# Complex Numbers And Quadratic Equations JEE Main PYQ - 2

**Total Time:** 25 Minute **Total Marks:** 40

## Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

### Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

# Complex Numbers And Quadratic Equations

**1.** The least positive integer n such that $1-\frac{2}{3}-\frac{2}{3^2}-\ldots\ldots-\frac{2}{3^{n-1}}<\frac{1}{100}$, is : **(+4, -1)**

**a.** 4

**b.** 5

**c.** 6

**d.** 7

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**2.** The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n=1$, is : **(+4, -1)**

**a.** 2

**b.** 3

**c.** 5

**d.** 6

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**3.** The complex number $z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to : **(+4, -1)**

**a.** $\sqrt{2}i\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$

**b.** $\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}$

**c.** $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

**d.** $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

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**4.** If $a\neq\pm b$ and are purely real, z ϵ complex number, $Re(az^2+bz)=a$ and $Re(bz^2+az)=b$ then number of value of z possible is **(+4, -1)**

**a.** 0

**b.** 1

c. 2

d. 3

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5. If the equations $x^2+2x+3=0$ and $ax^2+bx+c=0, a, b, c \in R$ have a common root, then $a:b:c$ is **(+4, -1)**

a. 1:02:03

b. 3:02:01

c. 1:03:02

d. 3:01:02

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6. Let $z_0$ be a root of the quadratic equation, $x^2+x+1=0$. If $z=3+6iz_0^{81}-3iz_0^{93}$ , then arg $z$ is equal to : **(+4, -1)**

a. $\frac{\pi}{4}$

b. $\frac{\pi}{3}$

c. 0

d. $\frac{\pi}{6}$

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7. If $z$ is a complex number of unit modulus and argument $\theta$, then $arg\left(\frac{1+z}{1+\bar{z}}\right)$ is equal to **(+4, -1)**

a. $-\theta$

b. $\frac{\pi}{2}-\theta$

c. $\theta$

d. $\pi-\theta$

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8. A complex number $z$ is the said to be unimodular if $|z|=1$. Suppose $z_1$ and $z_2$ are complex number such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and $z_2$ is not unimodular. **(+4, -1)**

Then the point $z_1$ lies on a :

**a.** Straight line parallel to x-axis

**b.** Straight line parallel to y-axis

**c.** Circle of radius 2

**d.** Circle of radius $\sqrt{2}$

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**9.** If the value of real number a> 0 for which $x^2$-5ax+1-0 and $x^2 - ax - 5 - 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then $\beta$ is equal to_______ **(+4, -1)**

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**10.** A triangle is formed by $X$-axis, $Y$-axis and the line $3x + 4y = 60$ Then the number of points $P(n, b)$ which lie strictly inside the triangle, where a is an integer and $b$ is a multiple of a, is ____ **(+4, -1)**

# Answers

1. **Answer: c**

### Explanation:

$1-2\left(\frac{1}{3^{1}}+\frac{1}{3^{2}} \ldots \ldots . \frac{1}{3^{n-1}}\right)$

### Concepts:

#### 1. Complex Number:

A Complex Number is written in the form

**a + ib**

where,

- "a" is a real number
- "b" is an imaginary number

The Complex Number consists of a symbol "i" which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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2. **Answer: b**

### Explanation:

$\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{n}=1 \ \left(\frac{-2\omega^{2}}{-2\omega}\right)^{n}=1 \ \omega^{n}=1$ least positive integer value of n is 3.

### Concepts:

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A Complex Number is written in the form

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### 3. Answer: d

**Explanation:**

$Z = \frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}} = \frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}$

$= \frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i} \times \frac{\frac{1}{2}-\sqrt{\frac{3}{2}i}}{\frac{1}{2}-\sqrt{3/2i}} = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$

Apply polar form,

$r\cos\theta = \frac{\sqrt{3}-1}{2}$

$r\sin\theta = \frac{\sqrt{3}+1}{2}$

Now, $\tan\theta = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

So, $\theta = \frac{5\pi}{12}$

So, the correct option is (D) : $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

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## 4. Answer: a

### Explanation:

$$a(x^2 - y^2) + bx = a \ldots..(i)$$
$$b(x^2 - y^2) + ax = b \ldots..(ii)$$
$$(i) - (ii)$$
$$(a - b)(x^2 - y^2) + (b - a)x = a - b \qquad (a \neq b)$$
$$\Rightarrow x^2 - y^2 - x = 1$$
$$(i) + (ii)$$
$$(a + b)(x^2 - y^2) + x(a + b) = a + b \qquad (a \neq -b)$$
$$\Rightarrow x^2 - y^2 + x = 1$$
$$\Rightarrow x = 0$$
$$\Rightarrow y^2 = -1$$

therefore, no complex number is possible.
The correct option is (A): 0

### Concepts:

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**a + ib**

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5. **Answer: a**

**Explanation:**

Given equations are $x^2+2x+3=0...(i)$
and $ax^2+bx+c=0...(ii)$
Since, E (i) has imaginary roots, so E (ii) will also
have both roots same as E (i).
Thus, $\frac{a}{1}=\frac{b}{2}=\frac{c}{3}$
Hence, a : b : c is 1 : 2 : 3.

**Concepts:**

1. **Complex Numbers and Quadratic Equations:**

Complex Number: Any number that is formed as a+ib is called a complex number. For example: 9+3i,7+8i are complex numbers. Here i = -1. With this we can say that i² = 1. So, for every equation which does not have a real solution we can use i = -1.

Quadratic equation: A polynomial that has two roots or is of the degree 2 is called a quadratic equation. The general form of a quadratic equation is y=ax²+bx+c. Here a≠0, b and c are the real numbers.

| Equations | Detailed Explanations |
|---|---|
| $3x^2 + 4x + 6 = 0$ | In this expression, the known values a = 3, b = 4 and c = 6; while x remains the unknown factor. |
| $2x^2 – 6x = 0$ | Here, the known factors a = 2 and b = 6. However, can you ascertain the value of c? Well, the value of c = 0 as it is not present. |
| $7x – 4 = 0$ | Here the value of a is equal to zero since the equation is not quadratic. |

6. **Answer: a**

### Explanation:

$z_0 = \omega$ or $\omega^2$ (where $\omega$ is a non-real cube root of unity)
$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$
z = 3 + 3i
$\Rightarrow \arg z = \frac{\pi}{4}$

### Concepts:

#### 1. Complex Numbers and Quadratic Equations:

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7. **Answer: c**

### Explanation:

The correct option is(C): θ.

Given, |z | = 1, arg 2 =$\theta$ $\therefore\ z = e^{i\theta}$
But $\bar{z} = \frac{1}{z}$
$\therefore arg\left(\frac{1+z}{1+\frac{1}{2}}\right) = arg(z) = \theta$

## Concepts:

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## 8. Answer: c

### Explanation:

$\left|\frac{z_1-2z_2}{2-z_1\bar{z}_2}\right| = 1$
$(z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$
$|z_1|^2 - 2z_1\bar{z}_2 - 2z_2\bar{z}_1 + 4|z_2|^2$
$= 4 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 + |z_1|^2 |z_2|^2$
$|z_1|^2 |z_2|^2 - |z_1|^2 - 4|z_2|^2 + 4 = 0$
$\left(|z_1|^2 - 4\right)\left(|z_2|^2 - 1\right) = 0$
$\Rightarrow |z_1| = 2$

## Concepts:

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9. **Answer: 13 - 13**

## Explanation:

Two equations have common root

$\therefore (4a)(26a) = (-6)^2 = 36$

$\Rightarrow a^2 = \frac{9}{26}$

$\therefore a = \frac{3}{\sqrt{26}}$

$\Rightarrow \beta = 13$

So, the correct answer is 13.

## Concepts:

### 1. Quadratic Equations:

A **polynomial** that has two roots or is of degree 2 is called a quadratic equation. The general form of a quadratic equation is **y=ax²+bx+c**. Here a≠0, b, and c are the **real n** umbers.

Consider the following equation **$ax^2+bx+c=0$**, where $a \neq 0$ and a, b, and c are real coefficients.

The solution of a quadratic equation can be found using the formula, **$x=((-b \pm \sqrt{(b^2-4ac)})/2a)$**

### Two important points to keep in mind are:

- A polynomial equation has at least one root.
- A polynomial equation of degree 'n' has 'n' roots.

**Read More:** Nature of Roots of Quadratic Equation

### There are basically four methods of solving quadratic equations. They are:

1. Factoring
2. Completing the square
3. Using Quadratic Formula
4. Taking the square root

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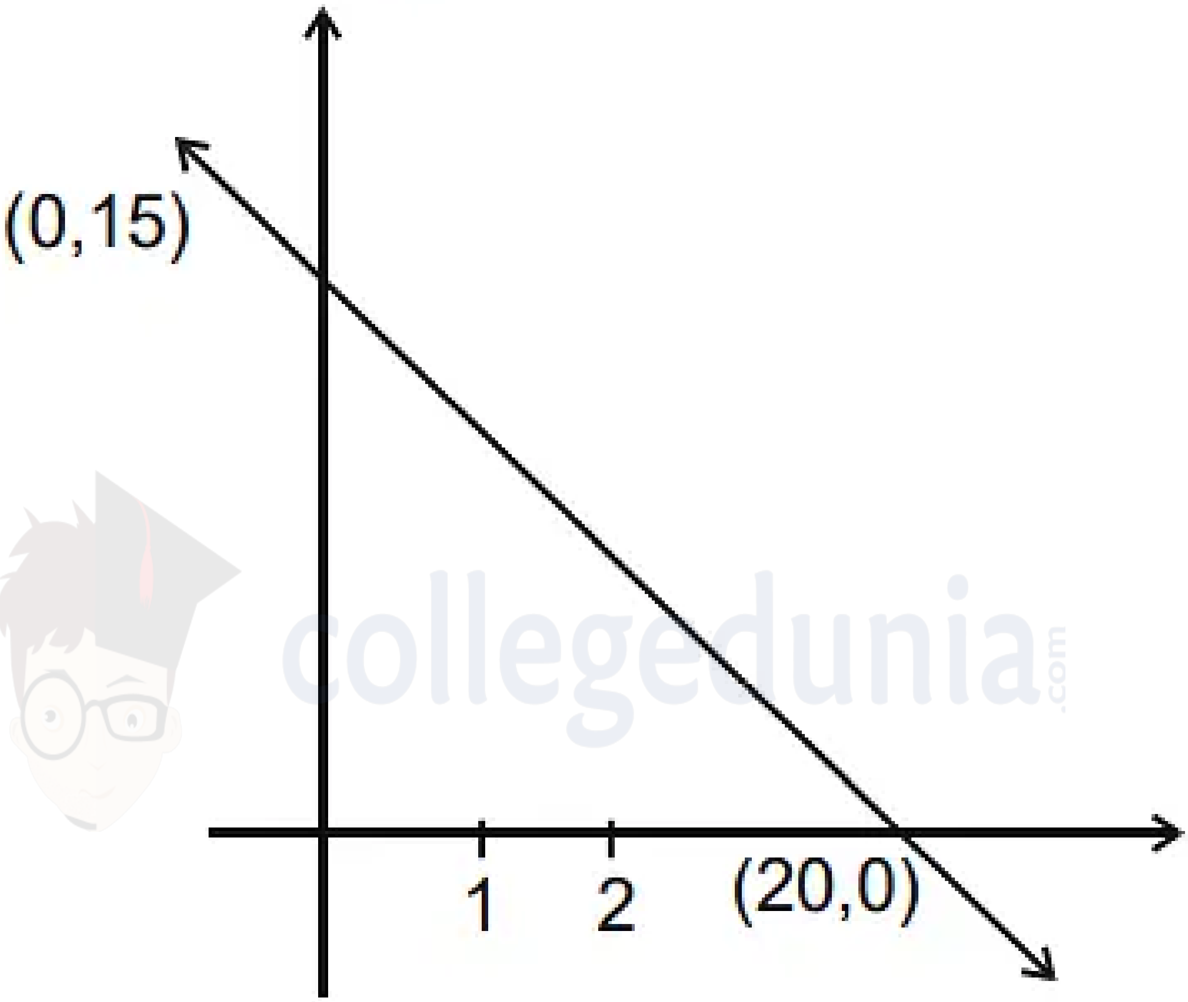
10. **Answer: 31 - 31**

### Explanation:

The correct answer is 31

$$\text{If } x = 1,\ y = \frac{57}{4} = 14.25$$




(1,1)(1,2)-(1,14)⇒14 pts.

If x=2,y=227=13.5

(2,2)(2,4)...(2,12)⇒6 pts.

If x=3,y=451=12.75

(3,3)(3,6)-(3,12)⇒4 pts.

If x=4,y=12

(4,4)(4,8)⇒2 pts.

If x=5·y=445=11.25

(5,5),(5,10)⇒2 pts.

If x=6,y=221=10.5

(6,6)⇒1pt

If x=7,y=439=9.75

(7,7)⇒1pt
If x=8,y=9
(8,8)⇒1pt
If x=9y=433=8.25⇒ no pt.
Total =31 pts.

## Concepts:

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Consider the following equation **$ax^2+bx+c=0$**, where $a\neq0$ and a, b, and c are real coefficients.

The solution of a quadratic equation can be found using the formula, **$x=((-b\pm\sqrt{(b^2-4ac)})/2a)$**

### Two important points to keep in mind are:

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1. Factoring
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