Series EF1GH/4

SET~3

रोल नं.

Roll No.

प्रश्न-पत्र कोड
Q.P. Code **65/4/3**

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

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निर्धारित समय : 3 घण्टे | अधिकतम अंक : 80

Time allowed : 3 hours | *Maximum Marks : 80*

नोट / NOTE :

(i) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।
Please check that this question paper contains **23** printed pages.

(ii) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।
Please check that this question paper contains **38** questions.

(iv) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
Please write down the serial number of the question in the answer-book before attempting it.

(v) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If A is a 3×4 matrix and B is a matrix such that A′B and AB′ are both defined, then the order of the matrix B is :

(a) 3×4 (b) 3×3

(c) 4×4 (d) 4×3

2. If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq units, then k is

(a) 12 (b) -2

(c) $-12, -2$ (d) $12, -2$

3. If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of $f(x)$ at $x = 0$ is :

(a) 6

(b) 5

(c) 3

(d) 2

4. If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then :

(a) $x = 1, \ y = 2$

(b) $x = 2, \ y = 1$

(c) $x = 1, \ y = -1$

(d) $x = 3, \ y = 2$

5. If a matrix $A = [1 \ \ 2 \ \ 3]$, then the matrix AA' (where A' is the transpose of A) is :

(a) 14

(b) $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}$

(d) $[14]$

6. The product $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$ is equal to :

(a) $\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$

(b) $\begin{bmatrix}(a+b)^2 & 0\\(a+b)^2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}a^2+b^2 & 0\\a^2+b^2 & 0\end{bmatrix}$

(d) $\begin{bmatrix}a & 0\\0 & b\end{bmatrix}$

7. Distance of the point (p, q, r) from y-axis is :

(a) q

(b) $|q|$

(c) $|q| + |r|$

(d) $\sqrt{p^2 + r^2}$

8. The solution set of the inequation $3x + 5y < 7$ is :

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

9. If $\int_0^a 3x^2\,dx = 8$, then the value of 'a' is :

(a) 2 (b) 4

(c) 8 (d) 10

10. The sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\sqrt{\frac{5}{21}}$ (b) $\frac{5}{\sqrt{21}}$

(c) $\sqrt{\frac{3}{21}}$ (d) $\frac{4}{\sqrt{21}}$

11. The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are :

(a) 2, 2 (b) 1, 3

(c) 2, 3 (d) 2, degree not defined

12. $\int e^{5 \log x}\,dx$ is equal to :

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$

(c) $5x^4 + C$ (d) $6x^5 + C$

13. A unit vector along the vector $4\hat{i} - 3\hat{k}$ is :

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$

(b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$

(d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

14. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?

(a) $(-2, 4)$ (b) $(3, 2)$

(c) $(-5, 6)$ (d) $(4, 2)$

15. If $y = \sin^2 (x^3)$, then $\frac{dy}{dx}$ is equal to :

(a) $2 \sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2 (x^3)$

16. The point (x, y, 0) on the xy-plane divides the line segment joining the points (1, 2, 3) and (3, 2, 1) in the ratio :

(a) 1 : 2 internally (b) 2 : 1 internally

(c) 3 : 1 internally (d) 3 : 1 externally

17. The events E and F are independent. If $P(E) = 0{\cdot}3$ and $P(E \cup F) = 0{\cdot}5$, then $P(E/F) - P(F/E)$ equals :

(a) $\frac{1}{7}$

(b) $\frac{2}{7}$

(c) $\frac{3}{35}$

(d) $\frac{1}{70}$

18. The integrating factor for solving the differential equation $x\frac{dy}{dx} - y = 2x^2$ is :

(a) e^{-y}

(b) e^{-x}

(c) x

(d) $\frac{1}{x}$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* The lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are perpendicular, when $\vec{b_1} \cdot \vec{b_2} = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by $\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$

20. *Assertion (A) :* All trigonometric functions have their inverses over their respective domains.

Reason (R) : The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If $xy = e^{x-y}$, then show that $\dfrac{dy}{dx} = \dfrac{y(x-1)}{x(y+1)}$.

22. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(b) Evaluate :

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

23. If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

24. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.

25. (a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3};\quad \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

OR

(b) The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{2}{(1-x)(1+x^2)}\,dx$$

27. (a) Evaluate :

$$\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}\,dx$$

OR

(b) Evaluate :

$$\int_{1}^{3} \{|(x-1)|+|(x-2)|\}\,dx$$

28. Solve the following linear programming problem graphically :

Maximise z = 5x + 3y

subject to the constraints

$3x + 5y \leq 15,$

$5x + 2y \leq 10,$

$x, y \geq 0.$

29. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

30. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$, $y(1) = 0$.

OR

(b) Find the general solution of the differential equation $e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$.

31. (a) Evaluate :

$$\int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

OR

(b) Evaluate :

$$\int_{-2}^{2} \frac{x^2}{1 + 5^x} \, dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the image of the point (2, – 1, 5) in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$.

OR

(b) Vertices B and C of Δ ABC lie on the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4}$. Find the area of Δ ABC given that point A has coordinates (1, –1, 2) and the line segment BC has length of 5 units.

33. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations

$x - y + 2z = 1$; $2y - 3z = 1$; $3x - 2y + 4z = 3$.

34. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

35. (a) If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b) \, R \, (c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(b) Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ? *1*

(ii) What is the probability that construction will be completed on time ? *1*

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? *2*

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? *2*

Case Study – 2

37. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim\limits_{h\to 0} \dfrac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4}, & x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at x = 1 ? *1*

(ii) What is L.H.D. of f(x) at x = 1 ? *1*

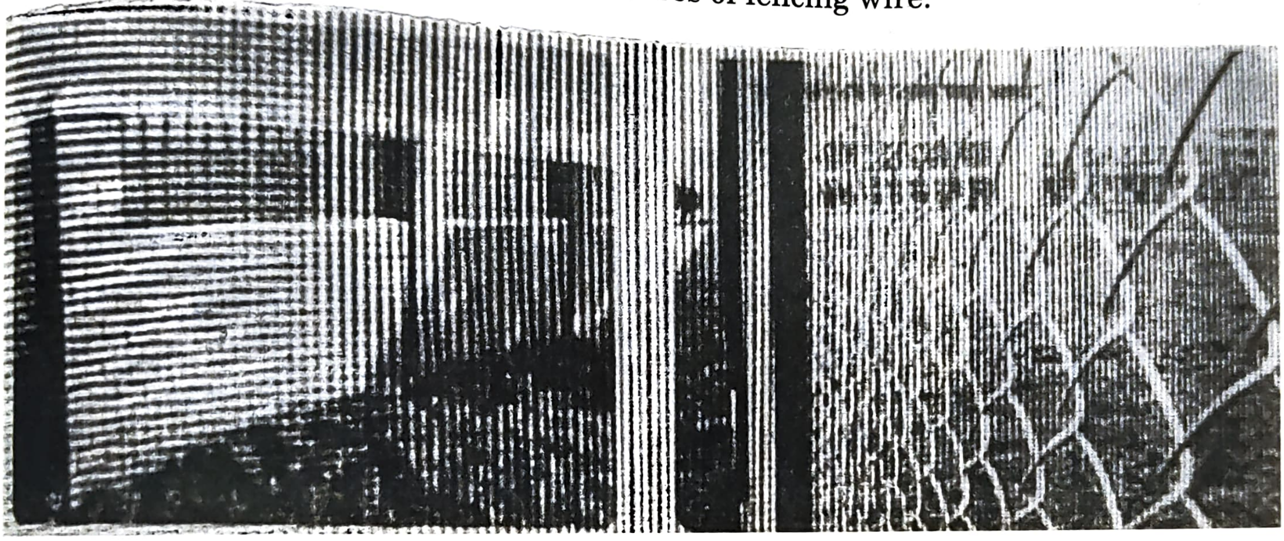
(iii) (a) Check if the function f(x) is differentiable at x = 1. 2

OR

(iii) (b) Find the $f'(2)$ and $f'(-1)$. 2

Case Study – 3

38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. 2

(ii) Determine the maximum value of A(x). 2