Complex Numbers And Quadratic Equations JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Complex Numbers And Quadratic Equations

1. For all $z \in C$ on the curve $C_1 : |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2 Then: **(+4, -1)**

a. the curve C_1 lies inside C_2

b. the curves C_1 and C_2 intersect at 4 points

c. the curve C_2 lies inside C_1

d. the curves C_1 and C_2 intersect at 2 points

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2. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2z \neq -i$ Then z lies on the circle of radius 2 and centre **(+4, -1)**

a. $(2, 0)$

b. $(0, 2)$

c. $(0, 0)$

d. $(0, -2)$

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3. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$ The set $S = \left\{z \in C : |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2\right\}$ represents a **(+4, -1)**

a. straight line with the sum of its intercepts on the coordinate axes equals -18

b. hyperbola with eccentricity 2

c. hyperbola with the length of the transverse axis 7

d. straight line with the sum of its intercepts on the coordinate axes equals 14

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4. Let $y(x) = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)\left(1 + x^8\right)\left(1 + x^{16}\right)$ Then $y' - y''$ at $x = -1$ is equal to : **(+4, -1)**

a. 976

b. 944

c. 496

d. 464

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5. Consider the lines L_1 and L_2 given by **(+4, -1)**

$L_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$

$L_2 : \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$

A line L_3 having direction ratios $1, -1, -2$, intersects L_1 and L_2 at the points P and Q respectively Then the length of line segment PQ is

a. $3\sqrt{2}$

b. 4

c. $2\sqrt{6}$

d. $4\sqrt{3}$

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6. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \left\{z \in C : |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2\right\}$ represents a **(+4, -1)**

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7. Let $y(x) = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)\left(1 + x^8\right)\left(1 + x^{16}\right)$. Then $y' - y''$ at $x = -1$ is equal to : **(+4, -1)**

a. 976

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b. 4

c. $2\sqrt{6}$

d. $4\sqrt{3}$

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9. Let $z = 1 + i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$ Then $\frac{12}{\pi}\arg(z_1)$ is equal to_____ **(+4, -1)**

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10. Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$ Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is _____ **(+4, -1)**

Answers

1. Answer: b

Explanation:

Let $w = z + \frac{1}{z} = 4e^{i\theta} + \frac{1}{4}e^{-i\theta}$

$\Rightarrow w = \frac{17}{4}\ cos\ \theta + i\frac{15}{4}\ sin\ \theta$

So locus of w is ellipse $\frac{x^2}{(\frac{17}{4})^2} + \frac{y^2}{(\frac{15}{4})^2} = 1$

Locus of z is circle $x^2 + y^2 = 16$

So intersect at 4 points

The Correct Option is (B): the curves C_1 and C_2 intersect at 4 points

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- “a” is a real number
- “b” is an imaginary number

The Complex Number consists of a symbol “i” which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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2. Answer: d

Explanation:

The correct answer is (D) : $(0, -2)$

$(z - 2i)(\overline{z} + 2i) = 4(z + i)(\overline{z} - i)$

$z\overline{z} + 4 + 2i(z - \overline{z}) = 4(z\overline{z} + 1 + i(\overline{z} - z))$

$3z\overline{z} - 6i(z - \overline{z}) = 0$

$x^2 + y^2 - 2i(2iy) = 0$

$x^2 + y^2 + 4y = 0$

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3. Answer: d

Explanation:

$\left((x - 2)^2 + (y - 3)^2\right) - \left((x - 3)^2 - (y - 4)^2\right) = 1 + 1$

$? \; x + y = 7$

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4. **Answer: c**

Explanation:

$y = \frac{1-x^{32}}{1-x}$? $y - xy = 1 - x^{32}$
$y' - xy' - y = -32x^{31}$
$y'' - xy'' - y' - y' = -(32)(31)x^{30}$
at $x = -1$? $y' - y'' = 496$

Concepts:

1. **Relations and functions:**

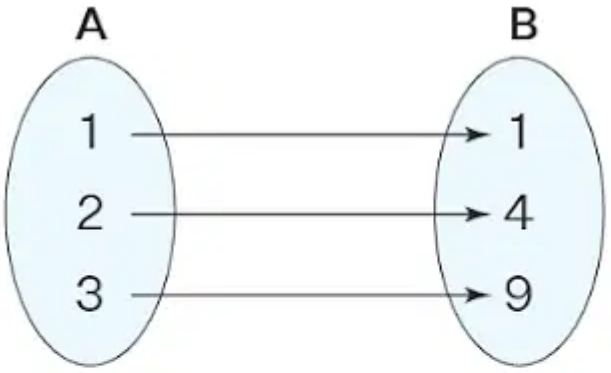
A **relation** R from a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

A relation f from a set A to a set B is said to be a **function** if every element of set A has one and only one image in set B. In other words, no two distinct elements of B have the same pre-image.

Representation of Relation and Function

Relations and functions can be represented in different forms such as arrow representation, algebraic form, set-builder form, graphically, roster form, and tabular form. Define a function f: A = {1, 2, 3} → B = {1, 4, 9} such that f(1) = 1, f(2) = 4, f(3) = 9. Now, represent this function in different forms.

1. Set-builder form - $\{(x, y): f(x) = y^2, x \in A, y \in B\}$
2. Roster form - {(1, 1), (2, 4), (3, 9)}
3. Arrow Representation



- Table Representation -

x	y
1	1
2	4
3	9

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5. Answer: c

Explanation:

The correct answer is (C) : $2\sqrt{6}$
Let P = (2λ+1,λ+3,2λ+2)
Let Q = $(\mu + 2, 2\mu + 2, 3\mu + 3)$
$\Rightarrow \frac{2\lambda-\mu-1}{1} = \frac{\lambda-2\mu-1}{-1}$
$= \frac{2\lambda-3\mu-1}{-2} \Rightarrow \lambda = \mu = 3$
$\Rightarrow P(7, 6, 8)$ and $Q(5, 8, 12)$
PQ = $2\sqrt{6}$

Concepts:

1. **Application of Derivatives:**

Various Applications of Derivatives-

Rate of Change of Quantities:

If some other quantity 'y' causes some change in a quantity of surely 'x', in view of the fact that an equation of the form y = f(x) gets consistently pleased, i.e, 'y' is a function of 'x' then the rate of change of 'y' related to 'x' is to be given by

$\frac{\triangle y}{\triangle x} = \frac{y_2 - y_1}{x_2 - x_1}$

This is also known to be as the Average Rate of Change.

Increasing and Decreasing Function:

Consider y = f(x) be a differentiable function (whose derivative exists at all points in the domain) in an interval x = (a,b).

- If for any two points x_1 and x_2 in the interval x such a manner that $x_1 < x_2$, there holds an inequality $f(x_1) \leq f(x_2)$; then the function f(x) is known as increasing in this interval.
- Likewise, if for any two points x_1 and x_2 in the interval x such a manner that $x_1 < x_2$, there holds an inequality $f(x_1) \geq f(x_2)$; then the function f(x) is known as decreasing in this interval.
- The functions are commonly known as strictly increasing or decreasing functions, given the inequalities are strict: $f(x_1) < f(x_2)$ for strictly increasing and $f(x_1) > f(x_2)$ for strictly decreasing.

Read More: Application of Derivatives

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6. **Answer: d**

Explanation:

$((x-2)^2+(y-3)^2)-((x-3)^2-(y-4)^2)=1+1$
$\Rightarrow x+y=7$

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7. Answer: c

Explanation:

The correct answer is (C) : 496
$y=\frac{1-x^{32}}{1-x}$
$\Rightarrow y-xy=1-x^{32}$
$y'-xy'-y=-32x^{31}$
$y''-xy''-y'-y'=-(32)(31)x^{30}$
at x=-1
$\Rightarrow y'-y''=496$

Concepts:

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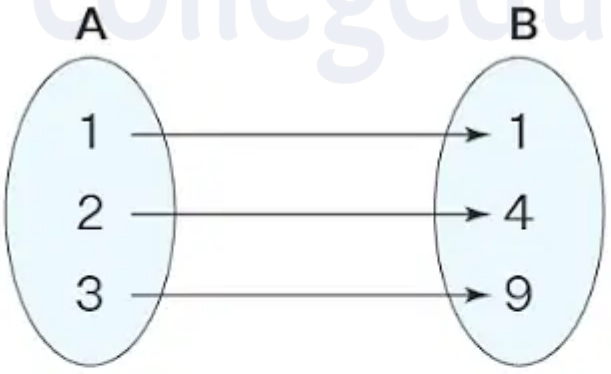

- Table Representation -

x	y
1	1
2	4
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---

8. **Answer: c**

Explanation:

$Let\ P = (2\lambda + 1, \lambda + 3, 2\lambda + 2)$

$Let\ Q = (\mu + 2, 2\mu + 2, 3\mu + 3)$

$\Rightarrow \frac{12\lambda - \mu - 1}{1} = \frac{-1\lambda - 2\mu + 1}{-1} = \frac{2\lambda - 3\mu - 1}{-2}$
$\Rightarrow \lambda = \mu = 3 \Rightarrow P(7, 6, 8)\ and,\ Q(5, 8, 12)$

$PQ = 2\sqrt{6}$

Concepts:

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$f(x_1) > f(x_2)$ for strictly decreasing.

Read More: Application of Derivatives

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9. Answer: 9 - 9

Explanation:

The correct answer is 9.

$z - 1 + i$

$z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$

$z_1 = \frac{1+i(1 \quad i)}{(1-i)(1-1-i)+\frac{1}{1+i}}$

$= \frac{1+i-i^2}{(1-i)(-i)+\frac{1-i}{2}}$

$= \frac{2+i}{-3i-1} = \frac{4+2i}{-3i-1}$

$= \frac{-(4+2i)(3i-1)}{(3i)^2-(1)^2}$

$\text{Arg}\,(z_1) = \frac{3\pi}{4}$

$\therefore \frac{12}{\pi}\arg(z_1) = \frac{3\pi}{4}$

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10. **Answer: 9 - 9**

Explanation:

The correct answer is 9.

Equation of plane is

$(x-2y-z-5)+b(x+y+3z-5)=0$

$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow b = 12$

$\therefore$ plane is $13x+10y+35z=65$

Distance from given point to plane $= 9$

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