CUET Mathematics Solution 2024
SET B

Ques 1.If A and B are symmetric matrices of the same order, then AB - BA is a:
(3) Skew-symmetric matrix
Explanation:

- A symmetric matrix has the property $A = A^T$ (transpose).
- When A and B are both symmetric, AB ≠ BA in general.
- However, $AB^T - BA^T = (AB - BA)^T$ (transpose both sides).
- Since the transpose of a difference is the difference of transposes, this becomes $(BA - AB)^T$.
- Due to the symmetry of A and B, $BA^T = B$ and $AB^T = A$, so the expression becomes $(B - A)^T$.
- Finally, the transpose of a skew-symmetric matrix is negative of itself: $(A^T)^{-1} (-A) = A$, so $(B - A)^T = -(B - A)$.

This implies AB - BA is a skew-symmetric matrix.

Ques 2. If A is a square matrix of order 4 and |A| = 4 then |2A| will be:
(2) 64
Explanation:

- The determinant of a constant multiple of a matrix is the product of the constant and the original determinant: |kA| = k |A|.
- Therefore, |2A| = 2 * |A| = 2 * 4 = 8.

However, there's a subtlety here. The determinant of a matrix can be positive, negative, or zero. In this case, we cannot definitively say whether |A| is positive or negative based on the given information.
If |A| is positive:

- Then, |2A| = 2 * |A| = 2 * 4 = 8.

If |A| is negative:

- Then, |2A| = 2 * |-A| = 2 * (-4) = -8. We discard this because the determinant cannot be negative.

Therefore, the most accurate answer we can provide is that |2A| will be either 8 or -8, depending on the sign of |A|. However, in most practical scenarios, determinants are assumed to be positive.

Ques 3. If [A] 3 * 2 [B] x y =[C] 3*1 then:*

(4) x = 3, y = 1

Explanation:

- For matrix multiplication to be valid, the number of columns in the first matrix (A) must equal the number of rows in the second matrix (B).
- In this case, we have 3 columns in A and x rows in B. So, x must be 3.
- After multiplication, the resulting matrix (C) has dimensions 3 (same as the number of rows in A) x 1 (since B has only 1 column).

Therefore, x = 3 and y = 1.

Ques 4. If a function f(x) = x ^ 2 + bx + 1 is increasing in the interval [1, 2], then the least value of b is:

(3) -2

Explanation:

- A function is increasing if its derivative f'(x) is positive for all x in the interval.
- f'(x) = 2x + b.
- For f(x) to be increasing in [1, 2], f'(x) > 0 for all x between 1 and 2.
- We need to find the smallest b that satisfies this condition.

Case 1: b = -2

- f'(x) = 2x - 2.
- f'(1) = 0 and f'(2) = 2.
- Since f'(1) = 0, the function neither increases nor decreases at x = 1. However, f'(2) is positive, indicating an increase at x = 2. This means the function starts flat and then increases within the interval.

Case 2: b < -2

- f'(x) = 2x + b < 0 for all x in [1, 2].

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- This implies the function is decreasing throughout the interval, contradicting the given condition.

Therefore, the least value of b for which f(x) is increasing in [1, 2] is b = -2.

Ques 5. Two dice are thrown simultaneously. If X denotes the number of fours, then the expectation of X will be:

(2) 1/3

Explanation:

- There are 6 possible outcomes for each die (1 to 6).
- There are

Ques 7. Objective Function Maximum Value:

We know the objective function Z = ax + by is maximized at points (8, 2) and (4, 6). We are also given that a >= 0, b >= 0, and ab = 25. We need to find the maximum value of Z.

Approach 1: Using the points

1. Since Z is maximized at these points, we can plug these coordinates into the equation and compare the results.
2. At (8, 2): Z = a(8) + b(2)
3. At (4, 6): Z = a(4) + b(6)

However, we don't have enough information to solve for a and b uniquely.

Here's why:

- We have two equations with two unknowns (a and b).
- However, both equations represent the maximum value of Z, so they will likely be equal (or very close due to rounding errors).

Therefore, this approach won't give us a definitive answer.

Approach 2: Using properties

1. We know ab = 25 and a >= 0, b >= 0. This implies both a and b can't be negative.
2. Since Z is maximized, we want to maximize the product of a and b (given ab = 25).
3. The maximum product of two non-negative numbers that multiply to 25 occurs when they are as close to equal as possible.

Solution:

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In this case, both a and b should be 5. This satisfies the condition ab = 25 and maximizes their product.

Maximum Value of Z:

- Substitute a = 5 and b = 5 into Z = ax + by: Z = 5(8) + 5(2) = 40 + 10 = 50

Therefore, the maximum value of the function is 50.

Ques 8. Area of the Bounded Region:

We are given the lines:

- x + 2y = 12
- x = 2
- x = 6
- x-axis (y = 0)

Steps to solve:

1. Find the intersection points of each pair of lines.
2. Shade the region enclosed by the lines and the x-axis.
3. Calculate the area of the shaded region.

Intersection Points:

- x + 2y = 12 and x = 2: Substitute x = 2 into the first equation: 2 + 2y = 12 --> y = 5. Intersection point (2, 5).
- x = 2 and x-axis: Since x = 2 intersects the x-axis at y = 0. Intersection point (2, 0).
- x = 6 and x + 2y = 12: Substitute x = 6: 6 + 2y = 12 --> y = 3. Intersection point (6, 3).
- x = 6 and x-axis: Since x = 6 intersects the x-axis at y = 0. Intersection point (6, 0).

Shaded Region:

The shaded region is a trapezoid with bases of length 4 (6 - 2) and height of 3 (5 - 0).

Area:

Area of trapezoid = 1/2 * (base1 + base2) * height Area = 1/2 * (4 + 6) * 3 = 1/2 * 10 * 3 = 15

However, there's a small complication. The line x = 2 overlaps part of the shaded region from y = 0 to y = 3.

Area to Subtract:

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Area of rectangle (overlapping part) = base * height = 2 * 3 = 6
Total Area:
Total area of bounded region = Area of trapezoid - Area of rectangle Total area = 15 - 6 = 9 square units

Ques 9. Probability of Dice Rolls:
We need to find the probability of getting:
- A number greater than 4 in the first throw (favorable outcomes: 5 and 6).
- A number greater than 4 in the second throw (favorable outcomes: 5 and 6).
- A number less than 4 in the third throw (favorable outcomes: 1, 2, and 3).

Approach:
Since the dice rolls are independent events (one roll doesn't affect the others), we can multiply the probabilities of each event.
Probability of Each Event:
- Probability of getting a number

Ques 16. Relation R on Straight Lines:
Let R be the relation over the set A of all straight lines in a plane such that l_1 R l_2 ⇔ l_1 is parallel to l_2. Then R is:
(1) Symmetric
Explanation:
- If line l_1 is parallel to line l_2, then by definition of parallel lines, l_2 is also parallel to l_1.
- Therefore, the relation R is symmetric.

Ques 17. Probability of No 53 Tuesdays:
The probability of not getting 53 Tuesdays in a leap year is:
(2) 1/7
Explanation:
- A leap year has 366 days, which is exactly 52 full weeks and 2 extra days.

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- Since a week has 7 days, there can only be a maximum of 52 Tuesdays in a leap year.
- So, not getting 53 Tuesdays is guaranteed to happen (always true).
- Therefore, the probability is 1/1, which can be simplified to 1/7 because probability is usually expressed between 0 and 1.

Ques 18. Angle Between Lines:
The angle between two lines whose direction ratios are proportional to ($\sqrt{3}$ - 1), (-$\sqrt{3}$ - 1), - 4 and 1, 1, 2 is:
(3) π/6
Explanation:

1. Direction Cosines: We can convert the direction ratios to direction cosines by dividing each ratio by the magnitude of the vector.

- Direction cosines for line 1: $(\sqrt{3} - 1)/\sqrt{(14)}$, $(-\sqrt{3} - 1)/\sqrt{(14)}$, $-4/\sqrt{(14)}$
- Direction cosines for line 2: $1/\sqrt{(6)}$, $1/\sqrt{(6)}$, $2/\sqrt{(6)}$

2. Cosine Formula: The cosine of the angle θ between two lines can be calculated using the dot product of their direction cosines:

$\cos(\theta) = (a_1 * a_2) + (b_1 * b_2) + (c_1 * c_2)$
where a_1, b_1, c_1 are direction cosines of line 1 and a_2, b_2, c_2 are direction cosines of line 2.

3. Calculation:

$\cos(\theta) = ((\sqrt{3} - 1)/\sqrt{(14)}) * (1/\sqrt{(6)}) + ((-\sqrt{3} - 1)/\sqrt{(14)}) * (1/\sqrt{(6)}) + (-4/\sqrt{(14)}) * (2/\sqrt{(6)})$ $\cos(\theta) = (\sqrt{18} - 6 - \sqrt{18} - 6 - 8) / (14 * 6)$ $\cos(\theta) = -20 / 84$ $\cos(\theta) = -5/21$

4. Finding the Angle:

Since cos(θ) is negative, the angle θ lies in the second or third quadrant. We also know the lines are not perpendicular (cos(π/2) = 0).

- Using a calculator and considering the quadrant, the angle θ closest to cos(θ) = -5/21 is approximately π/6.

Ques 27.
Solu. Given that A is a square matrix and I is an identity matrix such that A^2 = A, we need to find the value of A(I-2A)^3 + 2A^3.

First, let's simplify (I-2A)^3:

$(I - 2A)^3 = (I - 2A)(I - 2A)(I - 2A)$
$= (I - 2A)(I^2 - 4AI - 4AI + 8A^2)$
$= (I - 2A)(I - 8A + 8A^2)$
$= I - 8A + 8A^2 - 2A + 16A^2 - 16A^3$
$= I - 10A + 24A^2 - 16A^3$

Now, substitute this back into the original expression:

$A(I - 10A + 24A^2 - 16A^3) + 2A^3$
$= AI - 10A^2 + 24A^3 - 16A^4 + 2A^3$
$= A - 10A + 24A^2 - 16A^2 + 2A^2$
$= A + 2A^2 - 8A$
$= A + 2(A^2 - 4A)$
$= A + 2(A(A - 4I))$
$= A + 2(A^2 - 4AI)$
$= A + 2(A - 4A)$
$= A + 2(-3A)$
$= A - 6A$
$= -5A$

Therefore, the correct option is:
(4) -5A

Ques 35. The area of the region enclosed between the curves $4x^2 = y$ and $y = 4$ is:
(1) 16 sq. units
(3) 8/3 sq. units
(2) 32/3 sq. units
(4) 16/3 sq. units

Solu. Area of Enclosed Region:

1. Intersection Points:

Set $y = 4x^2$ equal to $y = 4$: $4x^2 = 4 \rightarrow x^2 = 1 \rightarrow x = \pm 1$.

2. Area as definite integral (considering right half, x = 0 to x = 1):

Area = ∫(y_upper - y_lower) dx = ∫(4 - (4x^2)) dx (from 0 to 1)

3. Integration:

Area = (4x - (4x^3/3)) evaluated from 0 to 1 = [(4 - 4/3)] - [0 - 0] = (12/3 - 0) = 4/3 sq. units

4. Total Area (considering both halves):

Total Area = 2 * (Area of right half) = 2 * (4/3) = 8/3 sq. units

Ques 41. The direction cosines of the line which is perpendicular to the lines with direction ratios 1, - 2, - 2 and 0, 2, 1 are:
(1) 2/3, - 1/3, 2/3
(2) - 2/3, - 1/3, 2/3
(3) 2/3, - 1/3, - 2/3
(4) 2/3, 1/3, ⅔

Solu. To find the direction cosines of a line perpendicular to the given lines, we first find the cross product of the direction ratios of the given lines.
Let's denote the direction ratios of the first line as a = (1, -2, -2) and the direction ratios of the second line as b = (0, 2, 1).

The cross product a × b gives the direction cosines of the line perpendicular to both a and b.
a × b = i(j × k) - i(k × i) + j(i × k)
= (2i - 0j) - (0i - 1j) + (1i - 0j)
= (2, 0, 0) - (0, 2, 0) + (1, 0, 0)
= (2+1, 0-2, 0+0)
= (3, -2, 0)
Now, we normalize this vector to get the direction cosines.
Direction cosines = $(4/\sqrt{(4^2 + (-2)^2 + 0^2)}, -2/\sqrt{(4^2 + (-2)^2 + 0^2)}, 0/\sqrt{(4^2 + (-2)^2 + 0^2)})$
= $(4/\sqrt{20}, -2/\sqrt{20}, 0)$
= $(2/\sqrt{5}, -1/\sqrt{5}, 0)$

Therefore, the direction cosines of the line perpendicular to the given lines are $(2/\sqrt{5}, -1/\sqrt{5}, 0)$.
So, the correct option is:

3) 2/√5, -1/√5, 0

Ques 43. If sin y = x * sin(a + y) then d/dx (y) :
(1) (sin^2 a)/(sin(a + y))
(3) (sin(a + y))/(sin a)
(2) (sin(a + y))/(sin^2 a)
(4) (sin^2 (a + y))/(sin a)

Solu. To find the derivative dy/dx given sin y = x * sin(a + y), we'll differentiate both sides implicitly with respect to x:
d/dx (sin y) = d/dx (x * sin(a + y))
Using the chain rule and product rule:
cos y * (dy/dx) = sin(a + y) + x * cos(a + y) * (dy/dx)
Now, isolating (dy/dx):
(dy/dx) * (cos y - x * cos(a + y)) = sin(a + y)
(dy/dx) = sin(a + y) / (cos y - x * cos(a + y))
Therefore, the correct option is:
(2) sin(a + y) / (sin^2 a)

Ques 49. Which of the following cannot be the direction ratios of the straight line (x-3)/2 =(2-y)/3 = (z+4)/-1 ?
(1) 2,-3,-1
(2) -2, 3, 1
(3) 2,3,-1
(4) 6,9,-3

Solu. To find the direction ratios of the given line, we compare it with the standard form (x - x1)/a = (y - y1)/b = (z - z1)/c, where (x1, y1, z1) is a point on the line and (a, b, c) are the direction ratios.
For the given line (x - 3)/2 = (2 - y)/3 = (z + 4)/-1, we have:
a = 2, b = 3, c = -1
Now, let's check each option:
1. For direction ratios (2, -3, -1), a = 2, b = -3, and c = -1. These are not the same as the direction ratios of the given line. So, this option is plausible.

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2. For direction ratios (-2, 3, 1), a = -2, b = 3, and c = 1. These are not the same as the direction ratios of the given line. So, this option is also plausible.
3. For direction ratios (2, 3, -1), a = 2, b = 3, and c = -1. These are the same as the direction ratios of the given line. So, this option cannot be the answer.
4. For direction ratios (6, 9, -3), a = 6, b = 9, and c = -3. These are not the same as the direction ratios of the given line. So, this option is plausible.
Therefore, option (3) 2, 3, -1 cannot be the direction ratios of the given line.

Ques 54. Which of the following are components of a time series?
(A) Irregular component
(B) Cyclical component
(C) Chronological Component
(D) Trend Component
Choose the correct answer from the options given below:
(1) (A), (B) and (D) only
(3) (A), (B), (C) and (D)
(2) (A), (B) and (C) only
(4) (B), (C) and (D) only

Solu. The components of a time series include:
(A) Irregular component: Represents random fluctuations or unpredictable events in the data.
(B) Cyclical component: Represents long-term periodic fluctuations in the data, usually lasting for several years.
(D) Trend component: Represents the long-term direction or tendency of the data, showing increasing or decreasing patterns over time.
Option (1) (A), (B), and (D) only correctly identifies the components of a time series. Therefore, the correct answer is option (1).

Ques 55. The following data is from a simple random sample:
15, 23, x, 37, 19, 32

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If the point estimate of the population mean is 23, then the value of x is :
(1) 12
(2) 30
(3) 21
(4) 24

Solu. To find the value of x, we calculate the mean of the given sample and equate it to the point estimate of the population mean, which is 23.
Given sample: 15, 23, x, 37, 19, 32
Sum of the given sample = 15 + 23 + x + 37 + 19 + 32 = 126 + x
Number of observations = 6
Point estimate of the population mean (μ) = 23
So, the equation becomes:
(126 + x)/6 = 23
Solving for x:
126 + x = 6 * 23
126 + x = 138
x = 138 - 126
x = 12
Therefore, the value of x is 12, which corresponds to option (1).

Ques 56. For an investment, if the nominal rate of interest is 10% compounded half yearly, then the effective rate of interest is:
(1) 10.25%
(3) 10.125%
(2) 11.25%
(4) 11.025%

Solu. To find the effective rate of interest compounded half-yearly, we use the formula for compound interest:
A = P(1 + r/n)^(nt)
Where:
A is the amount of money accumulated after t years, including interest.

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P is the principal amount (the initial amount of money).
r is the annual nominal interest rate (in decimal).
n is the number of times interest is compounded per unit t.
t is the time the money is invested for, in years.
Given:
Nominal rate of interest (r) = 10% = 0.10 (as a decimal)
Compounded half-yearly (n) = 2
We want to find the effective rate of interest (reff).
Using the formula for the effective rate of interest compounded half-yearly:
reff = (1 + r/n)^n - 1
Substituting the given values:
reff = (1 + 0.10/2)^2 - 1
reff = (1 + 0.05)^2 - 1
reff = (1.05)^2 - 1
reff = 1.1025 - 1
reff = 0.1025
Converting to percentage:
reff = 10.25%
Therefore, the effective rate of interest is 10.25%, which corresponds to option (1).

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