Marking Scheme

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Senior School Certificate Examination, 2024

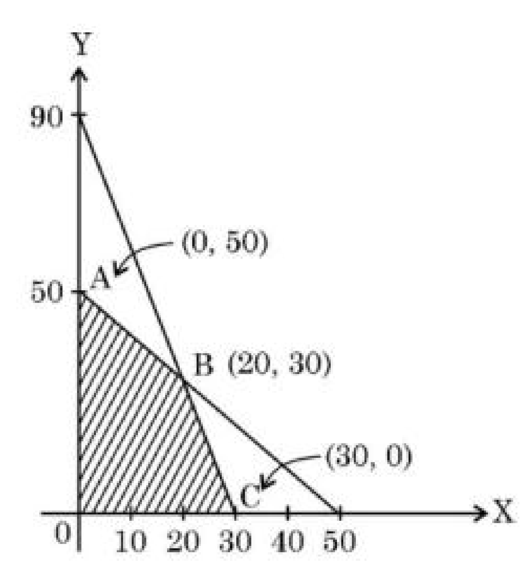
MATHEMATICS PAPER CODE 65/4/3

General Instructions:

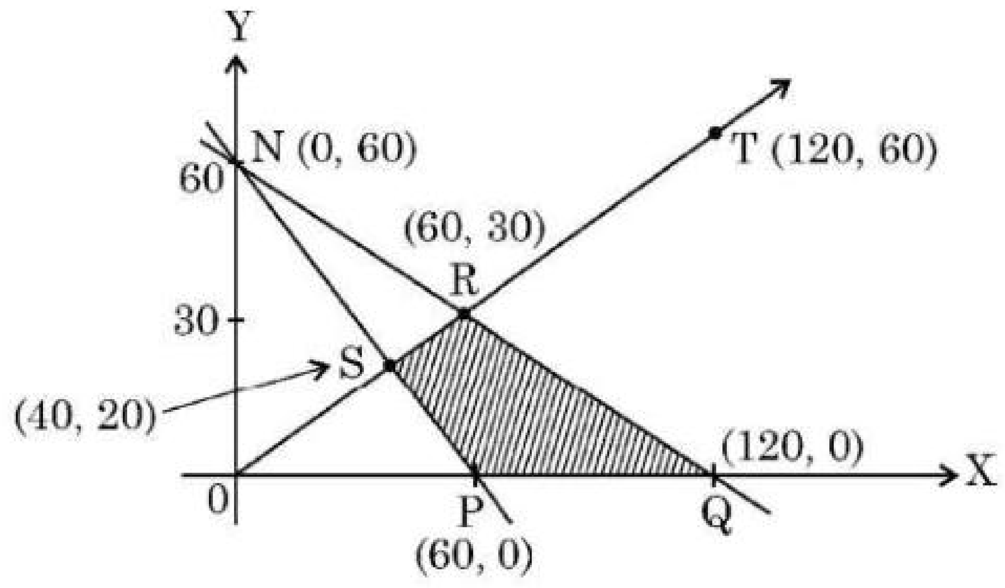
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after delibration and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**

10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vert\vec{a}\vert=1, \vert\vec{b}\vert=2$ and $\vec{a}.\vec{b}=\sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is: $(A)\ \frac{\pi}{6}$ $(B)\ \frac{\pi}{3}$ $(C)\ \frac{5\pi}{6}$ $(D)\ \frac{11\pi}{6}$	
Ans:	**(C)** $\frac{5\pi}{6}$	**1**
2.	The vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}-3\hat{j}-5\hat{k}$ and $\vec{c}=-3\hat{i}+4\hat{j}+4\hat{k}$ represents the sides of (A) an equilaterl triangle (B) an obtuse-angled triangle (C) an isosceles triangle (D) a right-angled triangle	
Ans:	**(D) a right-angled triangle**	**1**
3.	Let $\vec{a}$ be any vector such that $\vert\vec{a}\vert=a$. The value of $\vert\vec{a}\times\hat{i}\vert^2+\vert\vec{a}\times\hat{j}\vert^2+\vert\vec{a}\times\hat{k}\vert^2$ is: $(A)\ a^2$ $(B)\ 2a^2$ $(C)\ 3a^2$ $(D)\ 0$	
Ans:	**(B)** $2a^2$	**1**
4.	If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ and $A^2+7I=kA$, then value of k is : $(A)\ 1$ $(B)\ 2$ $(C)\ 5$ $(D)\ 7$	
Ans:	**(C) 5**	**1**
5.	Let $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ and $B=\frac{1}{3}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & \lambda\end{bmatrix}$. If AB= I, then value of λ is : $(A)\ \frac{-9}{4}$ $(B)\ -2$ $(C)\ \frac{-3}{2}$ $(D)\ 0$	
Ans:	$(B)\ -2$ OR $(C)\ \frac{-3}{2}$	**1**
6.	Derivative of x^2 with respect to x^3, is : $(A)\ \frac{2}{3x}$ $(B)\ \frac{3x}{2}$ $(C)\ \frac{2x}{3}$ $(D)\ 6x^5$	
Ans:	**(A)** $\frac{2}{3x}$	**1**

7.	The function $f(x) = \| x \| + \| x - 2 \|$ is (A) continuous, but not differentiable at $x = 0$ and $x = 2$. (B) differentiable but not continuous at $x = 0$ and $x = 2$. (C) continuous but not differentiable at $x = 0$ only. (D) neither continuous nor differentiable at $x = 0$ and $x = 2$.	
Ans:	**(A)** continuous, but not differentiable at $x = 0$ and $x = 2$	**1**
8.	The value of $\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta$ is : $(A)\ \pi+\sqrt{3}$ $(B)\ 3\sqrt{3}-\pi$ $(C)\ \sqrt{3}-\pi$ $(D)\ \pi-\sqrt{3}$	
Ans:	$(B)\ 3\sqrt{3}-\pi$	**1**
9.	The integrating factor of the differential equation $\frac{dy}{dx}+\frac{2}{x}y=0,\ (x\neq 0)$ is : $(A)\ \frac{2}{x}$ $(B)\ x^2$ $(C)\ e^{\frac{2}{x}}$ $(D)\ e^{\log(2x)}$	
Ans:	$(B)\ x^2$	**1**
10.	The lines $\frac{1-x}{2}=\frac{y-1}{3}=\frac{z}{1}$ and $\frac{2x-3}{2p}=\frac{y}{-1}=\frac{z-4}{7}$ are perpendicular to each other for p equal to : (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 2 (D) 3	
Ans:	**(C) 2**	**1**
11.	The maximum value of Z = 4x + y for a L.P.P. whose feasible region is given below is  $(A)\ 50$ $(B)\ 110$ $(C)\ 120$ $(D)\ 170$	
Ans:	**(C) 120**	**1**

12.	The probability distribution of a random variable X is: X: 0, 1, 2, 3, 4 P(X): 0.1, k, 2k, k, 0.1 where k is some unknown constant. The probability that the random variable X takes the value 2 is: $(A)\ \frac{1}{5}$ $(B)\ \frac{2}{5}$ $(C)\ \frac{4}{5}$ $(D)\ 1$	
Ans:	**(B)** $\frac{2}{5}$	**1**
13.	The function $f(x) = kx - \sin x$ is strictly increasing for (A) $k > 1$ (B) $k < 1$ (C) $k > -1$ (D) $k < -1$	
Ans:	**(A) k > 1**	**1**
14.	The cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is (A) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ (B) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (C) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (D) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$	
Ans:	(B) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$	**1**
15.	If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is (A) 0 (B) 5 (C) 10 (D) 25	
Ans:	**(D) 25**	**1**
16.	Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is: $(A)\ 7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ $(B)\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ $(C)\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ $(D)\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$	
Ans:	$(B)\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$	**1**

17.	If $A=\begin{bmatrix}2 & 1\\ -4 & -2\end{bmatrix}$, then the value of $I-A+A^2-A^3+....$ is $(A)\begin{bmatrix}-1 & -1\\ 4 & 3\end{bmatrix}$ $(B)\begin{bmatrix}3 & 1\\ -4 & -1\end{bmatrix}$ $(C)\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ $(D)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	
Ans:	$(A)\begin{bmatrix}-1 & -1\\ 4 & 3\end{bmatrix}$	**1**
18.	The integrating factor of the differential equation $(x+2y^2)\frac{dy}{dx}=y \quad (y>0)$ is : $(A)\ \frac{1}{x}$ $(B)\ x$ $(C)\ y$ $(D)\ \frac{1}{y}$	
Ans:	**(D)** $\frac{1}{y}$	**1**
	ASSERTION-REASON BASED QUSTIONS Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct nswer from the codes (A), (B), (C) and (D) as given below: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A). (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.	
19.	**Assertion (A) :** The relation $R=\{(x, y) : (x+y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation. **Reason (R) :** The number '2n' is composite for all natural numbers n.	
Ans:	**(C) Assertion (A) is true, but Reason (R) is false.**	**1**
20.	**Assertion (A) :** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.  **Reason (R) :** The optimal solution of a LPP having bounded feasible region must occur at corner points.	
Ans:	**(B) Both A and R are true but R is not the correct explanation of A.**	**1**

	SECTION B **In this section there are 5 very short answer type questions of 2 marks each.**	
21(a).	If $y = \cos^3(\sec^2 2t)$, find $\dfrac{dy}{dt}$.	
Sol.	$y = \cos^3(\sec^2 2t)$ $\Rightarrow \dfrac{dy}{dt} = 3\cos^2(\sec^2 2t)[-\sin(\sec^2 2t)] \times \dfrac{d(\sec^2 2t)}{dt}$ $\Rightarrow \dfrac{dy}{dt} = -3\cos^2(\sec^2 2t).\sin(\sec^2 2t) \times 2\sec 2t.\sec 2t \tan 2t.2$ $\therefore \dfrac{dy}{dt} = -12\cos^2(\sec^2 2t) \times \sin(\sec^2 2t) \times \sec^2 2t \times \tan 2t.$	½ 1 ½
	OR	
21(b).	If $x^y = e^{x-y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$.	
Sol.	$As, x^y = e^{x-y} \Rightarrow \log(x^y) = \log(e^{x-y})$ $\Rightarrow y\log x = (x-y) \Rightarrow y = \dfrac{x}{1+\log x}$ *Now*, Differentiating both the sides wrt x $\dfrac{dy}{dx} = \dfrac{(\log x + 1).1 - x(\frac{1}{x})}{(\log x+1)^2} = \dfrac{\log x}{(1+\log x)^2}$	1 1
22.	The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?	
Sol.	Given, $\dfrac{dV}{dt} = 6\text{ cm}^3/\text{sec}$. Since, $V = x^3$ $\dfrac{dV}{dt} = 3x^2\dfrac{dx}{dt} \Rightarrow 6 = 3x^2\dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt} = \dfrac{2}{x^2}\text{ cm/sec}$ Now, Surface Area = S = $6x^2 \Rightarrow \dfrac{dS}{dt} = 12x\dfrac{dx}{dt} = 3\text{ cm}^2/\text{sec}$	1 1
23	Show that the function f given by $f(x) = \sin x + \cos x$, is strictly decreasing in the interval $\left(\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right)$.	
Sol.	$f(x) = \sin x + \cos x \Rightarrow f'(x) = \cos x - \sin x$ $f'(x) = 0 \Rightarrow x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$ Thus, in the interval $\left(\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right)$ $f'(x) < 0$ $\therefore f$ is strictly decreasing function on $\left(\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right)$	1 1

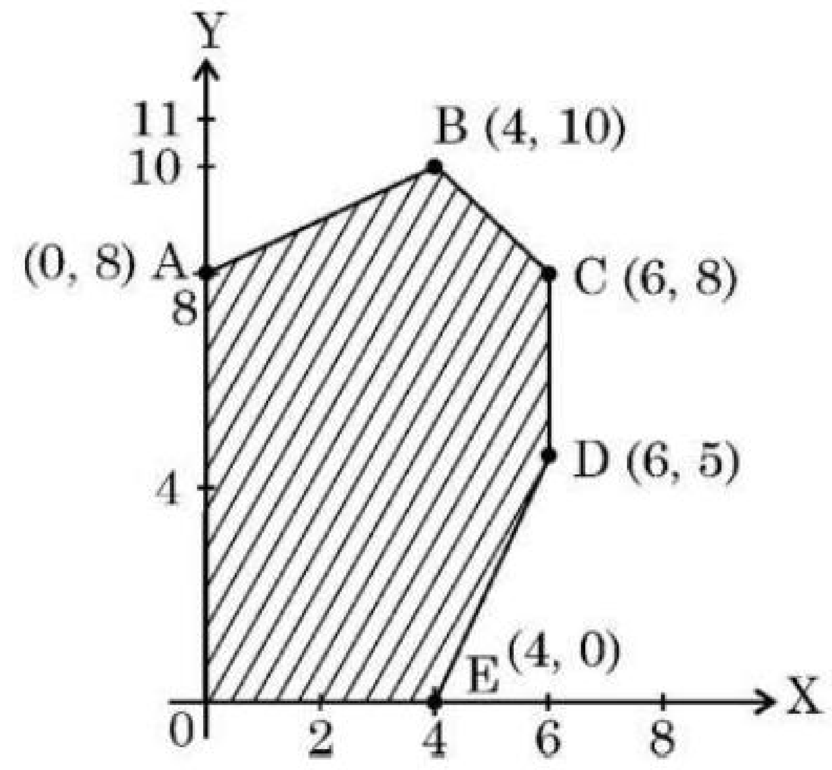
24(a).	Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2}<x<\frac{\pi}{2}$ in the simplest form.	
Sol.	$y=\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right]=\tan^{-1}\left[\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{(\cos\frac{x}{2}-\sin\frac{x}{2})^2}\right]$ $y=\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]=\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]=\left(\frac{\pi}{4}+\frac{x}{2}\right)$	½ 1 ½
	OR	
24(b).	Find the principal value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.	
Sol.	$\tan^{-1}(1)+\left[\pi-\cos^{-1}(\frac{1}{2})\right]-\sin^{-1}(\frac{1}{\sqrt{2}})=\frac{\pi}{4}+\left(\pi-\frac{\pi}{3}\right)-\frac{\pi}{4}$ $=\frac{2\pi}{3}$	1 ½ ½
25.	Find : $\int\frac{2x}{(x^2+1)(x^2-4)}\,dx$.	
Sol.	$I=\int\frac{2x}{(x^2+1)(x^2-4)}dx$ Put $x^2=t\Rightarrow 2xdx=dt$ $I=\int\frac{1}{(t+1)(t-4)}dt=\frac{1}{5}\int\frac{dt}{t-4}-\frac{1}{5}\int\frac{dt}{t+1}$ $I=\frac{1}{5}\log\lvert x^2-4\rvert-\frac{1}{5}\log\lvert x^2+1\rvert+c$ or $\frac{1}{5}\log\left\lvert\frac{x^2-4}{x^2+1}\right\rvert+c$	 ½ 1 ½
	SECTION C **In this section there are 6 short answer type questions of 3 marks each.**	
26.	Find $\frac{dy}{dx}$, if $y=(\cos x)^x+\cos^{-1}\sqrt{x}$ is given.	
Sol.	Let u $=(\cos x)^x\Rightarrow\frac{du}{dx}=(\cos x)^x(-x\tan x+\log(\cos x))$, v $=\cos^{-1}\sqrt{x}\Rightarrow\frac{dv}{dx}=\frac{-1}{2\sqrt{x-x^2}}$ Since, $y=u+v\Rightarrow\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}=(\cos x)^x(-x\tan x+\log(\cos x))+\frac{-1}{2\sqrt{x-x^2}}$	1 1 1

27(a).	Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.	
Sol.	$\frac{dy}{dx} = y\cot 2x \Rightarrow \int \frac{dy}{y} = \int \cot 2x dx$ $\Rightarrow \log\|y\| = \frac{1}{2}\log\|\sin 2x\| + \log c$ $y = c.\sqrt{\sin 2x}$ when $y(\frac{\pi}{4}) = 2$, gives $c = 2$ $\therefore\ y = 2\sqrt{\sin 2x}$ is the required Particular solution of given D.E.	**1** **1** ½ ½
	OR	
27(b).	Find the particular solution of the differential equation $(xe^{\frac{y}{x}} + y)\, dx = x\, dy$, given that $y = 1$ when $x = 1$.	
Sol.	$\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x} = f(\frac{y}{x})$ so, its a homogeneous differential equation Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ Now, $v + x\frac{dv}{dx} = e^{v} + v$ $\Rightarrow \int e^{-v} dv = \int \frac{1}{x} dx$ $\Rightarrow -e^{-v} = \log\|x\| + c \Rightarrow -e^{\frac{-y}{x}} = \log\|x\| + c$......(1) Now, $x = 1,\ y = 1$,gives $c = -e^{-1}$ Thus, $\log\|x\| + e^{\frac{-y}{x}} = e^{-1}$	1 ½ **1** ½
28.	Find : $\int \sec^3\theta\, d\theta$	
Sol.	$I = \int \sec^3\theta\, d\theta = \int \sec^2\theta.\sec\theta\, d\theta$ $I = \sec\theta\int\sec^2\theta\, d\theta - \int\left(\frac{d(\sec\theta)}{d\theta}\right)\left(\int\sec^2\theta\, d\theta\right) d\theta$ $I = \sec\theta\tan\theta - \int\sec^3\theta\, d\theta + \int\sec\theta\, d\theta$ $I = \frac{1}{2}\left(\sec\theta\tan\theta + \log\|\sec\theta + \tan\theta\| + c)\right)$	½ **1** ½ **1**

29(a).	A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.	
Sol.	Let E_1 be the event of lost card is King, E_2 be the event of lost card not a King and A be the event of drawing a King from remaining 51 cards. so, $P(E_1)=\frac{1}{13}, P(E_2)=\frac{12}{13}, P(A\vert E_1)=\frac{3}{51}, P(A\vert E_2)=\frac{4}{51}$ Now, Required probability is $P(E_1 \vert A)$, $P(E_1 \vert A)=\frac{P(A\vert E_1)\times P(E_1)}{P(A\vert E_1)\times P(E_1)+P(A\vert E_2)\times P(E_2)}=\frac{\frac{1}{13}\times\frac{3}{51}}{\frac{1}{13}\times\frac{3}{51}+\frac{12}{13}\times\frac{4}{51}}=\frac{1}{17}$	½ 1 ½ 1
	OR	
29(b).	A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.	
Sol.	Let P(1)=P(3)=P(5) = p, so P(2)=P(4)=P(6) =2 p As, $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1 \Rightarrow 9p = 1 \Rightarrow p = \frac{1}{9}$ P(Getting 6)=$\frac{2}{9}$,P(Not getting six)=$\frac{7}{9}$ Let X represents the Number of sixes Possible values of X are 0, 1 or 2 Now, $P(X=0)=\frac{7}{9}\times\frac{7}{9}=\frac{49}{81}, P(X=1)=2\times\frac{7}{9}\times\frac{2}{9}=\frac{28}{81}, P(X=2)=\frac{2}{9}\times\frac{2}{9}=\frac{4}{81}$ Required probability distribution of number of sixes is X: 0, 1, 2 $P(X)$: $\frac{49}{81}$, $\frac{28}{81}$, $\frac{4}{81}$ Mean of X = $\sum_{i=1}^{3} X_i P(X_i) = 0+\frac{28}{81}+\frac{8}{81}=\frac{36}{81}=\frac{4}{9}$	½ ½ 1½ ½

30.

The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure :



(i) If Z = 3x – 4y be the objective function, then find the maximum value of Z.

(ii) If Z = px + qy where p, q > 0 be the objective function. Find the condition on p and q so that maximum value of Z occurs at B(4, 10) and C(6, 8).

Sol.

(i)

corner points	$Z = 3x - 4y$
$A(0,8)$	-32
$B(4,10)$	-28
$C(6,8)$	-14
$D(6,5)$	-2
$E(4,0)$	12(Maximum)
$O(0,0)$	0

1½

Maximum value of Z is 12 at E when $x = 4,\ y = 0$

(ii) $Z_B = Z_C \Rightarrow 4p + 10q = 6p + 8q$ Thus, $p = q$ **1 +½**

31(a).

Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x}{1+\cos 2x+\sin 2x}\,dx$

Sol.

$I = \int_0^{\frac{\pi}{4}} \frac{x}{1+\cos 2x+\sin 2x}\,dx \quad ....(1)$

On applying $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we get $I = \int_0^{\frac{\pi}{4}} \frac{\frac{\pi}{4}-x}{1+\cos 2x+\sin 2x}\,dx ...(2)$ **½**

On adding Eq. (1) and (2), we get $2I = \frac{\pi}{4}\int_0^{\frac{\pi}{4}} \frac{1}{1+\cos 2x+\sin 2x}\,dx$ **½**

$I = \frac{\pi}{16}\int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x+\sin x\cos x}\,dx = \frac{\pi}{16}\int_0^{\frac{\pi}{4}} \frac{\sec^2 x dx}{1+\tan x}$ **1**

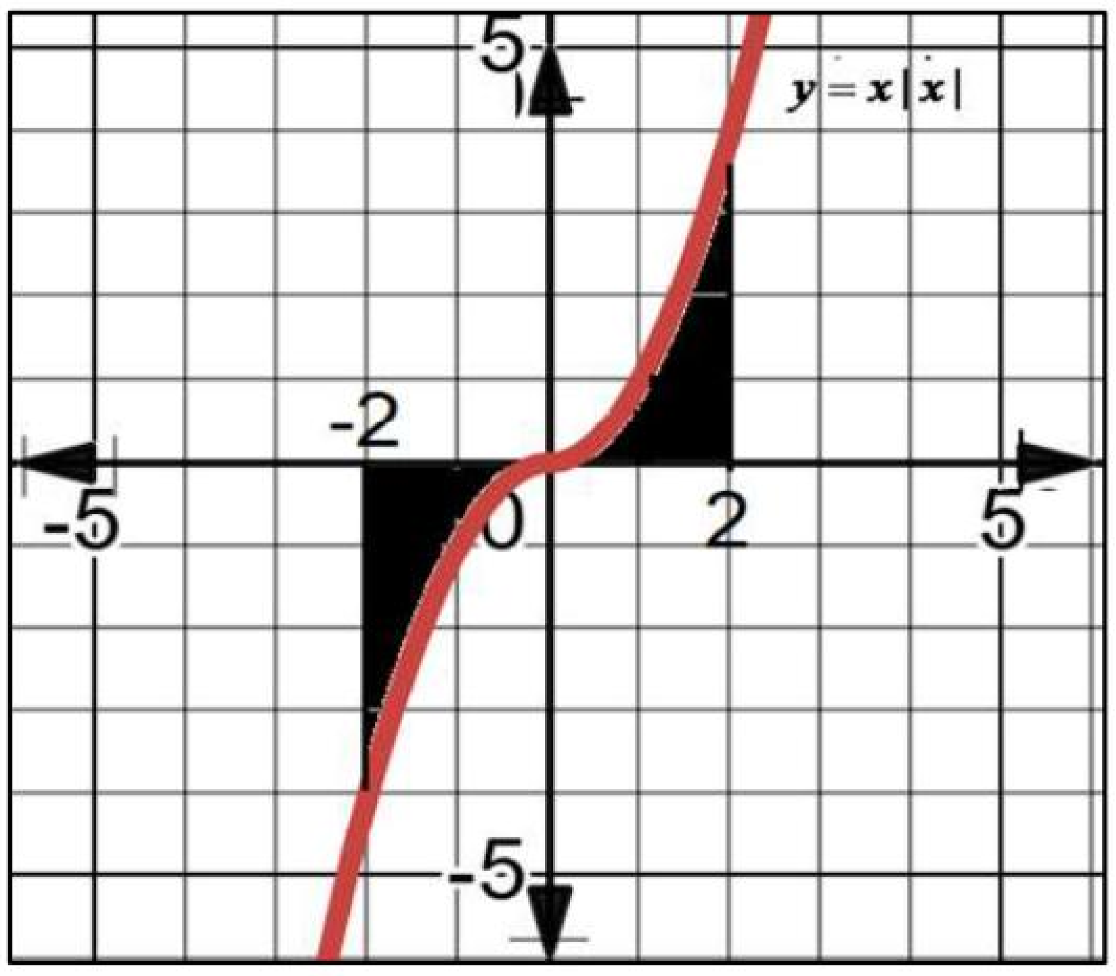
$I = \frac{\pi}{16}(\log|1+\tan x|)_0^{\frac{\pi}{4}}$ **½**

$I = \frac{\pi}{16}\log 2$ **½**

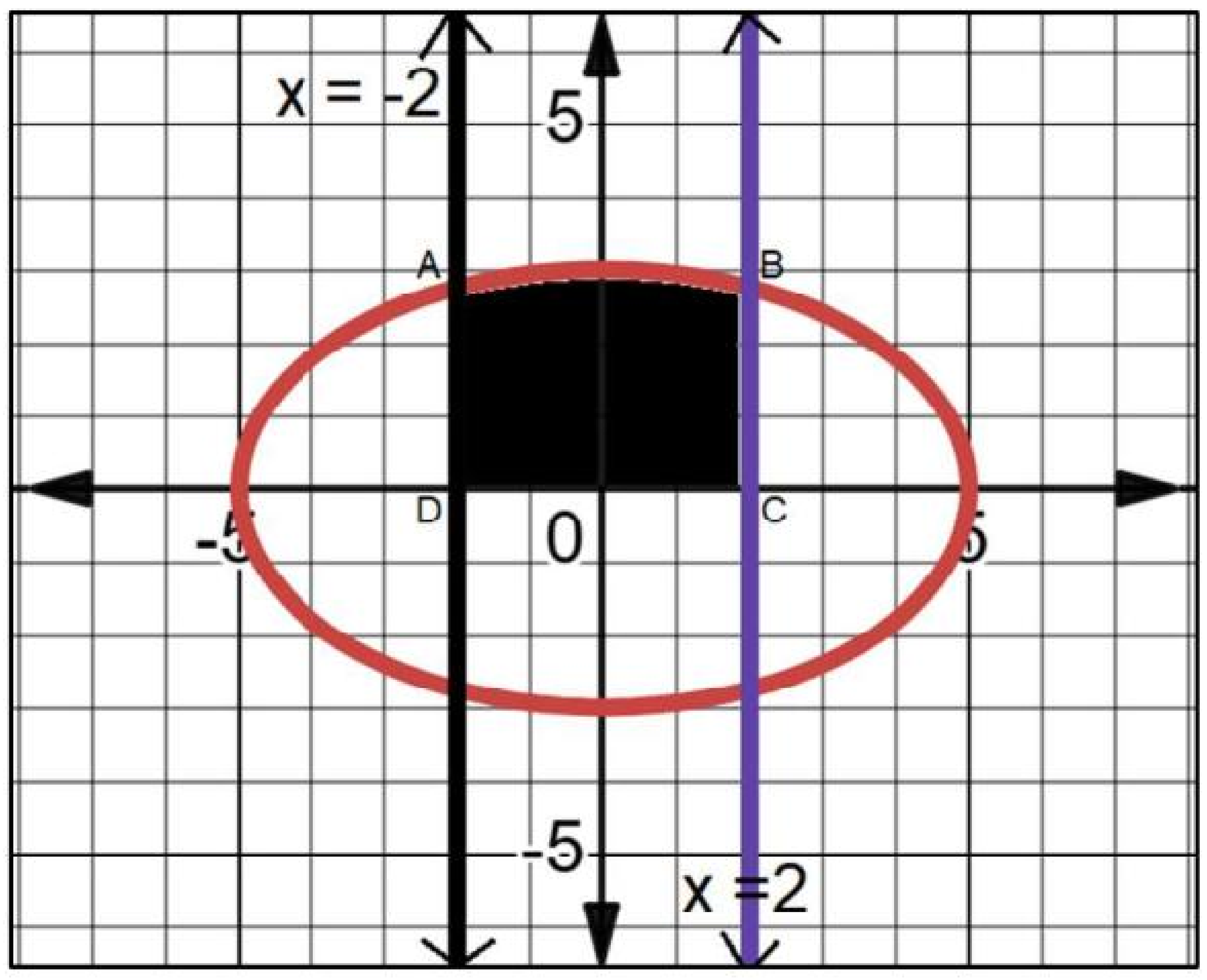
	OR	
31(b).	Find : $\int e^x \left(\frac{x}{\sqrt{1+x^2}}+\frac{1}{(1+x^2)^{\frac{3}{2}}}\right)dx$	
Sol.	$I=\int e^x \left(\frac{x}{\sqrt{1+x^2}}+\frac{1}{(1+x^2)^{\frac{3}{2}}}\right)dx$ Let $f(x)=\frac{x}{\sqrt{1+x^2}}$, $f'(x)=\frac{\sqrt{1+x^2}-x\frac{x}{\sqrt{1+x^2}}}{1+x^2}=\frac{1+x^2-x^2}{(1+x^2)\sqrt{1+x^2}}=\frac{1}{(1+x^2)^{\frac{3}{2}}}$ On applying $\int e^x(f(x)+f'(x))dx=e^x f(x)+c,\ \ I=e^x\frac{x}{\sqrt{1+x^2}}+c$	½ 1 ½ 1
	SECTION D **In this section there are 4 long answer type questions of 5 marks each.**	
32(a).	Let A = R – {5} and B = R – {1}. Consider the function f : A → B, defined by f(x) = $\frac{x-3}{x-5}$. Show that f is one-one and onto.	
Sol.	Let $f(x_1)=f(x_2)$, for some $x_1, x_2 \in A$ $\Rightarrow \frac{x_1-3}{x_1-5}=\frac{x_2-3}{x_2-5}$ $\Rightarrow (x_1-3)(x_2-5)=(x_2-3)(x_1-5)$ $\Rightarrow x_1=x_2$, So $\underline{f\ is\ one-one\ Function}$. Let $y=f(x)=\frac{x-3}{x-5} \Rightarrow y(x-5)=x-3$ $\Rightarrow yx-5y=x-3$ $\Rightarrow x=\frac{5y-3}{y-1}$, We observe that x is defined for all values of y except $y=1$, So, Range $=R-\{1\}$ and Co-domain is Given $R-\{1\}$ [As, $f:A\to B$] Since, Range = Co-domain, f is onto Function. Thus, f is one-one & onto function.	2 ½ 2 ½
	OR	
32(b).	Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where a – b + $\sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.	

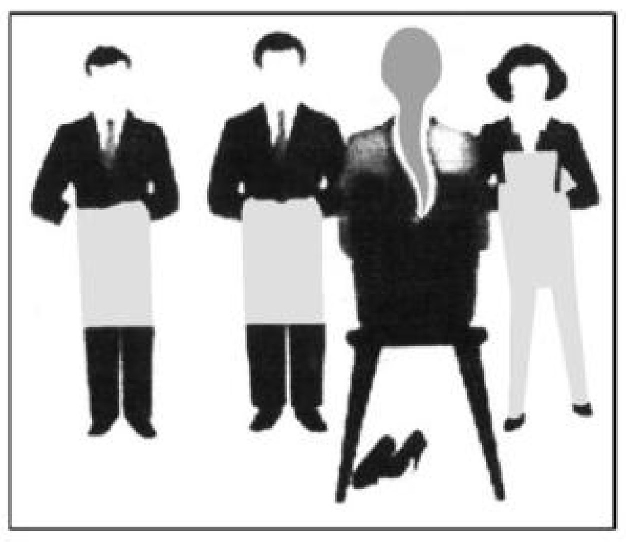
Sol.	Reflexive: For $a \in S$ $\Rightarrow a - a + \sqrt{2}$ is irrational number $\Rightarrow \sqrt{2}$ is irrational number $\Rightarrow (a, a) \in S$ Thus, S is Reflexive Relation. Symmetric: Let $(a,b) \in S \Rightarrow a - b + \sqrt{2}$ is irrational number but $b - a + \sqrt{2}$ may not be irrational number For example, $(\sqrt{2}, 1) \in S \Rightarrow \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1$ is irrational number $(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is not irrational number $\therefore (b, a) \notin S$, So S is NOT Symmetric Relation. Transitive: *Let* $(a,b) \in S \Rightarrow a - b + \sqrt{2}$ is irrational number $\&(b,c) \in S \Rightarrow b - c + \sqrt{2}$ is irrational number but $a - c + \sqrt{2}$ may not be irrational number *For* example, $(1, \sqrt{3}) \in S \Rightarrow 1 - \sqrt{3} + \sqrt{2}$ is irrational number $(\sqrt{3}, \sqrt{2}) \in S \Rightarrow \sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3}$ is irrational number But $(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is not irrational number $\therefore (a, c) \notin S$, *So S* is NOT Transitive Relation. Thus, S is Reflexive But Neither Symmetric nor Transitive Relation.	1 ½ 1 ½ 2
33(a).	Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).	
Sol.	Equation of the given line in standard form is $L_1 : \frac{x}{2} = \frac{y-3}{2} = \frac{z-1}{1}$ Equation of the line parallel to L_1 & passing through $(4,\ 0, -5)$ is $L_2 : \frac{x-4}{2} = \frac{y}{2} = \frac{z+5}{1}$ Vector Equation of Lines are $L_1 : \vec{r} = (0\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$ $L_2 : \vec{r} = (4\hat{i} + 0\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k})$	½ 1 ½

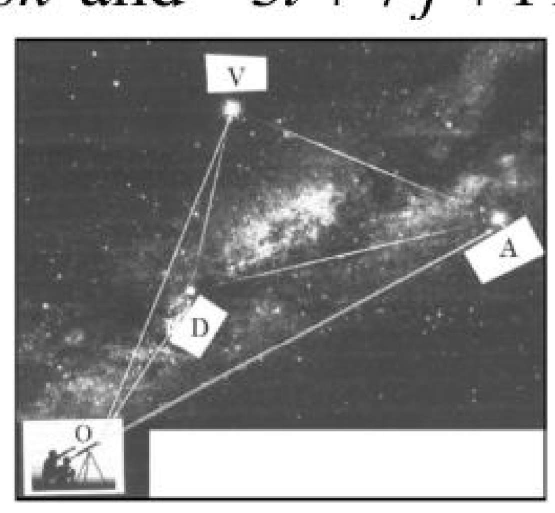
	Now, $\overrightarrow{a_2}-\overrightarrow{a_1}=(4\hat{i}+0\hat{j}-5\hat{k})-(0\hat{i}+3\hat{j}+\hat{k})=(4\hat{i}-3\hat{j}-6\hat{k})$ $\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$ $(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -3 & -6\\ 2 & 2 & 1\end{vmatrix}=9\hat{i}-16\hat{j}+14\hat{k}$ $\lvert\vec{b}\rvert=\sqrt{4+4+1}=3$ Thus, distance between the lines is $\text{S.D.}=\dfrac{\left\lvert(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}\right\rvert}{\lvert\vec{b}\rvert}=\dfrac{\sqrt{81+256+196}}{3}=\dfrac{\sqrt{533}}{3}$ units	1 ½ 1 ½
	OR	
33(b).	If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).	
	$L_1:\dfrac{x-1}{-3}=\dfrac{y-2}{2k}=\dfrac{z-3}{2}\Rightarrow$ direction ratio's of $L_1=<-3,2k,2>$ $L_2:\dfrac{x-1}{3k}=\dfrac{y-1}{1}=\dfrac{z-6}{-7}\Rightarrow$ direction ratio's of $L_2=<3k,1,-7>$ Since $L_1\perp L_2$, $-9k+2k-14=0\Rightarrow k=-2$ Thus, d.r.'s of $L_1=<-3,-4,2>$, d.r.'s of $L_2=<-6,1,-7>$ Now the vector perpendicular to both L_1 & L_2 is given by $\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3 & -4 & 2\\ -6 & 1 & -7\end{vmatrix}=26\hat{i}-33\hat{j}-27\hat{k}$ Thus, Equation of the required line is $\vec{r}=(3\hat{i}-4\hat{j}+7\hat{k})+\lambda(26\hat{i}-33\hat{j}-27\hat{k})$	½ ½ 1 2 1
34.	Find A^{-1}, if $A=\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & -1\\ 1 & 0 & 1\end{bmatrix}$. Hence, solve the following system of equations : $x+2y+z=5$ $2x+3y=1$ $x-y+z=8$	

Sol.	*For* Matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$, Adjoint of Matrix A is $\lvert A \rvert = -6 \neq 0$ so, A^{-1} exists. $adjA = \begin{pmatrix} 3 & -2 & -5 \\ -3 & 0 & 3 \\ -3 & 2 & -1 \end{pmatrix}$, Thus, $A^{-1} = \frac{-1}{6}\begin{pmatrix} 3 & -2 & -5 \\ -3 & 0 & 3 \\ -3 & 2 & -1 \end{pmatrix}$ *so*, Given equation can be written into a matrix equation as $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 8 \end{pmatrix} \Rightarrow X = (A^T)^{-1}.B = X = (A^{-1})^T.B$ $A^T \quad X \quad = \quad B$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{6}\begin{pmatrix} 3 & -3 & -3 \\ -2 & 0 & 2 \\ -5 & 3 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \\ 8 \end{pmatrix} = \frac{-1}{6}\begin{pmatrix} -12 \\ 6 \\ -30 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ $\therefore \boxed{x = 2, y = -1, z = 5}$	½ 2 ½ ½ 1 ½
35(a).	Sketch the graph of $y = x\lvert x \rvert$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.	
Sol.	 $\text{As, } y = x\lvert x \rvert = \begin{cases} -x^2, x \leq 0 \\ x^2, x \geq 0 \end{cases}$	**For correct figure 1 ½**

*These answers are meant to be used by evaluators.

	Area of the shaded region $= \int_{-2}^{2} y\ dx = 2\int_{0}^{2} y\ dx = 2\int_{0}^{2} x^2\ dx$ $= 2\left(\frac{x^3}{3}\right)_0^2$ $= 2\left(\frac{8}{3}\right) = \frac{16}{3}$	**1 ½** **1** **1**
	OR	
35(b).	Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.	
Sol.	 As, $9x^2 + 25y^2 = 225 \Rightarrow \boxed{y = \pm\frac{3}{5}\sqrt{5^2 - x^2}}$ Required Area $= \int_{-2}^{2} \frac{3}{5}\sqrt{5^2 - x^2}\ dx = \frac{6}{5}\int_{0}^{2}\sqrt{5^2 - x^2}\ dx$ $= \frac{6}{5}\left(\frac{x\sqrt{5^2 - x^2}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right)_0^2$ $= \frac{6}{5}\left(\frac{2\sqrt{21}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right)\right)$ $= \left(\frac{6\sqrt{21}}{5} + 15\sin^{-1}\left(\frac{2}{5}\right)\right)$	**For correct figure 1 ½** **1 ½** **1** **1**
	SECTION E **In this section there are 3 case-study based questions of 4 marks each.**	

36.	Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.  Based on the above information, answer the following questions: (i) What is the probability that at least one of them is selected ? (ii) Find $P(G\|\overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. (iii) Find the probability that exactly one of them is selected. OR (iii) Find the probability that exactly two of them are selected.	**1** **1** **2** **2**
Sol.	Given $P(\text{Rohit}) = \frac{1}{5}, P(\text{Jaspreet}) = \frac{1}{3}, P(\text{Alia}) = \frac{1}{4}$ (i) P(atleast one of them is selected) = 1 − P(no one is selected) $= 1-\left(\frac{4}{5}\times\frac{2}{3}\times\frac{3}{4}\right) = \frac{3}{5}$ (ii) $P(G\|\overline{H}) = \frac{P(G\cap\overline{H})}{P(\overline{H})} = \frac{1}{3}$ (iii) P(exactly one of them selected) $= P(R)\times P(\overline{J})\times P(\overline{A}) + P(\overline{R})\times P(J)\times P(\overline{A}) + P(\overline{R})\times P(\overline{J})\times P(A)$ $= \frac{6+12+8}{60} = \frac{13}{30}$ OR (iii) P(exactly two of them selected) $= P(R)\times P(J)\times P(\overline{A}) + P(R)\times P(\overline{J})\times P(A) + P(\overline{R})\times P(J)\times P(A)$ $= \frac{3+2+4}{60} = \frac{3}{20}$	**1** **1** **1** **1** **1** **1**
37.	A store has been selling calculators at Rs. 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by demand function $p = 450 - \frac{x}{2}$.	

	 Based on the above information, answer the following questions: (i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also verify the result. (ii) What rebate in price of calculator should the store give to maximise the revenue?	
Sol.	(i) Revenue by selling x items $= R(x) = x.p(x) = 450x - \frac{x^2}{2}$ $\frac{dR}{dx} = 450 - x$ For Maxima or Minima, $\frac{dR}{dx} = 0 \Rightarrow x = 450$ $\frac{d^2R}{dx^2} = -1 < 0$ (Revenue is Maximum at x = 450 units) (ii) At $x = 450, p = 450 - \frac{450}{2} = 225$ So, Rebate $= 350 - 225 =$ Rs.125 per calculator	½ 1 ½ 1 1
38.	An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0,0,0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}, 7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.  Based on the above information, answer the following questions: (i) How far is the star V from star A? (ii) Find a unit vector in the direction of $\overrightarrow{DA}$. (iii) Find the measure of $\angle$VDA. OR What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$?	1 1 2 2

Sol.	(i) $\overrightarrow{AV}$ = Position Vector of V − Position Vector of A $= -10\hat{i}+2\hat{j}+3\hat{k}$ Thus, $\lvert\overrightarrow{AV}\rvert = \sqrt{100+4+9} = \sqrt{113}$ units (ii) $\overrightarrow{DA}$ = Position Vector of A − Position Vector of D $= 5\hat{i}+2\hat{j}+4\hat{k}$ Unit vector in the direction of $\overrightarrow{DA} = \dfrac{5\hat{i}+2\hat{j}+4\hat{k}}{3\sqrt{5}}$ (iii) $\overrightarrow{DV} = -5\hat{i}+4\hat{j}+7\hat{k}$ $\angle VDA = \cos^{-1}\left(\dfrac{\overrightarrow{DV}.\overrightarrow{DA}}{\lvert\overrightarrow{DV}\rvert\lvert\overrightarrow{DA}\rvert}\right) = \cos^{-1}\left(\dfrac{11\sqrt{2}}{90}\right)$ *OR* (iii) $\overrightarrow{DV} = -5\hat{i}+4\hat{j}+7\hat{k}$ Projection of $\overrightarrow{DV}$ on $\overrightarrow{DA} = \left(\dfrac{\overrightarrow{DV}.\overrightarrow{DA}}{\lvert\overrightarrow{DA}\rvert}\right) = \dfrac{11\sqrt{5}}{15}$	**½** **½** **½** **½** **½** **1 ½** **½** **1 ½**