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JEE (MAIN) 2024

QUESTIONS & SOLUTIONS

SHIFT-1

DATE & DAY: 01st February 2024 & Thursday

PAPER-1

Duration: 3 Hrs.
Time: 09:00 - 12:00 IST

SUBJECT: MATHEMATICS

PART : MATHEMATICS

1. A bag contain 8 Balls, whose colours are either white or black, 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is.

(1) $\frac{2}{5}$ (2) $\frac{2}{7}$ (3) $\frac{1}{7}$ (4) $\frac{1}{5}$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. n(s) = there are 5 possible sample space.

$$P\left(\frac{A_1}{E}\right)=\frac{P(A_1).P\left(\frac{E}{A_1}\right)}{P(A_1).P\left(\frac{E}{A_1}\right)+P(A_2).P\left(\frac{E}{A_2}\right)+.......}$$

$$P\left(\frac{4B4W}{2B2W}\right)=\frac{P(4B4W)\times P\left(\frac{2B2W}{4B4W}\right)}{P(4B4W)\times P\left(\frac{2B2W}{4B4W}\right)+P(3B5W).P\left(\frac{2B2W}{3B5W}\right)+.....}$$

$$=\frac{\frac{1}{5}\times\frac{{}^4C_2\,{}^4C_2}{{}^8C_4}}{\frac{1}{5}\times\frac{{}^4C_2\times{}^4C_2}{{}^8C_4}+\frac{1}{5}\times\frac{{}^5C_2\,{}^3C_2}{{}^8C_4}\times 2+\frac{1}{5}\times\frac{{}^6C_2\,{}^2C_2}{{}^8C_4}\times 2}=\frac{36}{36+60+30}=\frac{36}{126}=\frac{6}{21}=\frac{2}{7}$$

2. The value of integral $\int_0^{\pi/4}\frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$ equals:

(1) $\frac{\sqrt{2}\pi^2}{8}$ (2) $\frac{\sqrt{2}\pi^2}{16}$ (3) $\frac{\sqrt{2}\pi^2}{32}$ (4) $\frac{\sqrt{2}\pi^2}{64}$

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. By property P– 6

$$\int_0^{\pi/8}\left(\frac{x}{\cos^4 2x+\sin^4 2x}+\frac{\frac{\pi}{4}-x}{\sin^4 2x+\cos^4 2x}\right)dx \Rightarrow \frac{\pi}{4}\frac{1}{2}\int_0^{\pi/4}\frac{d\theta}{\sin^4\theta+\cos^4\theta}=\frac{\pi}{8}\int_0^1\frac{1+t^2}{t^4+1}dt,$$

t = tanθ

$$=\frac{\pi}{8}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)_0^1=\frac{\pi^2}{16\sqrt{2}}$$

3. If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $C = ABA^T$, $X = A^TC^2A$, then det X is equal to

(1) 243 (2)729 (3) 27 (4) 891

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $A^TA = \begin{bmatrix} \sqrt{2} & -1 \\ 1 & \sqrt{2} \end{bmatrix}\begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 3I$

$x = A^TABA^TABA^TA$

$= (3I)\,B\,(3I)\,B\,(3I) = 27B^2 \Rightarrow |X| = (27)^2|B|^2 = 729$

4. If $\tan A = \dfrac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = (x^{-3} + x^{-2} + x^{-1})^{1/2}$ $0 < A, B, C < \dfrac{\pi}{2}$, then A + B is equal to

(1) C (2) $\pi - C$ (3) $2\pi - C$ (4) $\dfrac{\pi}{2} - C$

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. $\tan(A+B) = \dfrac{\dfrac{1+x}{\sqrt{x(x^2+x+1)}}}{1 - \dfrac{1}{x^2+x+1}}$

$= \dfrac{1+x}{\sqrt{x(x^2+x+1)}} \times \dfrac{(x^2+x+1)}{x(x+1)}$

$= \dfrac{\sqrt{x^2+x+1}}{\sqrt{x^3}} = \tan C$

5. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

(1) 47 (2) 53 (3) 51 (4) 43

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. Since rooms are identical so we can distribute in following way

	(1)	(2)	(3)	(4)
1 way = 1	0	0	0	5
$\frac{5!}{4!\,1!}$ ways = 5	0	0	1	4
$\frac{5!}{2!\,3!}$ ways = 10	0	0	2	3
$\frac{5!}{3!\,1!\,1!} \times \frac{1}{2!} = 10$	0	1	1	3
$\frac{5!}{1!\,2!\,2!\,\,2!} = 15$	0	1	2	2
$\frac{5!}{1!\,1!\,1!\,\,2!} \times \frac{1}{3!} = 10$	1	1	1	2

Total 51 ways

6. Let $S = \{z \in C : |z-1| = 1 \text{ and } (\sqrt{2}-1)(z+\bar{z}) - i(z-\bar{z}) = 2\sqrt{2}\}$. Let $z_1, z_2, \in S$ be such that $|z_1| = \max\limits_{z\in S} |z|$ and $|z_2| = \min\limits_{z\in S} . |z|$. Then $\left|\sqrt{2}z_1 - z_2\right|^2$ equals:

(1) 1 (2) 4 (3) 3 (4) 2

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $(\sqrt{2}-1)(z+\bar{z}) - i(z-\bar{z}) = 2\sqrt{2}$

$(\sqrt{2}-1)(2x) - i(2iy) = 2\sqrt{2}$

$(\sqrt{2}-1)x + y = \sqrt{2}$

and circle $|z-1| = 1$

$(x-1)^2 + y^2 = 1$

$(x-1)^2 + \left(\sqrt{2} - (\sqrt{2}-1)x\right)^2 = 1$

$(x-1)^2 + \left(\sqrt{2}(1-x) + x\right)^2 = 1$

$(x-1)^2 + 2(1-x)^2 + x^2 + 2x\sqrt{2}(1-x) = 1$

$3(x-1)^2 + x^2 + 2\sqrt{2}x(1-x) = 1$

$(-2\sqrt{2}+4)x^2 + x(-6+2\sqrt{2}) + 2 = 0$

$x = 1$ or $x = \frac{1}{2-\sqrt{2}}$

fore $x = 1 \Rightarrow y = 1$

and $x = \frac{1}{2-\sqrt{2}} \Rightarrow y = \sqrt{2} - \frac{1}{\sqrt{2}}$

So, $\left|\sqrt{2}z_1 - z_2\right|^2 = \left|\frac{1}{\sqrt{2}-1} + i - 1 - i\right|^2 = 2$

7. Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is

(1) 31 (2) 28 (3) 30 (4) 32

NTA Ans. (3)

Reso Ans. (3)

Sol. Median = 170

a, b, 125, 170, 190, 210, 230

mean deviation about median

$$\Rightarrow \frac{205}{7} = \frac{(170 - a) + (170 - b) + (170 - 125) + (0) + (190 - 170) + (210 - 170) + (230 - 170)}{7}$$

205 = 505 – (a + b)

a + b = 300

Now mean = $\frac{\Sigma x_i}{7} = \frac{925 + a + b}{7}$

now mean deviation mean = 175

$$\frac{(175 - a) + (175 - b) + (175 - 125) + (175 - 170) + (190 - 175) + (210 - 175) + (230 - 175)}{7} = 30$$

8. $\vec{a} = -5\hat{i} + \hat{j} - \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = (((\vec{a} \times \vec{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$. Then $\vec{c}.(-\hat{i} + \hat{j} + \hat{k})$ is equal to :

(1) –12 (2) –10 (3) –13 (4) –15

NTA Ans. (1)

Reso Ans. (1)

Sol. $(\vec{a} \times \vec{b}) \times \hat{i} = (\vec{a}.\hat{i})\vec{b} - (\vec{b}.\hat{i})\vec{a}$

$= -5\vec{b} - \vec{a} = \vec{p}(\text{let})$

$= -11\hat{j} + 23\hat{k}$

Now $\vec{c} = (\vec{p} \times \hat{i}) \times \hat{i}$

$= (\vec{p}.\hat{i})\hat{i} - (\hat{i}.\hat{i})\vec{p}$

$= 0 - (-11\hat{j} + 23\hat{k})$

11j – 23k

Now $\vec{c}(-\hat{i} + \hat{j} + \hat{k})$

11 – 23 = – 12

9. Let $S = \left\{x \in R : (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\right\}$. Then the number of elements in S is :

(1) 4 (2) 0 (3) 2 (4) 1

NTA Ans. (3)

Reso Ans. (3)

Sol. $\left(\sqrt{3}+\sqrt{2}\right)^x = t$

$t + \frac{1}{t} = 10 \Rightarrow t^2 - 10t + 1 = 0 \Rightarrow t = \frac{10 \pm \sqrt{96}}{2}$

$t = 5 \pm 2\sqrt{6} \Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^x = 5 + 2\sqrt{6} \Rightarrow x = \pm 2$

number of elements in S are 2

10. The area enclosed by the curves xy+4y=16 x+y=6 is equal to:

(1) 28–30 $\log_e 2$ (2) 30–28 $\log_e 2$ (3) 30–32 $\log_e 2$ (4) 32–30 $\log_e 2$

NTA Ans. **(3)**

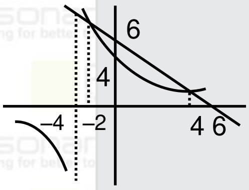
Reso Ans. **(3)**

Sol. (x + 4) y = 16

$\Rightarrow (x + 4)(6 - x) = 16$

$6x - x^2 + 24 - 4x - 16 = 0$

$x^2 - 2x - 8 = 0 \Rightarrow x = 4$ or -2



Area $= \int_{-2}^{4}\left((6-x) - \left(\frac{16}{x+4}\right)\right)dx = = 6(6) - \frac{1}{2}(16-4) - 16\left(\ell n(8) - \ell n2\right) = 30 - 32\,\ell n2$

11. Let f : R → R and g : R → R be defined as $f(x) = \begin{cases} \log_e x, x > 0 \\ e^{-x}, \quad x \le 0 \end{cases}$ and $g(x) = \begin{cases} x, \quad x \ge 0 \\ e^{x}, \quad x \le 0 \end{cases}$ then, gof: R→R is :

(1) one-one but not onto (2) neither one-one nor onto

(3) onto but not one-one (4) both one-one and onto

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $f(x) = \begin{cases} \ell nx & ; x > 0 \\ e^{-x} & ; x \le 0 \end{cases}$

$g(x) = \begin{cases} x & ; x \ge 0 \\ e^{x} & ; x < 0 \end{cases}$

Let h(x) = gof (x) = g(f(x))

$= \begin{cases} f(x) & ; f(x) \ge 0 \\ e^{f(x)} & ; f(x) < 0 \end{cases}$

$h(x) = \begin{cases} \ell nx & ; \quad x \ge 1 \\ e^{-x} & ; \quad x \le 0 \\ x & ; \quad 0 < x < 1 \end{cases}$

range of h(x) = $[0, \infty) \ne$ codomain and It is many- one so many – one & Into

12. If the system of equations
$2x+3y-z=5$
$x+\alpha y+3z=-4$
$3x-y+\beta z=7$
has infinitely many solutions, then 13 $\alpha\beta$ is equal to
(1) 1110 (2) 1120 (3) 1210 (4) 1220

NTA Ans. **(2)**
Reso Ans. **(2)**
Sol. Family of plane

$$\frac{2+\lambda}{3}=\frac{3+\lambda\alpha}{-1}=\frac{-1+3\lambda}{\beta}=\frac{5-4\lambda}{7}$$

$$19\lambda = 1 \Rightarrow \lambda = \frac{1}{19}$$

also

$$\alpha = \left(\frac{39}{19\times 3}(-1)-3\right)19$$

$$\alpha = -70 \text{ and } \beta = \left(\frac{-7+21}{19}\right)\frac{19}{97} = \frac{-112}{91} = \frac{-16}{3}$$

So $13\alpha\beta = 70 \times 16 = 1120$

13. For $0 < \theta < \pi/2$, if the eccentricity of the hyperbola $x^2 - y^2 \operatorname{cosec}^2\theta = 5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2 \operatorname{cosec}^2\theta + y^2=5$, then the value of θ is:
(1) $\frac{\pi}{6}$ (2) $\frac{5\pi}{12}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

NTA Ans. **(3)**
Reso Ans. **(3)**
Sol. Let e_1 eccentricity of ellipse and e_2 is eccentricity of hyperbola

$$e_1 = \sqrt{1-\sin^2\theta} = \cos\theta$$

$$e_2 = \sqrt{1+\sin^2\theta}$$

e_2 is $\sqrt{7}$ times of e_1

$$\sqrt{1+\sin^2\theta} = \sqrt{7}\cos\theta$$

$1 + \sin^2\theta = 7\cos^2\theta$
$2 = 8\cos^2\theta$

$$\cos^2\theta = \frac{1}{4}, \quad \cos\theta = \frac{1}{2}, \quad \cos\theta = \frac{-1}{2} \text{ (rejected)}$$

$$\theta = \frac{\pi}{3}$$

14. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1,\ y(0) = 1$. Then $\left(\frac{1}{2}+y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals :
(1) $\frac{4}{4+\sqrt{e}}$ (2) $\frac{3}{3-\sqrt{e}}$ (3) $\frac{2}{1+\sqrt{e}}$ (4) $\frac{1}{2-\sqrt{e}}$

NTA Ans. **(4)**
Reso Ans. **(4)**

Sol. put $x + y = t$

$$1+\frac{dy}{dx}=\frac{dt}{dx}$$

Now $\frac{dt}{dx}-1=2xt^3-xt-1$

$$\frac{dt}{dx}=2xt^3-xt$$

$$\frac{1}{t^3}\frac{dt}{dx}+\frac{x}{t^2}-2x$$

Put $\frac{1}{t^2}=u$

$$\frac{-2}{t^3}\frac{dt}{dx}=\frac{du}{dx}$$

$$\frac{-1}{2}\frac{du}{dx}+xu=2x$$

$$\frac{du}{dx}-2xu=-4x$$

I.F $= e^{-\int 2xdn} = e^{-x^2}$

Solve $u \cdot e^{x^2} = \int e^{-x^2}.(-4x)dx$

$$\frac{e^{-x^2}}{t^2}=\int e^{-x^2}(-4x)dx$$

$-x^2 = z$

$-2xdx = dz$

$$\frac{e^{-x^2}}{(x+y)^2}=\int 2e^z dz$$

$$\frac{e^{-x^2}}{(x+y)^2}=2e^z+c$$

$$\frac{e^{-x^2}}{(x+y)^2}=2e^{-x^2}+c$$

$$\frac{1}{(x+y)^2}=2+ce^{x^2}$$

at $x = 0$, $y = 1$

$\frac{1}{1} = 2 + c$

$c = -1$

Now $(x + y)^2 = \frac{1}{2 - e^{x^2}}$

at $x = \frac{1}{\sqrt{2}} \Rightarrow \left(y + \frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2 - e^{\frac{1}{2}}}$

$\left(y\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\right)^2 = \left(\frac{1}{2 - \sqrt{e}}\right)$

15. Let f(x): R→R be defined as

$$f(x) = \begin{cases} \frac{a - b\cos 2x}{x^2}; x < 0 \\ x^2 + cx + 2; 0 \le x \le 1 \\ 2x + 1 \quad ; x > 1 \end{cases}$$

If f is continuous everywhere in R and m is the number of points where f is NOT differential then $m + a + b + c$ equals :

(1) 1 (2) 4 (3) 3 (4) 2

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $f(0^-) = f(0) \Rightarrow 2b = 2 \Rightarrow b = 1$

$f(1) = f(1^+) \Rightarrow 3 + c = 3 \Rightarrow c = 0$

Now $f(x) = \begin{cases} \frac{1 - \cos 2x}{x^2}, & x < 0 \\ x^2 + 2, & 0 \le x \le 1 \end{cases}$

$2x + 1 \text{ , } 1 < x$

$\begin{cases} \frac{2\sin^2 x}{x^2}, & x < 0 \\ x^2 + 2, & 0 \le x \le 1 \end{cases}$

$2x + 1, 1 < x$

clearly differentiable everywhere so $m = 0$

$m + a + b + c = 2$

16. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is $\sqrt{14}$.

Then the square of the eccentricity $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(1) 3 (2) 7/2 (3) 3/2 (4) 5/2

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $\frac{2b^2}{a}=\sqrt{14}$ .....(1)

$e^2=1-\frac{b^2}{a^2}$

$\frac{1}{2}=1-\frac{\sqrt{14}}{a}.\frac{1}{2}$

$\frac{\sqrt{14}}{2a}=\frac{1}{2}$

$14 = a^2$

and $b^2 = 7$ from (1)

Now eccentricity of hyperbola = $\sqrt{1+\frac{b^2}{a^2}}$

$=\sqrt{\frac{3}{2}}$

17. Let 3, a, b, c be in A.P. and 3, a–1, b + 1 c + 9 be in G.P. Then, the arithmetic mean of a, b and c is:

(1) –4 (2) –1 (3) 13 (4) 11

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $2a = b + 3$ ....(1)

$2b = a + c$ .... (2)

$\frac{a-1}{3}=\frac{b+1}{a-1}=\frac{c+9}{b+1}$ .... (3)

$\frac{\frac{b+3}{2}-1}{3}=\frac{b+1}{\frac{b+3}{2}-1}=\frac{2b-a+9}{b+1}$

$\frac{b+1}{6}=\frac{(b+1)\times 2}{b+1}=\frac{2b-\frac{b+3}{2}+9}{b+1}$

$\frac{b+1}{6}=2$

$b=11$

$2b+2=\frac{4b-b-3+18}{2}$

$4b+4=3b+15$

$b=11$

Now, a =7, c = 22 – 7 = 15

Now A. M of a, b, c

$=\frac{a+b+c}{3}=\frac{7+11+15}{3}=11$

18. Let C: $x^2 + y^2=4$ and C': $x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of λ so that the circles C and C' intersect at two distinct points, is R-[a, b], then the point (8a+12, 16b-20) lies on the curve:

(1) $x^2+2y^2-5x+6y=3$ (2) $5x^2 - y = -11$

(3) $x^2 - 4y^2 = 7$ (4) $6x^2 + y^2= 42$

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $x^2 + y^2 = 4$, c_1 (0,0) , $r_1 = 2$

$x^2 + y^2 - 4\lambda x + 9 = 0$, $C_2(2\lambda,0)$, $r_2 = \sqrt{4\lambda^2 - 9}$

$|r_1 - r_2| < c_1c_2 < |r_1 + r_2|$

$\left|2-\sqrt{4\lambda^2-9}\right| < |2\lambda| < 2 + \left|2+\sqrt{4\lambda^2-9}\right|$

$4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2-9} < 4\lambda^2 < 4 + 4\lambda^2 - 9 + 4\sqrt{4\lambda^2-9}$

$-5 < 4\sqrt{4\lambda^2-9}$ ....(A)

$\lambda \in R$

and $5 < 4\sqrt{4\lambda^2-9}$

$25 < 16(4\lambda^2 - 9)$

$\frac{169}{64} < \lambda^2$

$\lambda \in \left(-\infty, \frac{-13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$.....(1)

Also $4\lambda^2 - 9 \le 0$

$\lambda \in \left(-\infty. \frac{-3}{2}\right] \cup \left[\left(\frac{3}{2}, \infty\right)\right]$.....(2)

Hence $\lambda \in R - \left[\frac{-13}{8}, \frac{13}{8}\right]$

(8a + 12), 16b − 20)

= (−1, 6)

which satisfies (4) option

19. If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2,\ \forall\ x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in:

(1) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ (2) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(3) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$ (4) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 2$

$\Rightarrow 25f(x) + 20f\left(\frac{1}{x}\right) = 5x^2 - 10$

$\Rightarrow 16f(x) + 20f\left(\frac{1}{x}\right) = \frac{4}{x^2} - 8$

$\Rightarrow 9f(x) = 5x^2 - 10 - \frac{4}{x^2} + 8$

$y = 9f(x)\ x^2 = 5x^4 - 2x^2 - 4$

$\frac{dy}{dx} = 20x^3 - 4x = 4x(5x^2 - 1)$

in $\left(-\sqrt{\frac{1}{5}}, 0\right)$ and $\left(\frac{1}{\sqrt{5}}, \infty\right)$ is increasing

20. If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1, then the sum of all possible values of λ is :

(1) 0 (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) $-2\sqrt{3}$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $S.D = \left|\frac{1}{\vec{b}_1 \times \vec{b}_2}\begin{vmatrix} \lambda-3 & 1 & -1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}\right|$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \hat{i}(3) - \hat{j}(-3) + 3\hat{k}$

$= \left|\frac{3(\lambda-\sqrt{3}) + 3 - 3}{\sqrt{9+9+9}}\right| = 1 \quad = \left|\frac{\lambda-\sqrt{3}}{\sqrt{3}} = 1 \Rightarrow \lambda - \sqrt{3} = \pm\sqrt{3}\right| \Rightarrow \lambda = 0, \ = 2\sqrt{3}$

21. If x=x(t) is the solution of the differential equation $(t+1)\,dx = (2x+(t+1)^4)dt$, $x(0) = 2$ then, x(1) equals

NTA Ans. **(14)**

Reso Ans. **(14)**

Sol. $\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^3$

$\text{I.F.} = e^{\int \frac{-2}{t+1}dt} = e^{-2\ell n(t+1)} = \frac{1}{(t+1)^2}$

$\frac{x}{(t+1)^2} = \int (t+1)dt \Rightarrow \frac{x}{(t+1)^2} = \frac{t^2}{2} + t + c$

At $t = 0, \quad x = 2$

$c = 2$

$\Rightarrow \frac{x}{(t+1)^2} = \frac{t^2}{2} + t + 2$

$t = 1 \Rightarrow x(1) = 14$

22. The number of elements in the set

$S=\{(x, y, z) : x, y, z \in Z, x+ 2y + 3z =42, x, y, z \geq 0\}$ equals

NTA Ans. **(169)**

Reso Ans. **(169)**

Sol. $z = 0, x + 2y = 42 \Rightarrow$ 22 Solutions

$z = 1, x + 2y = 39 \Rightarrow$ 20 Solutions

$z = 2, x + 2y = 36 \Rightarrow$ 19 Solutions ……….

Similarly $z = 14$ $x + 2y = 0 \Rightarrow$ 1 Solutions

number of solutions = $(1+2+3....... +22) - (3+6+...... +21) = 169$

23. If the coefficient of x^{30} in the expansion of $\left(1+\frac{1}{x}\right)^6 (1+x^2)^7 (1-x^3)^8 ; x \neq 0$ is α then $|\alpha|$ equals____.

NTA Ans. **(678)**

Reso Ans. **(678)**

Sol. Coefficient of x^{30} in

$= \left(1+\frac{1}{x}\right)^6 (1+x^2)^7 (1-x^3)^8$

$\Rightarrow$ coefficient x^{36} = $(1 + x)^6 (1 + x^2)^7 (1 - x^3)^8$

$= ({}^6C_0 + {}^6C_1 x + ... + {}^6C_6 x^6)(1 + {}^7C_1 x^2 + {}^7C_2 x^4 + ... + {}^7C_7 x^{14})(1 - {}^{\cdot}C_1 x^3 + {}^8C_2 x^6 + ... + {}^8C_8 x^{24})$

coefficient x^{36} with ${}^8C_8\, x^{24}$ is = ${}^6C_0 \times {}^7C_0 \times {}^8C_8 + {}^6C_2 \times {}^7C_5 \times {}^8C_8 + {}^6C_4 \times {}^7C_4 \times {}^8C_8$

${}^6C_6 \times {}^7C_3 \times {}^8C_8 = 7 + 15 \times 21 + 15 \times 35 + 35 = 882$

coefficient x^{36} with $\left(-{}^8C_7 x^{21}\right) = -\,{}^8C_7\,({}^7C_7 \times {}^6C_1 + {}^7C_6 \times {}^6C_3 + {}^7C_5 \times {}^6C_5)$

$= -\,8(6 + 140 + 126)$

$= -\,2176$

coefficient x^{36} with ${}^8C_6\, x^{18}$

$= {}^8C_6\,({}^7C_7 \times {}^6C_4 + {}^7C_6 \times {}^6C_6)$

$= 28(15 + 7)$

$= 616$

coefficient x^{36} with ${}^8C_5 x^{15} = 0$

so final coefficient $x^{36} = -\,678$

$|\alpha| = 678$

24. Let 3, 7, 11, 15, 403 and 2, 5, 8, 11,..., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to________

NTA Ans. **(6699)**

Reso Ans. **(6699)**

Sol. 3, 7, 11, ……, 403 C.D. = 4

2, 5, 8, ……., 404 C.D. = 3

LCM {4,3} = 12

11, 23, …… is sequence of common terms

$t_n = 11 + (n - 1)12 \le 403$

$n \le 33.6$

$n = 33$

Sum = $\frac{33}{2}[22 + 384] = 6699$

25. Let {x} denote the fractional part of x and f(x) = $\frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}\left(1-\{x\}\right)}{\{x\}-\{x\}^3}, x \neq 0$. If L and R respectively denotes the left hand limit and the right hand limit of f(x) at x=0, then $\frac{32}{\pi^2}\left(L^2 + R^2\right)$ is equal to __________

NTA Ans. **(18)**

Reso Ans. **(18)**

Sol. $L = \lim_{x\to 0^-} \frac{\cos^{-1}\left(1-(x+1)^2\right)\sin^{-1}\left(1-(1+x)\right)}{(x+1)(1-(x+1)^2)}$, {x} = x + 1

$= \lim_{x\to 0^-} \frac{\cos^{-1}(-x^2-2x)\sin^{-1}(-x)}{(x+1)(-x^2-2x)}$

$= \frac{\cos^{-1}(0)}{1}\frac{1}{2}(1) = \frac{\pi}{4}$

$R = \lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x(1-x^2)}$, {x} = x

$\lim_{x\to 0^+} \frac{\dfrac{-1(-2x)\frac{\pi}{2}}{2\sqrt{1-(1-x^2)^2}}}{1}$

$\lim_{x\to 0^+} \frac{\sqrt{2-x^2}}{1}\frac{\pi}{2} = \frac{\pi}{\sqrt{2}}$

$\frac{32}{\pi^2}\left(L^2+R^2\right) = \frac{32}{\pi^2}\left(\frac{\pi^2}{16}+\frac{\pi^2}{2}\right) = 18$

26. Let the line L: $\sqrt{2}$ x + y = α pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to ___________.

NTA Ans. **(72)**

Reso Ans. **(72)**

Sol. $x^2 + y^2 = 3$

$x^2 = 2y$

Solving weget $y^2 + 2y - 3 = 0$

y = – 3 or 1

So in first quadrant

Point of Intersection $(\sqrt{2}, 1)$

It satisfies line & $\sqrt{2}x + y = \alpha$

so α = 3

Let circles C_1 & C_2 and $(x-0)^2 + (y-\beta)^2 = 12$

Now line $\sqrt{2}x + y = 3$ touches above circles

So

$\left|\frac{0+\beta-3}{\sqrt{3}}\right| = 2\sqrt{3}$

So Q_1 (0, 9) & Q_2 (0, – 3) and P $(\sqrt{2}, 1)$

Area of $\Delta PQ_1Q_2 = \frac{1}{2}(9+3)\sqrt{2}$

$= 6\sqrt{2}$

square of Area = 72

Ans. 72

27. Let P= {z∈ C: |z+ 2– 3i |≤1) and Q= (z ∈ C: z(1+i) + $\bar{z}$ (1–i) ≤ – 8) .Let in P∩ Q, |z–3+2i| be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha+\beta\sqrt{2}$ where α, β are integers, then α+β equals________

NTA Ans. (36)

Reso Ans. (36)

Sol. $|z+2-3i| \le 1$

$(x+2)^2 + (y-3)^2 \le 1 \rightarrow$ Circle

With centre (–2, 3) & r = 1

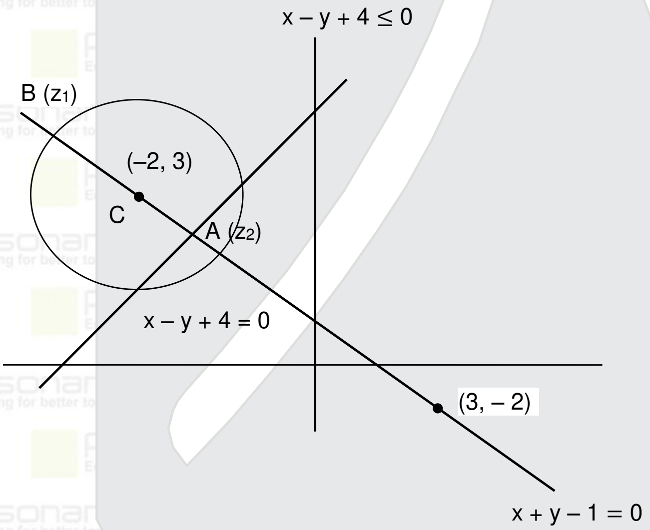
And

$z(1+i) + \bar{z}(1-i) \le -8$

x + ix + iy – y + x – xi – iy – y ≤ – 8

2x – 2y ≤ – 8

x – y + 4 = 0



line ⊥'r x – y + 4 = c

& passing through (3, –2) is x + y –1 =0

From fig. B represents z_1

And A represents z_2

Distance of line x – y + 4 = 0

Centre (–2, 3) = $\left|\frac{-2-3+4}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

Now line x + y – 1 = 0 can be written as

$$\frac{x+2}{-\frac{1}{\sqrt{2}}} = \frac{y-3}{\frac{1}{\sqrt{2}}} = \text{CA or CB}$$

For $CA = \frac{-1}{\sqrt{2}}$ & $CB = 1$

$$A = \left(\frac{-3}{2}, \frac{5}{2}\right)$$

$$B = \left(\frac{-1}{\sqrt{2}} - 2, \frac{+1}{\sqrt{2}} + 3\right)$$

Hence $|z_1|^2 = \left(2 + \frac{1}{\sqrt{2}}\right)^2 + \left(3 + \frac{1}{\sqrt{2}}\right)^2$

$$= 4 + \frac{1}{2} + 2\sqrt{2} + 9 + \frac{1}{2} + 3\sqrt{2}$$

$$= 14 + 5\sqrt{2}$$

& $2|z_2|^2 = 2\left(\frac{9}{4} + \frac{25}{4}\right) = 17$

Hence $|z_1|^2 + 2|z_2|^2 = 31 + 5\sqrt{2}$

So $\alpha + \beta = 36$

28. $\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\, dx}{(1+e^{\sin x})(1+\sin^4 x)} = \alpha\pi + \beta\log_e(3+\sqrt{2})$, where α, β are integers, then $\alpha^2 + \beta^2$ equals________

NTA Ans. ()

Reso Ans. ()

Sol. Using even odd property

$$= \int_0^{\frac{\pi}{2}} \left(\frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} + \frac{e^{\sin x} 8\sqrt{2}\cos x}{(e^{\sin x}+1)(1+\sin^4 x)} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+\sin^4 x)} dx \quad \text{Put } t = \sin x \;\; = 8\sqrt{2}\int_0^1 \frac{dt}{1+t^4} = 4\sqrt{2}\int_0^1 \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt - 4\sqrt{2t}\int_0^1 \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt$$

$$= 4\sqrt{2}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)_0^1 - 4\sqrt{2}\frac{1}{2\sqrt{2}}\ell n\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|_0^1 = 2\pi + 2\ell n(3+2\sqrt{2})$$

$\alpha^2 + \beta^2 = 8$

29. Let the line of the shortest distance between the lines

$L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and

$L_2 : \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

Intersect L_1 and L_2 at P and Q respectively. If (α, β, λ) is the mid point of the segment PQ then $2(\alpha+\beta+\lambda)$ is equal to ________.

NTA Ans. **(21)**

Reso Ans. **(21)**

Sol. Let point on L_1 is P(i $(1 + \lambda)$ + j$(2 - \lambda)$ + k$(3 + \lambda)$)

and point on L_2 is P(i $(4 + \mu)$ + j$(5 + \mu)$ + k$(6 - \mu)$)

$\overrightarrow{PQ}$ is perpendicular to L_1 and L_2

Vector $\overrightarrow{PQ}$ $i(\lambda - \mu - 3) + j(-3 - \lambda - \mu) + \hat{k}(-3 + \lambda + \mu)$

$\overrightarrow{PQ} \perp L_1$ then $(\lambda - \mu - 3) + (3 + \lambda + \mu) + (-3 + \lambda + \mu) = 0$

$3\lambda + \mu = 3$ ....(1)

$\overrightarrow{PQ} \perp L_2$ then $(\lambda - \mu - 3) + (-3 - \lambda - \mu) + (3 - \lambda - \mu) = 0$

$-\lambda - 3\mu = 3$

$\Rightarrow \mu = \frac{-3}{2}$ & $\lambda = \frac{3}{2}$ ....(2)

$P\left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$

$Q\left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

mid point of PQ = $\left(\frac{5}{2}, 2, 6\right)$

now $2(\alpha + \beta + \gamma) = 21$

30. Let A = {1,2, 3, ..., 20}. Let R_1 and R_2 two relation on A such that

R_1 = {(a, b): b is divisible by a}

R_2 = {(a, b): a is an integral multiple of b}.

Then, number of elements in $R_1 - R_2$ is equal to ____________

NTA Ans. **(46)**

Reso Ans. **(46)**

Sol. R_1 = {(1, 1) (1, 2) .....(1, 20) (2, 2) (2, 4) ..... (2, 20)

(3, 3) (3, 6) (3, 9) (3, 12) (3, 15) (3, 18)

(4, 4) (4, 8) (4, 12) (4, 16) (4, 20)

(5, 5) (5, 10) (5, 15) (5, 20) (6, 6) (6, 12) (6, 18)

(7, 7) (7, 14)

(8, 8) (8,16)

(9, 9) (9, 18)

(10, 10) (10, 20)

(11, 11)......... (20, 20)

$n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 + 2 + 10 = 66$

$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$

$= 66 - 20 = 46$