$\boxed{293}$ $\boxed{\triangledown\text{III}}$

Total No. of Questions – 24
Total No. of Printed Pages – 3

Regd.
No.

Part – III
MATHEMATICS, Paper-II(B)
(English Version)

Time : 3 Hours]   *[Max. Marks : 75*

Note : This question paper consists of **three** sections A, B, C.

SECTION – A $10 \times 2 = 20$

I.   Very short answer type questions :

(i)   Attempt **all** questions.

(ii)   Each question carries **two** marks.

1.   Find the equation of the circle for which the points (4, 2), (1, 5) are the end points of a diameter.

2.   Find the value of k if the points (4, 2) and (k, –3) are conjugate points with respect to the circle $x^2 + y^2 - 5x + 8y + 6 = 0$.

3.   Find the equation of the radical axis of the circles $x^2 + y^2 + 4x + 6y - 7 = 0$, $4(x^2 + y^2) + 8x + 12y - 9 = 0$.

4.   Find the equation of the tangent to the parabola $x^2 - 4x - 8y + 12 = 0$ at $\left(4, \dfrac{3}{2}\right)$.

5. Find the product of lengths of the perpendiculars from any point on the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ to its asymptotes.

6. Evaluate : $\displaystyle\int \dfrac{e^x(1+x)}{\cos^2(xe^x)}\, dx$ on $I \subset R \setminus \{x \in R : \cos(xe^x) = 0\}$.

7. Evaluate : $\displaystyle\int \dfrac{dx}{(x+1)(x+2)}$.

8. Evaluate : $\displaystyle\int_0^1 \dfrac{dx}{\sqrt{3-2x}}$.

9. Evaluate : $\displaystyle\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x \, dx$.

10. Form the differential equation corresponding to $y = cx - 2c^2$, where c is a parameter.

<div align="center">SECTION – B $5 \times 4 = 20$</div>

II. Short answer type questions :

(i) Attempt any **five** questions.

(ii) Each questions carries **four** marks.

11. If a point P is moving such that the lengths of tangents drawn from P to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ are in the ratio $2 : 3$, then find the equation of the locus of P.

12. Find the equation of the circle passing through the points of intersection of the circles $x^2 + y^2 - 8x - 6y + 21 = 0$, $x^2 + y^2 - 2x - 15 = 0$ and $(1, 2)$.

13. Find the equation of the ellipse referred to its major and minor axes as the co-ordinate axes X, Y–respectively with latus rectum of length 4 and distance between foci $4\sqrt{2}$.

14. Show that the locus of the feet of the perpendicular drawn from foci to any tangent of the ellipse is the auxiliary circle.

15. Find the equations of the tangents to the hyperbola $x^2 - 4y^2 = 4$ which are

    (i) Parallel

    (ii) Perpendicular to the line $x + 2y = 0$.

16. Find the area of one of the curvilinear triangles bounded by $y = \sin x$, $y = \cos x$ and X-axis.
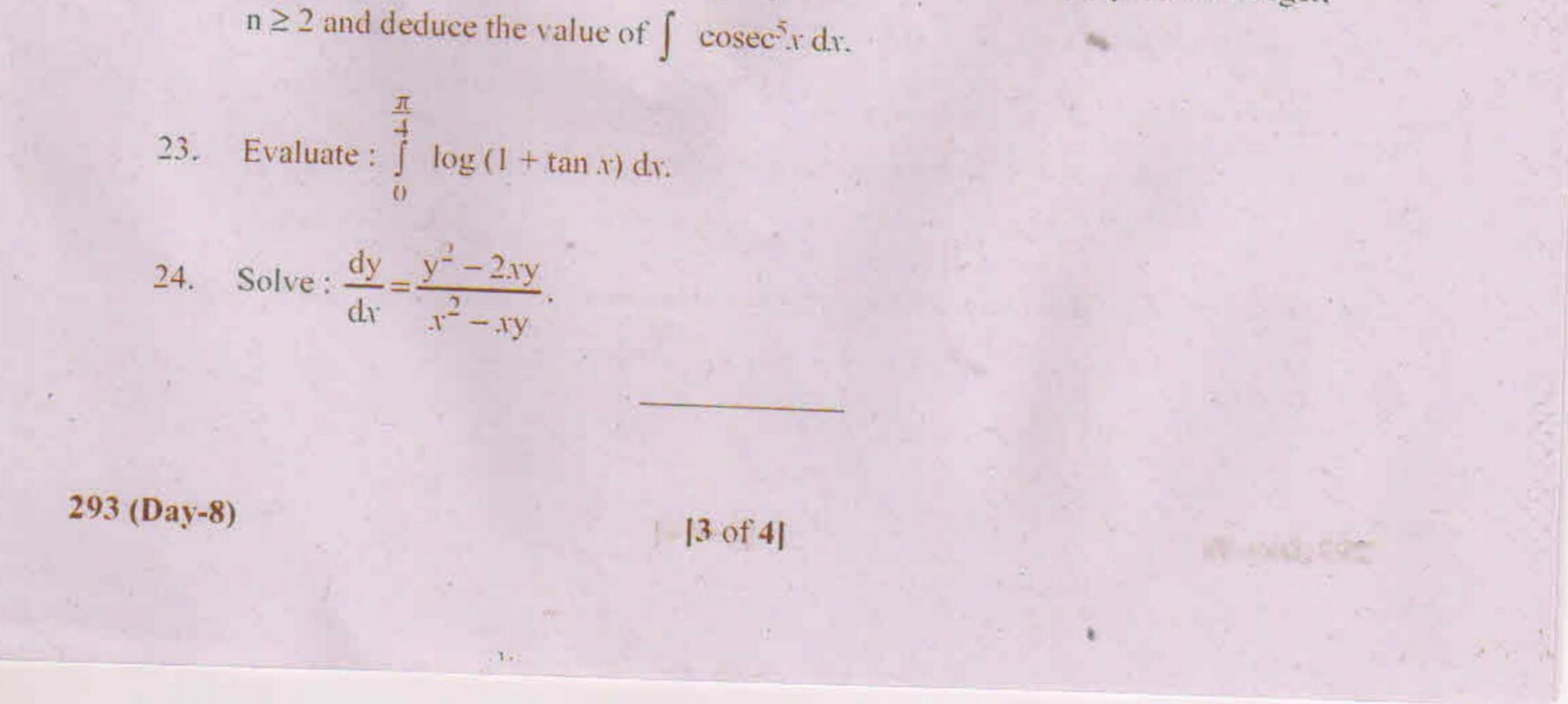
17. Solve : $x(x - 1) \dfrac{dy}{dx} - y = x^3(x - 1)^3$

SECTION – C

$5 \times 7 = 35$

III. Long answer type questions :

(i) Attempt any **five** questions.

(ii) Each question carries **seven** marks.

18. Show that the following four points $(1, 1)$, $(-6, 0)$, $(-2, 2)$, $(-2, -8)$ are concyclic and find the equation of the circle on which they lie.

19. Find the transverse common tangents of the circles $x^2 + y^2 - 4x - 10y + 28 = 0$ and $x^2 + y^2 + 4x - 6y + 4 = 0$.

20. Find the equation of the parabola whose focus is $S(3, 5)$ and vertex is $A(1, 3)$.

21. Evaluate : $\displaystyle\int \dfrac{\cos x + 3 \sin x + 7}{\cos x + \sin x + 1}\, dx$.

22. Obtain the reduction formula for $I_n = \displaystyle\int \csc^n x\, dx$, n being a positive integer, $n \geq 2$ and deduce the value of $\displaystyle\int \csc^5 x\, dx$.

23. Evaluate : $\displaystyle\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx$.

24. Solve : $\dfrac{dy}{dx} = \dfrac{y^2 - 2xy}{x^2 - xy}$.

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