Marking Scheme

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Senior School Certificate Examination, 2024

MATHEMATICS PAPER CODE 65/5/3

General Instructions:

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after delibration and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**

10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME – 65/5/3

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A **(Questions nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)**	
1.	The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is : (A) $\frac{\vec{p}+3\vec{q}}{4}$ (B) $\frac{\vec{p}+3\vec{q}}{8}$ (C) $\frac{5\vec{p}+3\vec{q}}{4}$ (D) $\frac{5\vec{p}+3\vec{q}}{8}$	
Ans	**(D)** $\frac{5\vec{p}+3\vec{q}}{8}$	**1**
2.	For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is : (A) 0 (B) 10 (C) $\mathbb{R}-\{10\}$ (D) $\mathbb{R}-\{-10\}$	
Ans	**(D)** $\mathbb{R}-\{-10\}$	**1**
3.	The angle which the line $\frac{x}{1}=\frac{y}{-1}=\frac{z}{0}$ makes with the positive direction of Y-axis is : (A) $\frac{5\pi}{6}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$	
Ans	**(B)** $\frac{3\pi}{4}$	**1**

4.	The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line : $\vec{r} = (2+\lambda)\hat{i} + \lambda\hat{j} + (2\lambda-1)\hat{k}$ is (A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ (B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$ (C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$	
Ans	**(D)** $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$	**1**
5.	If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$, then value of x for which $A^2 = B$ is : (A) –2 (B) 2 (C) 2 or –2 (D) 4	
Ans	**(A) –2**	**1**
6.	Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is : (A) –60 units/sec (B) 60 units/sec (C) –70 units/sec (D) –140 units/sec	
Ans	**(A) –60 units/sec**	**1**
7.	Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$, where p is a constant. The value of p for which $f'(0) = 1$ is : (A) $\mathbb{R}$ (B) 1 (C) 0 (D) –1	
Ans	**(D) –1**	**1**

8.	If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then : (A) $A \subset B$, but $A \neq B$ (B) $A = B$ (C) $A \cap B = \phi$ (D) $P(A) = P(B)$	
Ans	**(D) $P(A) = P(B)$**	**1**
9.	A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is : (A) injective but not surjective. (B) surjective but not injective. (C) both injective and surjective. (D) neither injective nor surjective.	
Ans	**(D) neither injective nor surjective**	**1**
10.	If A is a square matrix of order 3 such that the value of $\lvert \text{adj} \cdot A \rvert = 8$, then the value of $\lvert A^T \rvert$ is : (A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 8 (D) $2\sqrt{2}$	
Ans	**(D) $2\sqrt{2}$**	**1**
11.	If $\int_{-2}^{3} x^2\, dx = k \int_{0}^{2} x^2\, dx + \int_{2}^{3} x^2\, dx$, then the value of k is : (A) 2 (B) 1 (C) 0 (D) $\frac{1}{2}$	
Ans	**(A) 2**	**1**
12.	The value of $\int_{1}^{e} \log x\, dx$ is : (A) 0 (B) 1 (C) e (D) e log e	
Ans	**(B) 1**	**1**

13.	The area bounded by the curve $y = \sqrt{x}$, Y-axis and between the lines y = 0 and y = 3 is : (A) $2\sqrt{3}$ (B) 27 (C) 9 (D) 3	
Ans	**(C) 9**	**1**
14.	The order of the following differential equation $\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^5 = 4\log\left(\frac{d^4y}{dx^4}\right)$ is : (A) not defined (B) 3 (C) 4 (D) 5	
Ans	**(C) 4**	**1**
15.	If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is : (A) –4 (B) 1 (C) 3 (D) 4	
Ans	**(D) 4**	**1**
16.	Find the matrix A^2, where $A = [a_{ij}]$ is a 2 × 2 matrix whose elements are given by a_{ij} = maximum (i, j) – minimum (i, j) : (A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	
Ans	**(C)** $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$	**1**

17.	Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is : (A) $\sin x\, e^{\sin^2 x}$ (B) $\cos x\, e^{\sin^2 x}$ (C) $-2\cos x\, e^{\sin^2 x}$ (D) $-2\sin^2 x \cos x\, e^{\sin^2 x}$	
Ans	**(C) $-2\cos x\, e^{\sin^2 x}$**	**1**
18.	The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to : (A) 2 (B) 1 (C) 0 (D) –2	
Ans	**(A) 2**	**1**
(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
	Assertion – Reason Based Questions **Direction :** In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options : (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	**Assertion (A)** : Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$. **Reason (R)** : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.	
Ans	**(C) Assertion (A) is true, but Reason (R) is false**	**1**

20.	**Assertion (A) :** The vectors $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$, $\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$, $\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ represent the sides of a right angled triangle. **Reason (R) :** Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.	
Ans	**(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A)**	**1**
	SECTION-B **(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)**	
21.	Simplify : $\cos^{-1}x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]; \frac{1}{2} \le x \le 1$	
Ans	**Let** $x = \cos\theta$, $\cos^{-1}x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]$ $= \theta + \cos^{-1}\left[\frac{\cos\theta}{2} + \frac{\sqrt{3}}{2} \times \sin\theta\right]$ $= \theta + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \theta\right)\right] = \theta + \frac{\pi}{3} - \theta = \frac{\pi}{3}$	**½** **1** **½**
22.	(a) Find : $\int \cos^3 x\, e^{\log \sin x}\, dx$ **OR** (b) Find : $\int \frac{1}{5 + 4x - x^2}\, dx$	

Ans	(a) $\int \cos^3 x\, e^{\log \sin x} dx = \int \cos^3 x \cdot \sin x\, dx$, Assuming $\cos x = t$ and $\sin x\, dx = -dt$ $= -\int t^3 dt$ $= -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$ **Or** (b) $\int \frac{1}{5+4x-x^2} dx = \int \frac{1}{3^2-(x-2)^2} dx$ $= \frac{1}{6}\log\left\|\frac{1+x}{5-x}\right\| + C$	1 ½ ½ 1 1
23.	The surface area of a cube increases at the rate of 72 cm^2/sec. Find the rate of change of its volume, when the edge of the cube measures 3 cm.	
Ans	Let edge of cube be 'x cm' $S = 6x^2, \frac{dS}{dt} = 72 cm^2/sec \Rightarrow 12x\frac{dx}{dt} = 72 \Rightarrow \frac{dx}{dt} = \frac{6}{x}$ Volume, $V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2\frac{dx}{dt} = 3 \times x^2 \times \frac{6}{x} = 18x, \left.\frac{dV}{dt}\right]_{x=3} = 54 cm^3/sec$ $\therefore$ Volume is increasing at the rate of 54 cm^3/sec	 ½ + ½ ½ + ½
24.	Find the vector equation of the line passing through the point (2, 3, –5) and making equal angles with the co-ordinate axes.	
Ans	(a) $\cos\alpha = \cos\beta = \cos\gamma = l \Rightarrow l^2 + l^2 + l^2 = 1 \Rightarrow 3l^2 = 1, \therefore l = 1/\sqrt{3}$ Direction cosines of the line are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ $\Rightarrow$ the direction ratios are 1,1,1 $\therefore$ Vector equation of line is: $\vec{r} = 2i + 3j - 5k + \lambda(\hat{i} + \hat{j} + \hat{k})$	1 ½ ½

25.	(a) Verify whether the function f defined by $f(x) = \begin{cases} x\sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ or not. **OR** (b) Check for differentiability of the function f defined by $f(x) = \lvert x-5 \rvert$, at the point $x = 5$.	
Ans	**(a)** $\lim_{x\to 0} f(x) = \lim_{x\to 0} x.\sin\frac{1}{x} = 0 \times \text{Finite value in } [-1,1] = 0 = f(0)$	$1\frac{1}{2}$
	$\therefore$ **f(x) is a continuous function.**	$\frac{1}{2}$
	Or	
	(b) $\text{LHD} = \lim_{x\to 5^-} \frac{\lvert x-5\rvert - 0}{x-5} = \lim_{x\to 5^-} \frac{-(x-5)}{x-5} = -1$	1
	$\text{RHD} = \lim_{x\to 5^+} \frac{\lvert x-5\rvert - 0}{x-5} = \lim_{x\to 5^+} \frac{(x-5)}{x-5} = 1$	$\frac{1}{2}$
	$\text{LHD} \neq \text{RHD}, \therefore f(x)$ **is not differentiable at** $x = 5$	$\frac{1}{2}$

SECTION-C

(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)

26.	(a) Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$ **OR** (b) Find : $\int \frac{2x+1}{(x+1)^2 (x-1)} dx$	
Ans	**(a) Let** $I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$ **---------- (i)**	

	$\Rightarrow I=\int_0^{\pi}\frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)}+e^{-\cos(\pi-x)}}dx=\int_0^{\pi}\frac{e^{-\cos x}}{e^{-\cos x}+e^{\cos x}}dx$ --------- (ii) Adding (i) and (ii), we get $2I=\int_0^{\pi}dx=x]_0^{\pi}=\pi,\ \therefore I=\frac{\pi}{2}$ **Or** (b) $\int\frac{2x+1}{(x+1)^2(x-1)}dx=-\frac{3}{4}\int\frac{1}{x+1}dx+\frac{1}{2}\int\frac{1}{(x+1)^2}dx+\frac{3}{4}\int\frac{1}{x-1}dx$ $=-\frac{3}{4}\log\lvert x+1\rvert-\frac{1}{2(x+1)}+\frac{3}{4}\log\lvert x-1\rvert+C$ or, $=\frac{3}{4}\log\left\lvert\frac{x-1}{x+1}\right\rvert-\frac{1}{2(x+1)}+C$	$1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$
27.	If $y=(\tan^{-1}x)^2$, show that $(x^2+1)^2\frac{d^2y}{dx^2}+2x(x^2+1)\frac{dy}{dx}=2$.	
Ans	$y=(\tan^{-1}x)^2\Rightarrow\frac{dy}{dx}=\frac{2\tan^{-1}x}{1+x^2}$ $\Rightarrow(1+x^2)\frac{dy}{dx}=2\tan^{-1}x$, differentiating with respect to 'x' $\Rightarrow(1+x^2)^2\frac{d^2y}{dx^2}+2x(x^2+1)\frac{dy}{dx}=2$	$1\frac{1}{2}$ $1\frac{1}{2}$
28.	(a) Find the particular solution of the differential equation $\frac{dy}{dx}-2xy=3x^2e^{x^2}$; $y(0)=5$. **OR** (b) Solve the following differential equation : $x^2\,dy+y(x+y)\,dx=0$	
Ans	**(a) Given differential equation is a linear order differential equation with:** $P=-2x,\ Q=3x^2e^{x^2}$ Integrating Factor $=e^{\int-2xdx}=e^{-x^2}$ The general solution is : $y\cdot e^{-x^2}=\int e^{-x^2}\cdot 3x^2e^{x^2}dx+C\Rightarrow y\cdot e^{-x^2}=x^3+C$	$\frac{1}{2}$ 1 1

	Putting $x=0, y=5$, we get, $C=5$ $\therefore$ The Particular solution is: $y\cdot e^{-x^2}=x^3+5$ or $y=\left(x^3+5\right)e^{x^2}$ **Or** (b) $x^2dy+y\left(x+y\right)dx=0 \Rightarrow \frac{dy}{dx}=-\frac{y}{x}-\left(\frac{y}{x}\right)^2$ Putting $\frac{y}{x}=v \Rightarrow y=vx,\ \frac{dy}{dx}=v+x\frac{dv}{dx}$ $v+x\frac{dv}{dx}=-v-v^2$, separating the variable and integrating $\int\frac{1}{v^2+2v}dv=-\int\frac{1}{x}dx$ $\Rightarrow\int\frac{1}{\left(v+1\right)^2-1}dv=-\int\frac{1}{x}dx$ $\Rightarrow\frac{1}{2}\log\left\vert\frac{v}{v+2}\right\vert=\log\left\vert\frac{C}{x}\right\vert$ The solution of the differential equation is, $\left\vert\frac{y}{y+2x}\right\vert=\frac{C^2}{x^2}$ or $x^2y=k\left(y+2x\right)$	½ ½ ½ ½ ½ ½ ½
29.	(a) Find $\frac{dy}{dx}$, if $(\cos x)^y=(\cos y)^x$. **OR** (b) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.	
Ans	(a) Taking 'log' on both sides of $\left(\cos x\right)^y=\left(\cos y\right)^x$, we get $y\log\cos x=x\log\cos y$ $\Rightarrow\frac{dy}{dx}\log\cos x+y\left(-\tan x\right)=\log\cos y+x\left(-\tan y\right)\frac{dy}{dx}$ $\Rightarrow\frac{dy}{dx}=\frac{\log\cos y+y\tan x}{\log\cos x+x\tan y}$ **Or** (b) Let $x=\sin A, y=\sin B \therefore A=\sin^{-1}x,\ B=\sin^{-1}y$	 1 1½ ½ ½

	$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ $\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$ $\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Rightarrow \cot\left(\frac{A-B}{2}\right)=a \Rightarrow A-B=2\cot^{-1}a$, $\therefore \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$, **differentiating with respect to 'x'.** $\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$	1 ½ 1
30.	Find the projection of vector $(\vec{b}+\vec{c})$ on vector $\vec{a}$, where $\vec{a}=2\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{k}$.	
Ans	$\vec{b}+\vec{c}=2\hat{i}+3\hat{j}+2\hat{k}$ **Projection of** $(\vec{b}+\vec{c})$ **on** $\vec{a}=\frac{(\vec{b}+\vec{c})\cdot\vec{a}}{\lvert\vec{a}\rvert}=\frac{4+6+2}{\sqrt{4+4+1}}$ $=4$	1 1½ ½
31.	An urn contains 3 red and 2 white marbles. Two marbles are drawn one by one with replacement from the urn. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.	
Ans	**Let X = Number of white marbles** **X:** 0, 1, 2 **P(X):** $9/25$, $12/25$, $4/25$ **Mean** $=0\times\frac{9}{25}+1\times\frac{12}{25}+2\times\frac{4}{25}=\frac{20}{25}=\frac{4}{5}$	1 1½ ½

SECTION-D

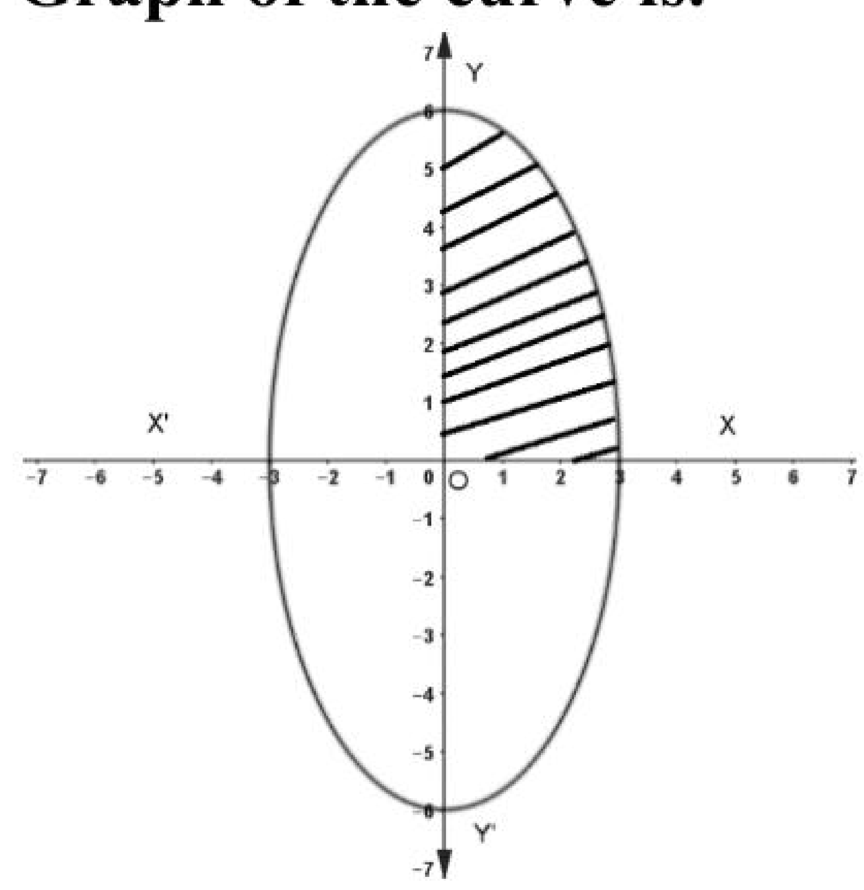
(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)

32. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

Ans

The given equation can be written as: $\frac{x^2}{9}+\frac{y^2}{36}=1$, **which is an ellipse.**

Graph of the curve is.



Correct Graph — 1

Area of the curve

$$=4\times\frac{6}{3}\int_0^3\sqrt{9-x^2}\,dx$$ — $1\frac{1}{2}$

$$=8\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$ — $1\frac{1}{2}$

$$=18\pi$$ — 1

33. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point (2, 3, –8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

OR

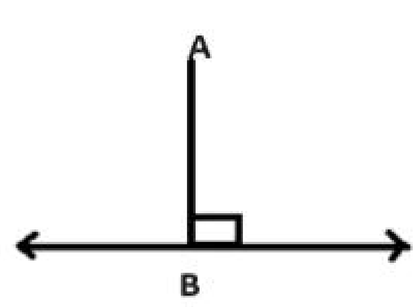
(b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through (2, –1, 1) and parallel to $\frac{x}{1}=\frac{y}{1}=\frac{z}{3}$

$L_2 : \vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

Ans

(a) The standard form of the equation of the line is $\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}$ — ½



Let foot of the perpendicular from the point $A(2,3,-8)$ **to the given line be** $B(-2\lambda+4, 6\lambda, -3\lambda+1)$ — ½

D-ratios of AB is: $-2\lambda+2, 6\lambda-3, -3\lambda+9$ — 1

As AB is perpendicular to the given line: $-2(-2\lambda+2)+6(6\lambda-3)-3(-3\lambda+9)=0$

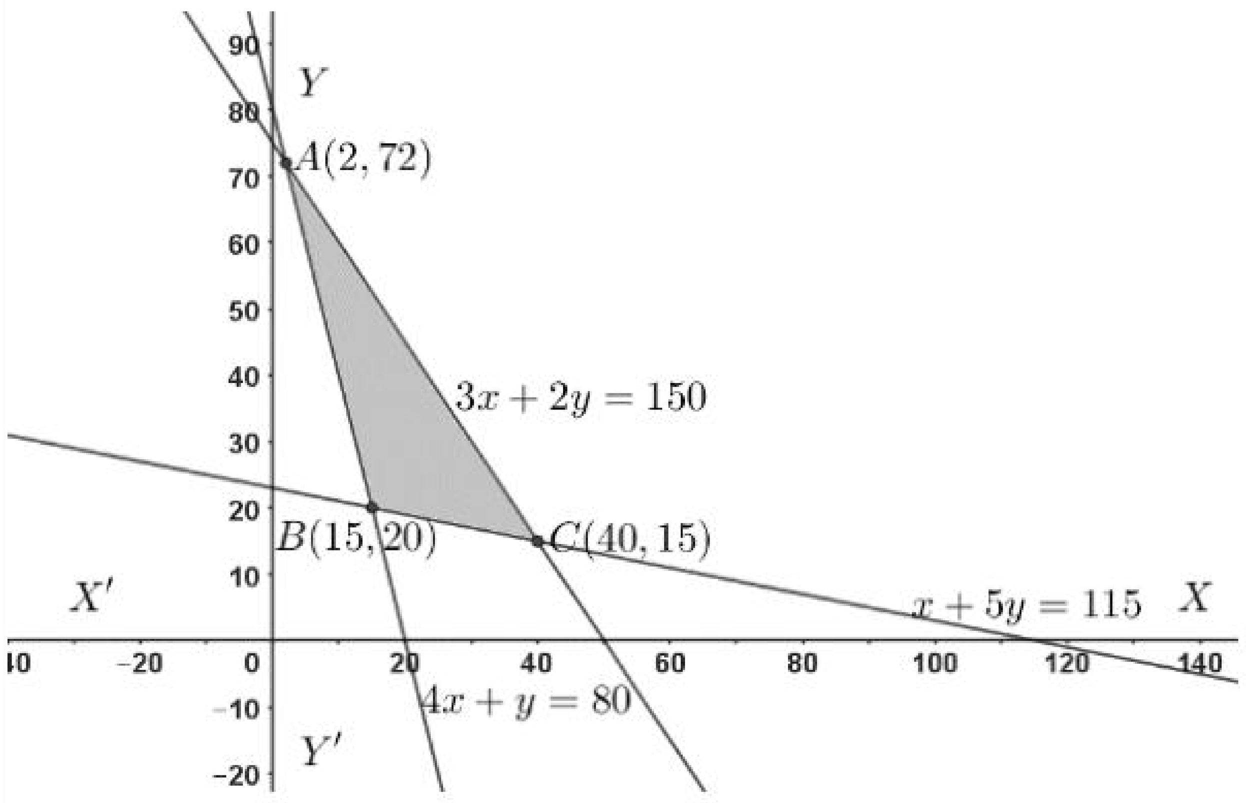
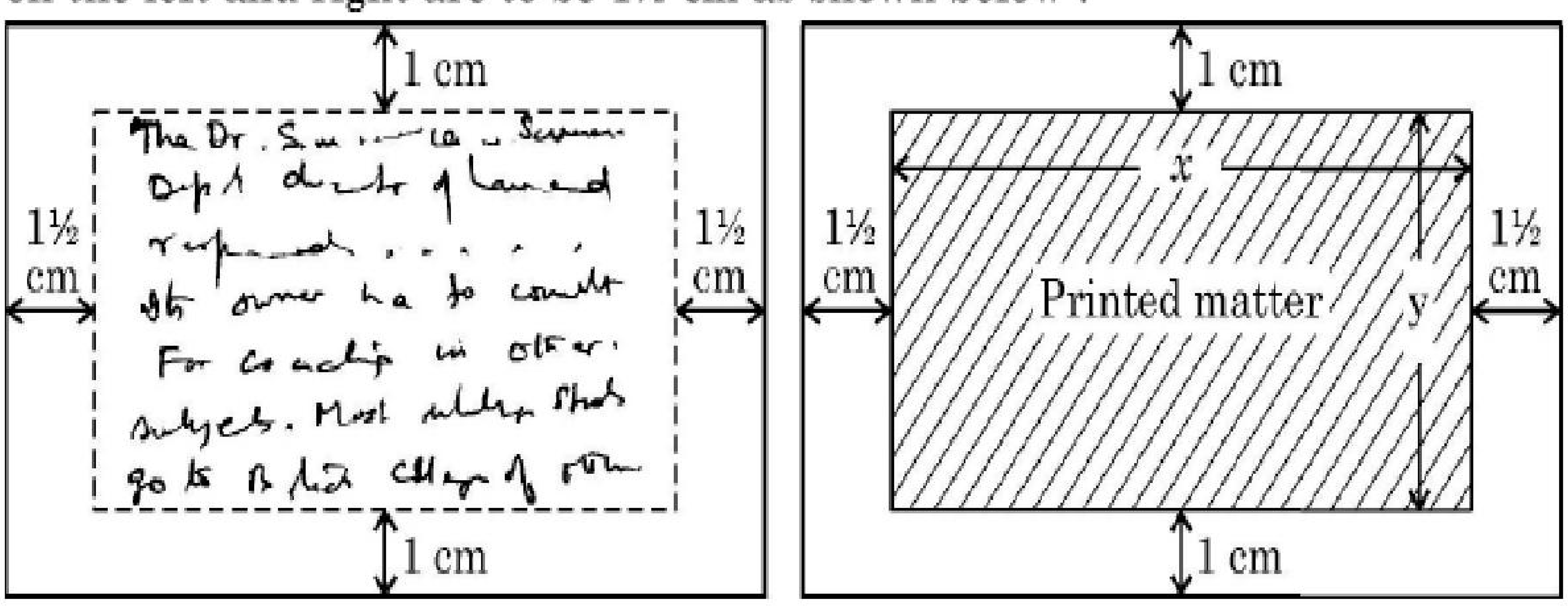
$\Rightarrow \lambda = 1$ — 1

$\therefore$ **Foot of the perpendicular is:** $B(2,6,-2)$ — 1

Perpendicular distance $= AB = 3\sqrt{5}$ — 1

	Or	
	(b) Equation of L_1 : $\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(\hat{i} + \hat{j} + 3\hat{k})$	1
	Equation of L_2 : $\vec{r} = (\hat{i} + \hat{j} - 2\hat{k}) + \mu(2\hat{j} - \hat{k})$	½
	Taking $\vec{a}_1 = 2\hat{i} - \hat{j} + \hat{k},\ \vec{b}_1 = \hat{i} + \hat{j} + 3\hat{k}$ $\vec{a}_2 = \hat{i} + \hat{j} - 2\hat{k},\ \vec{b}_2 = 2\hat{j} - \hat{k}$	½
	$\vec{a}_2 - \vec{a}_1 = -\hat{i} + 2\hat{j} - 3\hat{k},\ \vec{b}_1 \times \vec{b}_2 = -7\hat{i} + \hat{j} + 2\hat{k}$	½+1
	Shortest Distance $= \frac{\lvert(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\rvert}{\lvert\vec{b}_1 \times \vec{b}_2\rvert} = \frac{1}{\sqrt{6}}$	1½
34.	(a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations : $x + 2y - 3z = 1$ $2x - 3z = 2$ $x + 2y = 3$ **OR** (b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations : $x + 2y - 3z = -4$ $2x + 3y + 2z = 2$ $3x - 3y - 4z = 11$	
Ans	**(a)** $\lvert A\rvert = 1(6) - 2(3) - 3(4) = -12 \neq 0$, $\therefore$ A^{-1} exist	1
	$\text{adj}A = \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$	2
	$\therefore A^{-1} = -\frac{1}{12}\begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$	½

	The given system of equations can be written as $AX = B,\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12}\begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1/2 \\ 2/3 \end{bmatrix}$ $\therefore$ **The solution of the given system of equations is:** $x = 2, y = \frac{1}{2}, z = \frac{2}{3}$ **Or** **(b)** $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} = \begin{bmatrix} 67 & 0 & 0 \\ 0 & 67 & 0 \\ 0 & 0 & 67 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ **Solution of the system of equations is given by:** $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix},$ $\therefore\ x = 3, y = -2, z = 1$	$1\frac{1}{2}$ $2\frac{1}{2}$ 1 $1\frac{1}{2}$
35.	Solve the following L.P.P. graphically : Minimise $Z = 6x + 3y$ Subject to constraints $4x + y \geq 80;$ $x + 5y \geq 115;$ $3x + 2y \leq 150$ $x, y \geq 0$	

Ans	 **Correct Graph** **Corner Points** / **Value of $Z = 6x + 3y$** $A(2,72)$: $Z = 228$ $B(15,20)$: $Z = 150$ $C(40,15)$: $Z = 285$ $\text{Min}(Z) = 150$ at $x = 15, y = 20$	$3\frac{1}{2}$ 1 $\frac{1}{2}$
36.	A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1½ cm as shown below :  On the basis of the above information, answer the following questions : (i) Write the expression for the area of the visiting card in terms of x. (ii) Obtain the dimensions of the card of minimum area.	
Ans	**(i) Let $A(x)$ be the area of the visiting card then,** As $xy = 24$, $A(x) = (x+3)(y+2) = 2x + 3y + xy + 6 = 2x + \frac{72}{x} + 30$ **(ii)** $A'(x) = 2 - \frac{72}{x^2}$ and $A''(x) = \frac{144}{x^3}$, solving $A'(x) = 0 \Rightarrow x = 6$ is the critical point. $A''(6) = \frac{144}{6} > 0$, $\therefore$ Area of the card is minimum at $x = 6, y = 4$ **The dimension of the card with minimum area is Length = 9 cm, Breadth = 6 cm**	2 1 $\frac{1}{2}$ $\frac{1}{2}$

37.	A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month. Based on the above information, answer the following questions : (i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1)$, $P(E_2)$. (ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A\|E_1)$ and $P(A\|E_2)$. (iii) Find the probability of customer paying second month's bill in time. **OR** (iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.	
Ans	**(i)** $P(E_1)=\frac{7}{10}=0.7,\ P(E_2)=\frac{3}{10}=0.3$ **(ii)** $P(A\|E_1)=0.8, P(A\|E_2)=0.4$ **(iii)** $P(A)=P(E_1)\cdot P(A\|E_1)+P(E_2)\cdot P(A\|E_2)=0.7\times0.8+0.3\times0.4=0.68 \text{ or } \frac{17}{25}$	1 1 2

	Or **(iii)** $P(A) = \frac{P(E_1)\cdot P(A\|E_1)}{P(E_1)\cdot P(A\|E_1)+P(E_2)\cdot P(A\|E_2)} = \frac{14}{17}$	2
38.	(a) Students of a school are taken to a railway museum to learn about railways heritage and its history.  An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by $R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$ On the basis of the above information, answer the following questions : (i) Find whether the relation R is symmetric or not. (ii) Find whether the relation R is transitive or not. (iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it. **OR** (b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.	

Ans	**(a) (i) Let $\left(l_1, l_2\right) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow \left(l_2, l_1\right) \in R$, $\therefore$ R is a symmetric relation**	1
	(ii) Let $\left(l_1, l_2\right), \left(l_2, l_3\right) \in R \Rightarrow l_1 \parallel l_2$, $l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow \left(l_1, l_3\right) \in R$, $\therefore$ R is a transitive relation	1
	(iii) The set is $\{l : l \text{ is a line of type } y = 3x + c, c \in R\}$	2
	Or	
	(b) Let $\left(l_1, l_2\right) \in R \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow \left(l_2, l_1\right) \in R$, $\therefore$ R is a symmetric relation	2
	Let $\left(l_1, l_2\right), \left(l_2, l_3\right) \in R \Rightarrow l_1 \perp l_2, l_2 \perp l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow \left(l_1, l_3\right) \notin R$, $\therefore$ R is not a transitive relation	2
	<u> Due to printing error Part (a) or Part(b), both parts be taken as independent questions of 4 marks each</u>**	