Complex Numbers And Quadratic Equations JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Complex Numbers And Quadratic Equations

1. The sum of all real values of x satisfying the equation $(x^2-5x+5)^{x^2+4x-60}=1$ is **(+4, -1)**

   a. 3

   b. -4

   c. 6

   d. 5

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2. For all complex numbers z of the form 1 + iα, $\alpha \,\epsilon$ R, if z = x + iy, then : **(+4, -1)**

   a. $y^2-4x+2=0$

   b. $y^2+4x-4=0$

   c. $y^2-4x+4=0$

   d. $y^2+4x+2=0$

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3. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2+bx+1=0\,(a\neq 0, a, b\in R)$, then the equation, $x\left(x+b^3\right)+\left(a^3?3abx\right)$ = 0 has roots : **(+4, -1)**

   a. $\alpha^{3/2}$ and $\beta^{3/2}$

   b. $\alpha\,\beta^{1/2}$ and $\alpha^{1/2}\,\beta$

   c. $\sqrt{\alpha\,\beta}$ and $\alpha\,\beta$

   d. $\alpha^{-\frac{3}{2}}$ and $\beta^{-\frac{3}{2}}$

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4. If, for a positive integer n, the quadratic equation, $x(x+1)+(x+1)(x+2)+ \ldots.+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to : **(+4, -1)**

   a. 9

b. 10

c. 11

d. 12

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5. If m is chosen in the quadratic equation $(m^2+1)x^2 - 3x + (m^2+1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is : **(+4, -1)**

a. $8\sqrt{3}$

b. $4\sqrt{3}$

c. $10\sqrt{5}$

d. $8\sqrt{5}$

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6. If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3\,\sin^2(A+B) - 10\,\sin(A+B)?\cos(A+B) - 25\cos^2(A+B)$ is : **(+4, -1)**

a. -10

b. 10

c. -25

d. 25

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7. If $\frac{z-\alpha}{z+\alpha}$ $(\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is : **(+4, -1)**

a. 1

b. 2

c. $\sqrt{2}$

d. $\frac{1}{2}$

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8. If z be a complex number satisfying $|Re(z)| + |Im(z)| = 4$, then $|z|$ cannot be : **(+4, -1)**

a. $\sqrt{7}$

b. $\sqrt{\frac{17}{2}}$

c. $\sqrt{10}$

d. $\sqrt{8}$

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9. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can ofter 5 fruits to Anil is______ **(+4, -1)**

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10. Let $a \in R$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$ If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is_____ **(+4, -1)**

Answers

1. **Answer: a**

Explanation:

$\left(x^2-5x+5\right)^{x^2+4x-60}=1=\left(x^2-5x+5\right)^0$
$\Rightarrow x^2+4x-60=0\left[a^x=a^y \Rightarrow x=y\ \ if\, a \neq 1,0,-1\right]$
$x=-10,6$
& base $x^2-5x+5=0$ or 1 or -1
If $x^2-5x+5=0$

$x^2-5x+5=1$
$\therefore\ x=4,1$

$x^2-5x+5=-1$
$\therefore\ x=2,3$
$x=3$ does not satisfy eqn.

But it will not satisfy original equation .
Hence solutions are - $10,6,4,1,2$
So, sum of solutions $=-10+6+4+1+2=3$

Concepts:

1. Complex Numbers and Quadratic Equations:

Complex Number: Any number that is formed as a+ib is called a complex number. For example: 9+3i,7+8i are complex numbers. Here i = -1. With this we can say that i^2 = 1. So, for every equation which does not have a real solution we can use i = -1.

Quadratic equation: A polynomial that has two roots or is of the degree 2 is called a quadratic equation. The general form of a quadratic equation is $y=ax^2+bx+c$. Here $a\neq0$, b and c are the real numbers.

Equations	Detailed Explanations
$3x^2 + 4x + 6 = 0$	In this expression, the known values a = 3, b = 4 and c = 6; while x remains the unknown factor.
$2x^2 - 6x = 0$	Here, the known factors a = 2 and b = 6. However, can you ascertain the value of c? Well, the value of c = 0 as it is not present.
$7x - 4 = 0$	Here the value of a is equal to zero since the equation is not quadratic.

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2. Answer: b

Explanation:

$(1+i\alpha)^2 = x + iy$

$1 - \alpha^2 + 2i\alpha = x + iy$

so $x = 1 - \alpha^2, y = 2\alpha$

putting $\alpha = y/2$

$x = 1 - \left(\frac{y}{2}\right)^2$

$\Rightarrow y^2 + 4x - 4 = 0$

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3. Answer: a

Explanation:

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = -\frac{b}{a} also \frac{1}{\sqrt{\alpha\beta}} = \frac{1}{a}$

$\Rightarrow \sqrt{\alpha} + \sqrt{\beta} = -b$

now $x\left(x + b^3\right) + a^3 - 3abx$

$= x^2 + \left(b^3 - 3ab\right)x + a^3$

$= x^2 + b\left(b^2 - 3a\right)x + a^3$

$= x^2 - \left(\sqrt{\alpha} + \sqrt{\beta}\right)\left\{\alpha + \beta + 2\sqrt{\alpha\beta} - 3\sqrt{\alpha\beta}\right\}x + \alpha\beta\sqrt{\alpha\beta}$

$= x^2 - \left(\alpha\sqrt{\alpha} + \beta\sqrt{\beta}\right) + \alpha\beta\sqrt{\alpha\beta}$

$\Rightarrow$ roots are $\alpha\sqrt{\alpha}$ and $\beta\sqrt{\beta}$

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4. **Answer: c**

Explanation:

Rearranging equation, we get

$nx^2 + \{1 + 3 + 5 + ... + (2n-1)\}x + \{1.2 + 2.3 + ... + (n-1)n\} = 10n$

$\Rightarrow nx^2 + n^2x + \frac{(n-1)n(n+1)}{3} = 10n$

$\Rightarrow x^2 + nx + \left(\frac{n^2-31}{3}\right) = 0$

Given difference of roots = 1

$\Rightarrow |\alpha - \beta| = 1$

$\Rightarrow$ D = 1

$\Rightarrow n^2 - \frac{4}{3}\left(n^2 - 31\right) = 1$

So, $n = 11$

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5. Answer: d

Explanation:

$SOR = \frac{3}{m^2+1} \Rightarrow (S.O.R)_{max} = 3$

when $m = 0$

$\alpha + \beta = 3$

$\alpha\beta = 1$

$|\alpha^2 - \beta^2| = ||\alpha - \beta| (\alpha^2 + \beta^2 + \alpha\beta)|$

$= \left|\sqrt{(\alpha - \beta)^2 - \alpha\beta} \left((\alpha + \beta)^2 - \alpha\beta\right)\right|$

$= |\sqrt{9 - 4} (9 - 1)|$

$= \sqrt{5} \times 8$

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6. Answer: c

Explanation:

$3x^2 - 10x - 25 = 0$

$\tan A + \tan B = \frac{10}{3}$

$\tan A + \tan B = -\frac{23}{3}$

$\tan(A + B) = \frac{\tan A + \tan}{1}$

$= \frac{\frac{10}{3}}{1 + \frac{23}{3}}$

$= \frac{10}{28} = \frac{5}{14}$

Divide and multiply by $\cos^2 \times (A + B)$

$3\tan^2(A + B) - 10\tan(A + B) - 25\left(\cos^2(A + B)\right)$

$3\frac{25}{196} - 10\left(\frac{5}{14}\right) - 25\left(\cos^2(A + B)\right)$

$\frac{75 - 700 - 4500}{196}\left(\cos^2(A + B)\right)$

$-\frac{5525}{196}\left(\frac{1}{1 + \tan^2(A + B)}\right)$

$-\frac{5525}{196}\left(\frac{1}{1 + \frac{25}{196}}\right)$

$= \frac{-5521}{221}$

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7. **Answer: b**

Explanation:

$$\frac{z-\alpha}{z+\alpha}+\frac{\bar{z}-\alpha}{\bar{z}+\alpha}=0$$
$$z\bar{z}+z\alpha-\alpha\bar{z}-\alpha^2+z\bar{z}-z\alpha+\bar{z}\alpha-\alpha^2=0$$
$$|z|^2=\alpha^2, a=\pm 2$$

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8. **Answer: a**

Explanation:

$z = x + iy \qquad |x| + |y| = 4$
$|z| = \sqrt{x^2 + y^2} \Rightarrow |z|_{min} = \sqrt{8} \,\&\, |z|_{max} = 4 = \sqrt{16}$
So |z| cannot be $\sqrt{7}$

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9. **Answer: 6860 - 6860**

Explanation:

The correct answer is 6860
7 Red apple(RA),5 white apple(WA), 8 oranges (O)
5 fruits to be selected (Note:- fruits taken different)
Possible selections :- $(2O, 1RA, 2WA)$ or $(2O,$

$2RA, 1WA)$ or $(3O, 1RA, 1WA)$

$\Rightarrow {}^8C_2{}^7C_1{}^5C_2 + {}^8C_2{}^7C_2{}^5C_1 + {}^8C_3{}^7C_1{}^5C_1$

$\Rightarrow 1960 + 2940 + 1960$

$\Rightarrow 6860$

Concepts:

1. Quadratic Equations:

A polynomial that has two roots or is of degree 2 is called a quadratic equation. The general form of a quadratic equation is **y=ax²+bx+c**. Here a≠0, b, and c are the real n umbers.

Consider the following equation **ax²+bx+c=0**, where a≠0 and a, b, and c are real coefficients.

The solution of a quadratic equation can be found using the formula, **x=((-b±√(b²-4ac))/2a)**

Two important points to keep in mind are:

- A polynomial equation has at least one root.
- A polynomial equation of degree 'n' has 'n' roots.

Read More: Nature of Roots of Quadratic Equation

There are basically four methods of solving quadratic equations. They are:

1. Factoring
2. Completing the square
3. Using Quadratic Formula
4. Taking the square root

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10. Answer: 45 - 45

Explanation:

The correct answer is 45.

$$x^2 + 60^{\frac{1}{4}}x + a = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$\alpha + \beta = -60^{\frac{1}{4}} \& \alpha\beta = a$

Given $\alpha^4 + \beta^4 = -30$

$\Rightarrow (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = -30$

$\Rightarrow \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2a^2 = -30$

$\Rightarrow \left\{60^{\frac{1}{2}} - 2a\right\}^2 - 2a^2 = -30$

$\Rightarrow 60 + 4a^2 - 4a \times 60^{\frac{1}{2}} - 2a^2 = -30$

$\Rightarrow 2a^2 - 4.60^{\frac{1}{2}}a + 90 = 0$

Product $= \frac{90}{2} = 45$

Concepts:

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