breadth of rectangle is 50 cm.	1 1 1 1 ½ ½
Q35	Using integration, find the area of the region enclosed between the curve $y = \sqrt{4-x^2}$ and the lines x = −1, x = 1 and the x-axis.	
Ans	 Required area $= 2\int_0^1 \sqrt{4-x^2}\,dx$ $= 2\left[\dfrac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\dfrac{x}{2}\right]_0^1$ $= \sqrt{3} + \dfrac{2\pi}{3}$	For correct figure 1 mark **1** **2** **1**

These answers are meant to be used by evaluators.

SECTION E **Questions no. 36 to 38 are case study based questions carrying 4 marks each.**		
Q36	A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.   Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports. In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000. Based on the above information, answer the following questions : (i) Express the given information algebraically using matrices. *1* (ii) Check whether the system of matrix equations so obtained is consistent or not. *1* (iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2* **OR** (iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*	
Ans(i)	Let No. of girl child scholarships = x No. of meritorious achievers = y x + y = 50	

*These answers are meant to be used by evaluators.

	$3000x + 4000y = 180000$ or $3x + 4y = 180$ $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$	1
Ans(ii)	$\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1 \neq 0$ $\therefore$ system is consistent.	1
Ans (iii)(a)	Let $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$ $AX = B \Rightarrow X = A^{-1}B$ $X = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 180 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$ $\Rightarrow x = 20, y = 30$	½ 1 ½
	OR	
Ans (iii)(b)	Required expenditure = ₹ [30(3000) + 20(4000)] = ₹ 1,70,000	1 1
Q37	Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.   A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :	

These answers are meant to be used by evaluators.

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

(i) Express the probability distribution given above in the form of a probability distribution table. *1*

(ii) Find the value of k. *1*

(iii) (a) Find the mean number of hours spent by the student. *2*

OR

(iii) (b) Find $P(1 < X < 6)$. *2*

Ans(i) *1*

X	1	2	3	4	5	6
P(X)	k	4k	9k	8k	10k	12k

1

Ans(ii)

$k + 4k + 9k + 8k + 10k + 12k = 1$

$\Rightarrow k = \frac{1}{44}$ 1

Ans (iii) (a)

Mean $= \sum x_i p_i = k + 8k + 27k + 32k + 50k + 72k$ 1

$= 190k$

$= \frac{190}{44}$ or $\frac{95}{22}$ 1

OR

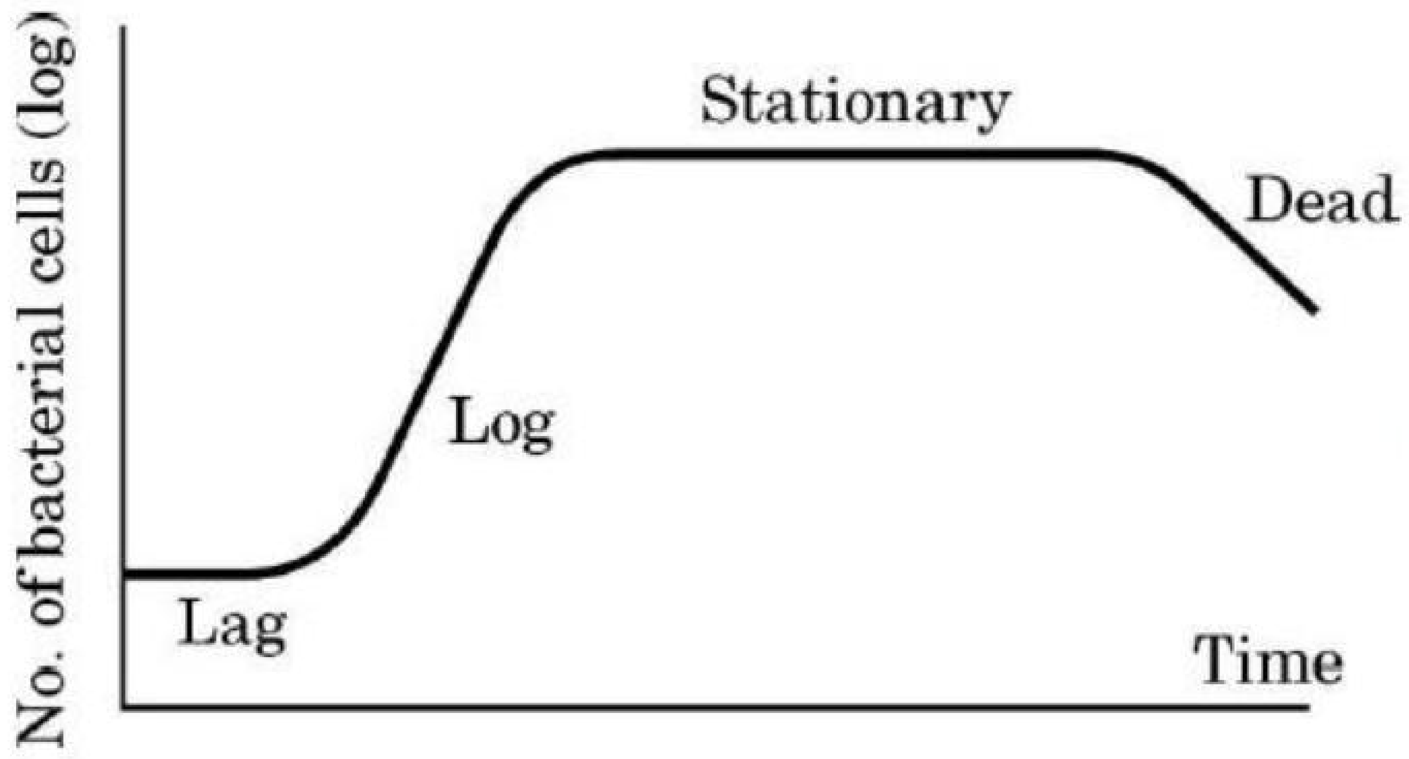
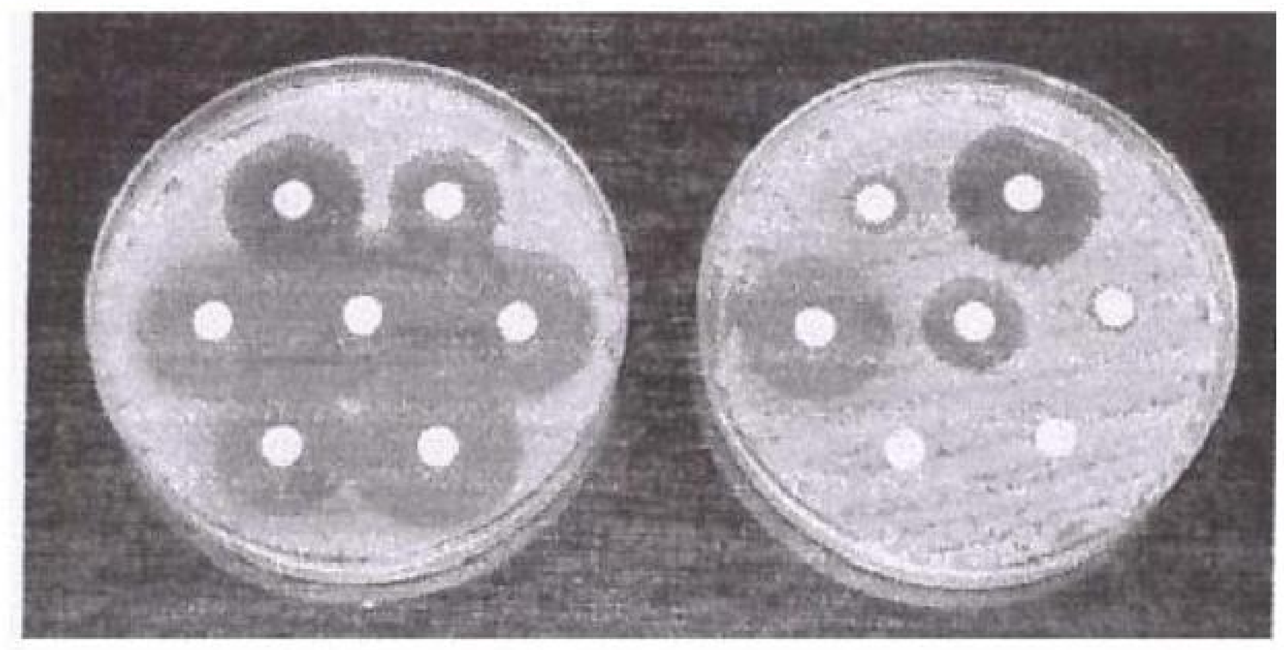
Ans (iii)(b)

$P(1 < X < 6) = 4k + 9k + 8k + 10k$ 1

$= 31k$

$= \frac{31}{44}$ 1

*These answers are meant to be used by evaluators.

Q38	A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.   The differential equation representing the growth of bacteria is given as : $\frac{dP}{dt}$ = kP, where P is the population of bacteria at any time 't'. Based on the above information, answer the following questions : (i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. *2* (ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. *2*	
Ans(i)	$\frac{dP}{dt} = kP \Rightarrow \int \frac{dP}{P} = \int k\,dt$ $\Rightarrow \log P = kt + C$ or $P = e^{kt+C}$	1 1
Ans(ii)	$\log P = kt + C$ when $t = 0, P = 1000 \Rightarrow C = \log 1000$ when $t = 1, P = 2000 \Rightarrow \log 2000 = k + \log 1000$ $\Rightarrow k = \log 2$	½ ½ 1

These answers are meant to be used by evaluators.