DATE & DAY: 01st February 2024 & Thursday

PAPER-1

Duration: 3 Hrs.
Time: 03:00 PM - 06:00 PM

SUBJECT: MATHEMATICS

ADMISSIONS OPEN FOR CLASS 12+
ACADEMIC SESSION 2024-25

TARGET: JEE (ADV.) 2024
For Class XII Passed Student
VISHESH COURSE
MODE: OFFLINE/ONLINE
CLASS STARTS: 08th APRIL, 2024

TARGET: JEE (MAIN) 2024
For Class XII Passed Student
ABHYAAS COURSE
MODE: OFFLINE/ONLINE
CLASS STARTS: 08th APRIL, 2024

SCHOLARSHIP ON THE BASIS OF JEE (MAIN) 2024 %ILE/AIR

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1. If the domain of the function \[ f(x) = \sqrt{\frac{x^2 - 25}{4 - x^2}} + \log_6\left(\frac{x^2 + 2x - 15}{x^2 + 1}\right) \] is \((-\infty, a) \cup [b, \infty)\), then \(a^2 + \beta^3\) is equal to:

(1) 140
(2) 175
(3) 125
(4) 150

NTA (4)
Reso. (4)

Sol. \(x^2 - 25 \geq 0\)
\(x \in (-\infty, -5] \cup [5, \infty)\) \hspace{1cm} (i)

\(4 - x^2 \neq 0\)
\(x \neq \pm 2\) \hspace{1cm} (ii)

\(x^2 + 2x - 15 > 0\)
\((x - 3)(x + 5) > 0\)
\(x \in (-\infty, -5) \cup (3, \infty)\) \hspace{1cm} (iii)

\(x \in (i) \cap (ii) \cap (iii)\)
\(x \in (-\infty, -5) \cup [5, \infty)\)
\(\Rightarrow a = -5, \beta = 5\)
\(a^2 + \beta^3 = 25 + 125 = 150\)

2. If \(z\) is a complex number such that \(|z| \geq 1\), then the minimum value of \(z + \frac{1}{2}(3 + 4i)\) is:

(1) 2
(2) \frac{5}{2}
(3) \frac{3}{2}
(4) 3

NTA (3)
Reso. Bonus (There is a correction \(|z| \leq 1\) then Ans. (3))

Sol.
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\[
\begin{align*}
    \left| z - \left( -\frac{3}{2}, -2i \right) \right|_{\text{min}} &= PA = OP - r \\
    &= \sqrt{\frac{9}{4} + 4 - 1} = \frac{5}{2} - 1 = \frac{3}{2}
\end{align*}
\]

3. Consider a \( \Delta ABC \) where \( A(1,3,2) \), \( B(-2,8,0) \) and \( C(3,6,7) \). If the angle bisector of \( \angle BAC \) meets the line \( BC \) at \( D \), then the length of the projection of the vector \( \overrightarrow{AD} \) on the vector \( \overrightarrow{AC} \) is:

- (1) \( \frac{37}{2\sqrt{38}} \)
- (2) \( \sqrt{19} \)
- (3) \( \frac{39}{2\sqrt{38}} \)
- (4) \( \frac{\sqrt{38}}{2} \)

4. Consider the relations \( R_1 \) and \( R_2 \) defined as \( aR_1b \iff a^2 + b^2 = 1 \) for all \( a, b \in \mathbb{R} \) and \( \langle a, b \rangle R_2(c, d) \iff a + d = b + c \) for all \( \langle a, b \rangle, \langle c, d \rangle \in \mathbb{N} \times \mathbb{N} \)

- (1) \( R_1 \) and \( R_2 \) both are equivalence relations
- (2) Only \( R_1 \) is an equivalence relation
- (3) Only \( R_2 \) is an equivalence relation
- (4) Neither \( R_1 \) nor \( R_2 \) is an equivalence relation

\[
\begin{align*}
\text{Projection of } \overrightarrow{AD} \text{ on } \overrightarrow{AC} &\Rightarrow \overrightarrow{AC} = \overrightarrow{AD}. \overrightarrow{AC} = \frac{-1 + 12 + 15}{2} \overrightarrow{AC} = \frac{37}{2\sqrt{38}}
\end{align*}
\]
Symmetric : (a, b) R (c, d) \(\Rightarrow a + d = b + c\)
\[\Rightarrow d + a = c + b\]
\[\Rightarrow c + b = d + a\]
\[\Rightarrow (c, d) R (a, b)\] True

Transitive 
(a, b) R (c, d) \(\Rightarrow a + d = b + c \ldots (1)\)
(c, d) R (e, f) \(\Rightarrow c + f = d + e \ldots (2)\)
\[\Rightarrow a + f = b + e \text{ by } (1) \text{ and } (2)\]
\[\Rightarrow (a, b) R (e, f) \text{ transitive}\]
:. \(R_2\) is equivalence

5. Let the system of equations 
\[x + 2y + 3z = 5, 2x + 3y + z = 9, 4x + 3y + \lambda z = \mu\] have infinite number of solutions.
Then \(\lambda + 2\mu\) is equal to:

\[
\begin{align*}
\text{NTA} & \quad (2) \\
\text{Reso.} & \quad (2)
\end{align*}
\]

Sol. System of equation’s are
\[
\begin{align*}
x + 2y + 3z &= 5 \\
2x + 3y + z &= 9 \\
4x + 3y + \lambda z &= \mu
\end{align*}
\]

have infinite many solutions only if \(\Delta = 0\) and \(\Delta_1 = 0, \Delta_2 = 0 \& \Delta_3 = 0\)

\[
\begin{vmatrix}
1 & 2 & 3 \\
2 & 3 & 1 \\
4 & 3 & \lambda
\end{vmatrix} = 0
\]
\[
3\lambda + 18 + 8 - 36 - 3 - 4\lambda = 0
\]
\[
\lambda = -13
\]

Now \(\Delta_1 = \begin{vmatrix}
5 & 2 & 3 \\
9 & 3 & 1 \\
\mu & 3 & -13
\end{vmatrix}\)
\[
= 5(-22) - 9(-35) + \mu(-7)
\]
\[
= -210 + 315 - 7\mu
\]
\[
= 105 - 7\mu = 7(15 - \mu)
\]

\[
\Delta_2 = \begin{vmatrix}
1 & 5 & 3 \\
2 & 9 & 1 \\
4 & \mu & -13
\end{vmatrix}
\]
\[
= 4(-22) - \mu(-5) - 13(-1)
\]
\[
= -88 + 5\mu + 13
\]
\[
= 5\mu - 75
\]
\[ \Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & \mu \end{vmatrix} = 4(3) - 3(-1) + \mu(-1) = (15 - \mu) \]

since \( \mu = 15 \), all \( \Delta_1 = \Delta_2 = \Delta_3 = 0 \)

So equations have infinite many solutions for \( \lambda = -13 \) & \( \mu = 15 \)

now \( \lambda + 2\mu = -13 + 30 = 17 \)

6. If \( \int_0^3 \cos^4 x \, dx = a\pi + b\sqrt{3} \), where \( a \) and \( b \) are rational numbers, then \( 9a + 8b \) is equal to:

\[ \begin{align*}
(1) & \quad 2 \\
(2) & \quad 1 \\
(3) & \quad 3 \\
(4) & \quad \frac{3}{2}
\end{align*} \]

NTA \( (1) \)

Reso. \( (1) \)

Sol. \[ \int_0^{\pi/3} \left( \frac{1 + \cos 2x}{2} \right)^2 \, dx = \int_0^{\pi/3} \left( 1 + 2\cos 2x + \frac{1 + \cos 4x}{2} \right) \, dx \]

\[ = \frac{1}{8} \int_0^{\pi/3} (3 + 4\cos 2x + \cos 4x) \, dx \]

\[ = \frac{1}{8} \left( 3x + 2\sin 2x + \frac{1}{4} \sin 4x \right) \bigg|_0^{\pi/3} \]

\[ = \frac{1}{8} \left( \pi + 2\sin \frac{2\pi}{3} + \frac{1}{4} \sin \frac{4\pi}{3} \right) \]

\[ = \frac{1}{8} \left( \pi + \sqrt{3} - \frac{\sqrt{3}}{2} \right) \]

\[ = \frac{1}{8} \left( \pi + \frac{7\sqrt{3}}{8} \right) \]

\[ \Rightarrow a = \frac{1}{8}, \quad b = \frac{7}{64} \]

\[ 9a + 8b = \frac{9}{8} + \frac{7}{8} = 2 \]
7. Let \( \alpha \) and \( \beta \) be the roots of the equation \( px^2 + qx + r = 0 \), where \( p \neq 0 \). If \( p \), \( q \) and \( r \) be the consecutive terms of a non constant G.P. and \( \frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4} \), then the value of \((\alpha - \beta)^2\) is:

- (1) 8
- (2) 9
- (3) \( \frac{20}{3} \)
- (4) \( \frac{80}{9} \)

8. Let Ajay will not appear in JEE exam with probability \( p = \frac{2}{7} \), while both Ajay and Vijay will appear in the exam with probability \( q = \frac{1}{5} \). Then the probability, that Ajay will appear in the exam and Vijay will not appear is:

- (1) \( \frac{9}{35} \)
- (2) \( \frac{3}{35} \)
- (3) \( \frac{24}{35} \)
- (4) \( \frac{18}{35} \)

B : – Event that Vijay appear in Exam

Given \( p = P(\overline{A}) = \frac{2}{7} \) and \( q = P(\overline{A} \cap B) = P(A). P(B) = \frac{1}{5} \)

\[ P(A) = 1 - p = \frac{5}{7} \]

\[ P(B) = \frac{7}{25} \]

Hence \( P(A \cap \overline{B}) = P(A). P(\overline{B}) \)

\[ = \frac{5}{7} \cdot \frac{18}{25} = \frac{18}{35} \]
9. Let P be a point on the ellipse \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \). Let the line passing through P and parallel to y-axis meet the circle \( x^2 + y^2 = 9 \) at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4: 3 as P moves on the ellipse, is:

(1) \( \frac{21}{13} \)  
(2) \( \frac{23}{\sqrt{13}} \)  
(3) \( \frac{7}{\sqrt{13}} \)  
(4) \( \frac{19}{11} \)

10. Consider 10 observations \( x_1, x_2, \ldots, x_{10} \) such that \( \sum_{i=1}^{10} (x_i - \alpha) = 2 \) and \( \sum_{i=1}^{10} (x_i - \beta)^2 = 40 \) where \( \alpha, \beta \) are positive integers. Let the mean and the variance of the observations be \( \frac{6}{5} \) and \( \frac{84}{25} \) respectively. Then \( \frac{\beta}{\alpha} \) is equal to

(1) \( 2 \)  
(2) \( 1 \)  
(3) \( \frac{5}{2} \)  
(4) \( \frac{3}{2} \)
\[ \sum_{i=1}^{10} (x_i - \beta) = 8 \]
\[ \Rightarrow \frac{1}{10} \sum_{i=1}^{10} (x_i - \beta) = \frac{8}{10} \]
\[ \Rightarrow \frac{1}{10} \sum_{i=1}^{10} (x_i - \beta) = \frac{4}{5} \]
\[ \Rightarrow \sum_{i=1}^{10} (x_i - \beta) = \pm \frac{4}{5} \]

**Case-I** When \( \sum_{i=1}^{10} (x_i - \beta) = 8 \)

By mean \[ \frac{6}{5} = \beta + \frac{1}{10} \sum_{i=1}^{10} (x_i - \beta) \]
\[ \Rightarrow \frac{6}{5} = \beta + \frac{8}{10} \Rightarrow \beta = \frac{2}{5} \] (Not integer)

**Case-II** When \( \sum_{i=1}^{10} (x_i - \beta) = -8 \)

By mean \[ \frac{6}{5} = \beta - \frac{8}{10} \Rightarrow \beta = 2 \]

Hence \( \frac{\beta}{\alpha} = 2 \) Ans.

11. Let \( f(x) = |2x^2 + 5x| - 3 \), \( x \in \mathbb{R} \). If \( m \) and \( n \) denote the number of points where \( f \) is not continuous and not differentiable respectively, then \( m + n \) is equal to:

1. 5
2. 3
3. 2
4. 0

**NTA** (2)
**Reso.** (2)

**Sol.**
\[ f(x) = |2x^2 + 5x| - 3 \]
\[ 2x^2 + 5x - 3 = (2x-1)(x+3) \]
f\( (x) \) is continuous for \( x \in \mathbb{R} \) and non-differentiable at

\[ x = \pm \frac{1}{2} \]

\[ \Rightarrow m = 0, n = 3 \]
\[ m + n = 3 \]
12. The number of solutions of the equation \(4 \sin^2 x - 4 \cos^3 x + 9 - 4 \cos x = 0; \ x \in [-2\pi, 2\pi]\) is:

(1) 0   (2) 3   (3) 1   (4) 2

NTA (1)  
Reso. (1)  
Sol.  

\[4-4\cos^2 x - 4\cos^3 x - 4\cos x + 9 = 0\]

\[4\cos^3 x + 4\cos^2 x + 4\cos x - 13 = 0\]

\[(\cos^2 x + \frac{1}{2})^2 + \frac{3}{4} = \frac{13}{4} \sec x\]

L.H.S \in [1, 3]  
R.H.S \in \left[ -\infty, \frac{13}{4}\right] \cup \left[ \frac{13}{4}, \infty\right]  

Number of solution = 0

13. Let the locus of the midpoint of the chords of the circle \(x^2 + (y - 1)^2 = 1\) drawn from the origin intersect the line \(x + y = 1\) at \(P\) and \(Q\). Then, the length of \(PQ\) is:

(1) \(\frac{1}{2}\)   (2) 1   (3) \(\frac{1}{\sqrt{2}}\)   (4) \(\sqrt{2}\)

NTA (3)  
Reso. (3)  
Sol.  

Let mid point is \((x_1, y_1)\)

\(x^2 + y^2 - 2y = 0\)

\(xx_1 + yy_1 - (y + y_1) = x_1^2 + y_1^2 - 2y_1\)

It is passing through origin

So, \(0 + 0 -(0 + y_1) = x_1^2 + y_1^2 - 2y_1\)

\(-y_1 = x_1^2 + y_1^2 - 2y_1\)

\(\Rightarrow x_1^2 + y_1^2 - y_1 = 0\)

Locus is \(x^2 + y^2 - y = 0\) \(\ldots (1)\)

\(\therefore\) it intersects the line \(x + y = 1\)

so put \(x = (1 - y)\) is equation \((1)\)

\((1 - y)^2 + y^2 - y = 0\)

\(2y^2 - 3y + 1 = 0\)

\((y - 1)(2y - 1) = 0\)
14. Let $\alpha$ be a non-zero real number. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f(0) = 2$ and $\lim_{x \to -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in \mathbb{R}$, then $f(-\log_2 3)$ is equal to _______.

(1) 7  (2) 9  (3) 3  (4) 5

Reso. (2) or Bonus

Sol. $f'(x) = \alpha f(x) + 3$

$\Rightarrow \frac{f'(x)}{f(x)} = 1$

$\Rightarrow \frac{1}{\alpha} \log_e |\alpha f(x) + 3| = x + c$

given $f(0) = 2$

$\Rightarrow \frac{1}{\alpha} \log_e |2\alpha + 3| = c$

$\Rightarrow \frac{1}{\alpha} \log_e |\alpha f(x) + 3| = x + \frac{1}{\alpha} \log_e |2\alpha + 3|$

$\Rightarrow \frac{1}{\alpha} \log_e |\alpha f(x) + 3| = x$

$\Rightarrow |\alpha f(x) + 3| = e^{ax}$ \hspace{1cm} (1)

Since $\lim_{x \to -\infty} f(x) = 1$

Case-I $\alpha > 0$ then by equation (1)

$\Rightarrow 3 + \alpha = 0 \quad \Rightarrow \alpha = -3$ (ambiguity)

$\Rightarrow |f(x) - 1| = e^{-3x}$

$\Rightarrow f(x) = 1 \pm e^{-3x}$

$\Rightarrow f(-\log_2 3) = 1 \pm e^{3\log_2 3}$

$= 1 \pm 8 = 9$ or $-7$

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Case-II $\alpha < 0$ then by (1)

\[
\left| \frac{\alpha + 3}{2\alpha + 3} \right| = \infty \text{ not defined}
\]

15. Let P and Q be the points on the line \( \frac{x + 3}{8} = \frac{y - 4}{2} = \frac{z + 1}{2} \) which are at a distance of 6 units from the point R (1, 2, 3). If the centroid of the triangle PQR is \((\alpha, \beta, \gamma)\), then \(\alpha^2 + \beta^2 + \gamma^2\) is:

\[
\begin{align*}
(1) & \quad 18 \\
(2) & \quad 24 \\
(3) & \quad 26 \\
(4) & \quad 36
\end{align*}
\]

NTA (1)  
Reso. (1)

Sol. Let \( \frac{x + 3}{8} = \frac{y - 4}{2} = \frac{z + 1}{2} = \lambda \)

\[
\Rightarrow \text{coordinated of any point on it is } P (8\lambda - 3, 2\lambda + 4, 2\lambda - 1)
\]

Its distance from R(1, 2, 3) is 6 unit

\[
(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36
\]

\[
(64 + 4 + 4)\lambda^2 + (16 + 4 + 16) - (64 - 8 + 16)\lambda = 36
\]

\[
72\lambda^2 - 72\lambda = 0
\]

\[
\lambda = 0 \text{ or } 1
\]

Hence P (−3, 4, −1) & Q(5, 6, 1)

Centroid of \( \triangle PQR \) is \((\alpha, \beta, \gamma) = (1, 4, 1)\)

\[
\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 18
\]

16. The value of \( \int_0^1 (2x^3 - 3x^2 - x + 1)^{1/3} \) dx is equal to:

\[
\begin{align*}
(1) & \quad -1 \\
(2) & \quad 2 \\
(3) & \quad 0 \\
(4) & \quad 1
\end{align*}
\]

NTA (3)  
Reso. (3)

Sol. \[
I = \int_0^1 (2x^3 - 3x^2 - x + 1)^{1/3} \) dx
\]

\[
= \int_0^1 (2x - 1)(x^2 - x - 1)^{1/3} \) dx
\]

\[
= \int_0^1 (2(1 - x) - 1)(1 - x)^2 - (1 - x) - 1^{1/3} \) dx
\]

\[
= \int_0^1 (1 - 2x)(x^2 - x - 1)^{1/3} \) dx
\]
\[ I = -1 \]
\[ 2I = 0 \]
\[ I = 0 \]

17. If the mirror image of the point \( P(3, 4, 9) \) in the line \( \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda \) is \( (\alpha, \beta, \gamma) \) then 14 \((\alpha, \beta, \gamma)\) is:

\[
\begin{align*}
(1) & \quad 102 \\
(2) & \quad 138 \\
(3) & \quad 132 \\
(4) & \quad 108
\end{align*}
\]

Sol.
Let \( \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda \)

Let foot of perpendicular from \( P(3, 4, 9) \) on line is \( M \)

\[ M(1 + 3\lambda, -1 + 2\lambda, 2 + \lambda) \]

direction ratio of \( PM : 3, -2, 2 \)

direction ratio of line \( L : 3, 2, 1 \)

Now \( PM \perp L \implies 3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0 \)

\[ 9\lambda - 6 + 4\lambda - 10 + \lambda - 7 = 0 \]

\[ 14\lambda = 23 \implies \lambda = \frac{23}{14} \]

Now \( M \) is mid point of \( PQ \)

\[ \frac{\alpha + 3}{2} = 1 + 3\lambda \]

\[ \frac{\beta + 4}{2} = -1 + 2\lambda \]

\[ \frac{\gamma + 9}{2} = 2 + \lambda \]

\[ \therefore \frac{\alpha + \beta + \gamma + 16}{2} = 2 + 6\lambda = \frac{166}{14} \]

\[ \alpha + \beta + \gamma = \frac{166}{7} - 16 = \frac{54}{7} \]

\[ \therefore 14(\alpha + \beta + \gamma) = 108 \]
18. Let \( S_n \) denote the sum of the first \( n \) terms of an arithmetic progression. If \( S_{10} = 390 \) and the ratio of the tenth and the fifth terms is \( 15 : 7 \), then \( S_{15} - S_5 \) is equal to:

(1) 800  
(2) 890  
(3) 790  
(4) 690

NTA (3)  
Reso. (3)

Sol.  
\[ S_n = \frac{n}{2} (2a + (n - 1)d) \]
and  
\[ T_n = a + (n - 1)d \]

Now  
\[ \frac{T_5}{T_{10}} = \frac{7}{15} \Rightarrow \frac{a + 4d}{a + 9d} = \frac{7}{15} \Rightarrow 8a = 3d \quad (1) \]

Also  
\[ S_{10} = 5(2a + 9d) = 390 \]

\[ \Rightarrow 2a + 9d = 78 \]

\[ \Rightarrow 2a + 24a = 78 \]

\[ \Rightarrow a = 3 & d = 8 \]

Hence  
\[ S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8) \]

\[ = 45 + 15 \times 14 \times 4 - 15 - 5 \times 16 \]

\[ = 45 + 840 - 15 - 80 \]

\[ = 885 - 95 \]

\[ = 790 \]

19. Let \( m \) and \( n \) be the coefficients of seventh and thirteenth terms respectively in the expansion of

\[ \left( \frac{1}{3} x^3 + \frac{1}{2} \right) \]

Then \( \left( \frac{n}{m} \right)^{13} \) is equal to:

(1) \( \frac{1}{9} \)  
(2) \( \frac{1}{4} \)  
(3) \( \frac{4}{9} \)  
(4) \( \frac{9}{4} \)

NTA (4)  
Reso. (4)

Sol.  
\[ m = 18C_6 \left( \frac{1}{3} \right)^{12} \left( \frac{1}{2} \right)^6 \]

\[ n = 18C_{12} \left( \frac{1}{3} \right)^6 \left( \frac{1}{2} \right)^{12} = \frac{18C_{12}}{(12^6)} \]

\[ \frac{n}{m} = \left( \frac{18}{12} \right)^6 \frac{18C_{12}}{18C_6} = \frac{3^6}{2^6} \]

\[ \left( \frac{n}{m} \right)^3 = \left( \frac{9}{4} \right)^3 \]
20. Let \( f(x) = \begin{cases} \frac{x-1}{2x}, & \text{x is even} \\ x, & \text{x is odd} \end{cases} \) if for some \( a \in \mathbb{N} \), \( f(f(a))) = 21 \) then \( \lim_{x \to a} \left( \frac{\lfloor x \rfloor}{a} - \frac{x}{a} \right) \) where \( \lfloor t \rfloor \) denotes the greatest integer less than or equal to \( t \), is equal to:

(1) 169  
(2) 121  
(3) 225  
(4) 144

NTA  
Reso. (4)

Sol. Let \( a \) is even then \( f(a) = a - 1 \) (odd)

\( f(f(a)) = f(a-1) = 2a - 2 \) (even)

\( f(f(f(a))) = 2a - 3 \)

\( \Rightarrow 2a - 3 = 21 \Rightarrow a = 12 \)

Now \( \lim_{x \to a} \left( \frac{\lfloor x \rfloor}{12} - \frac{x}{12} \right) \) if \( x < 12 \)

= 144 - 0 = 144

If \( a \) is odd then \( f(f(a)) = 21 \Rightarrow a \notin \mathbb{N} \)

21. Three points \( O(0, 0) \), \( P(a, a^2) \), \( Q(-b, b^2) \), \( a > 0, b > 0 \), are on the parabola \( y = x^2 \). Let \( S_1 \) be the area of the region bounded by the line \( PQ \) and the parabola, and \( S_2 \) be the area of the triangle \( OPQ \). If the minimum value of \( \frac{S_1}{S_2} = \frac{m}{n} \), \( \gcd(m, n) = 1 \), then \( m + n \) is equal to ________.

NTA  
Reso. (7)

Sol.

Equation of line \( PQ \) is

\[ y - a^2 = \frac{b^2 - a^2}{b - a} (x - a) \]

\[ \Rightarrow y - a^2 = (a - b)(x - a) \]

\[ \Rightarrow y = (a - b)x + ab \]

\[ S_1 = \int_{-b}^{a} [(a - b)x + ab - x^2] \, dx \]
\[
\frac{1}{2}\left(a^2 - a^2b + b^2 - ab^2 + b^3\right) - \frac{1}{2}\left(ab^2 - b^3 - ab^2 + b^3\right) = \frac{1}{2}a^3 + \frac{1}{2}b^3 - \frac{1}{2}ab(a + b) = \frac{1}{6}(a + b)^3
\]

Also area \( S_2 = \frac{1}{2}(a^2b + ab^2) = \frac{1}{2}ab(a + b) \)

\[
\Rightarrow \frac{S_1}{S_2} = \frac{1}{3} \left( \frac{a + b}{ab} \right) = \frac{1}{3} \left( \frac{b + a}{a + b} + 2 \right)
\]

\[
\Rightarrow \left( \frac{S_1}{S_2} \right)_{\text{min}} = \frac{1}{3} (2 + 2) = \frac{4}{3} \Rightarrow m + n = 7
\]

22. The sum of squares of all possible values of \( k \), for which area of the region bounded by the parabolas \( 2y^2 = kx \) and \( ky^2 = 2(y-x) \) is maximum, is equal to ________

NTA (8)

Reso. (8)

Sol. On solving \( 2y^2 = kx \) and \( ky^2 = 2(y-x) \) we have \( ky^2 = 2y - 2 \left( \frac{2y^2}{k} \right) \)

\[
(k^2 + 4) y^2 - 2ky = 0
\]

\[
\Rightarrow y = 0 \quad \text{and} \quad y = \frac{2k}{k^2 + 4}
\]

\[
Ky^2 = 2(y-x) \quad \text{or}
\]

\[
\int_0^{2k/(k^2 + 4)} \left[ \left( \frac{2y - ky^2}{2} \right) - \left( \frac{2y^2}{k} \right) \right] dy = \left( \frac{y^2}{2} - \frac{ky^3}{6} - \frac{2y^3}{3k} \right)_{0}^{2k/(k^2 + 4)}
\]
23. If \( y = \frac{\sqrt{x+1}}{\sqrt{x} + x + \sqrt{x}} \), then \( 96y(y/6) \) is equal to ________.

**Reso. (105)**

**Sol.**

\[
y = \frac{(\sqrt{x+1})(\sqrt{x} - 1)(\sqrt{x} + x + 1)}{(x + \sqrt{x} + 1)} + \frac{1}{15} (3\cos^5x - 5\cos^3x)
\]

\[
y' = 1 + \frac{1}{15} (3\cos^5x - 5\cos^3x)
\]

\[
y'' = 1 + \frac{1}{15} (3\cos^5x - 5\cos^3x)
\]

\[
y'(\pi/6) = 1 + \frac{3}{4} = \frac{32}{32} = \frac{35}{32}
\]

\[
96y'(\pi/6) = 105
\]

24. If \( \frac{dx}{dy} = \frac{1 + x - y^2}{y} \), \( x(1) = 1 \), then \( 5x(2) \) is equal to ________.

**Reso. (5)**

**Sol.**

\[
\frac{dx}{dy} = \frac{1 + x - y^2}{y} \Rightarrow \frac{dx}{1 - y^2} = -\frac{1}{y} dy
\]

I.F. = \( e^{-\ln y} = \frac{1}{y} \)

solution is

\[
x = \int \frac{1 - y^2}{y} dy + C
\]

\[
\frac{x}{y} = \int \left( \frac{1}{y^2} - 1 \right) dy + C
\]

---

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\[ \frac{x}{y} = -1 - y + C \]
\[ x = -1 - y^2 + Cy \]

\[ x(1) = 1 \Rightarrow 1 = -1 - 1 + C \]
\[ C = 3 \]

\[ x(2) = -1 - 4 + 6 = 1 \]

25. Let \( f : (0, \infty) \rightarrow \mathbb{R} \) and \( F(x) = \int_{0}^{x} f(t)dt \). If \( F(x^2) = x^4 + x^5 \), then \( \sum_{r=1}^{12} f(r^2) \) is equal to \[
\text{NTA} \quad (219) \\
\text{Reso.} \quad (219) \\
\text{Sol.} \\
\begin{align*}
F'(x) &= xf(x) \quad \text{and} \quad 2xF' \left( x^2 \right) = 4x^3 + 5x^4 \\
F'(x^2) &= 4x^2 + 5x^3 \\
2F'(x) &= 4x + 5x^2 \\
\Rightarrow 2x + \frac{5}{2}x^2 &= xf(x) \\
\Rightarrow f(x) &= 2 + \frac{5}{2}x^2 \\
\Rightarrow f(x^2) &= 2 + \frac{5}{2}x \\
\sum_{r=1}^{12} f(r^2) &= \sum_{r=1}^{12} \left( 2 + \frac{5}{2}r \right) \\
&= 24 + \frac{5}{2} \sum_{r=1}^{12} r \\
&= 24 + \frac{5}{2} \times 12 \times 13 \\
&= 24 + 195 = 219
\end{align*}
\]

26. Let \( \triangle ABC \) be an isosceles triangle in which \( A \) is at \((-1,0)\), \( \angle A = \frac{2\pi}{3} \), \( AB=AC \) and \( B \) is on the positive \( x \)-axis. If \( BC = 4\sqrt{3} \) and the line \( BC \) intersects the line \( y=x+3 \) at \((\alpha, \beta)\), then \( \frac{\beta^4}{\alpha^2} \) is \[
\text{NTA} \quad (36) \\
\text{Reso.} \quad (36) \\
\]

\[ 2F'(x) = 4x + 5x^2 \]
Let B(p, 0)
\[ AB = p + 1, \quad P > 1 \]
Hence coordinate of C are \[ \left( -\frac{1}{2} p - \frac{3}{2}, \frac{\sqrt{3}}{2} p + 1 \right) \]
\[ \Rightarrow BC^2 = \left( -\frac{3}{2} (p + 1) \right)^2 + \left( \frac{\sqrt{3}}{2} (p + 1) \right)^2 \]
\[ \Rightarrow BC = \sqrt{3}(p + 1) = 4\sqrt{3} \Rightarrow p = 3 \]
Thus equation of BC is \[ y = -\frac{1}{\sqrt{3}}(x - 3) \]
Solve with line \[ y = x + 3 \] we have \[ x + 3 = -\frac{1}{\sqrt{3}}(x - 3) \]
\[ (\sqrt{3} + 1)x = 3(1 - \sqrt{3}) \]
\[ \Rightarrow x = -3 \left( \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \right) = -\frac{3}{2} \left( 4 - 2\sqrt{3} \right) \]
\[ \Rightarrow \alpha = -3(2 - \sqrt{3}) \]
& \[ \beta = -3 + 3\sqrt{3} \]
\[ \Rightarrow \frac{\beta^4}{\alpha^2} = \frac{8\left(\sqrt{3} - 1\right)^4}{9\left(2 - \sqrt{3}\right)^2} = \frac{9\left(4 - 2\sqrt{3}\right)^2}{\left(2 - \sqrt{3}\right)^2} = 36 \]

27. Let \( A = I_2 - 2MM^T \), where \( M \) is a real matrix of order \( 2 \times 1 \) such that the relation \( M^TM = I_1 \) holds. If \( \lambda \) is a real number such that the relation \( AX = \lambda X \) holds for some non-zero real matrix \( X \) of order \( 2 \times 1 \), then the sum of squares of all possible values of \( \lambda \) is equal to

**NTA (2) Reso. (2)**

**Sol.** Let \( M = \begin{bmatrix} a \\ b \end{bmatrix} \)
\[ \Rightarrow M^TM = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a^2 + b^2 \end{bmatrix} = I_1 \ (given) \]
Hence \( a^2 + b^2 = 1 \)

Let \( a = \cos \theta \), \( b = \sin \theta \)

then \( A = I_2 - 2 \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix} \)

\[
A = \begin{bmatrix} 1 & -2\cos^2 \theta \\ -2\sin \theta \cos \theta & 1 \end{bmatrix} - \begin{bmatrix} 2\cos^2 \theta & 2\sin \theta \cos \theta \\ 2\sin \theta \cos \theta & 2\sin^2 \theta \end{bmatrix}
\]

\[
= \begin{bmatrix} 1 - 2\cos^2 \theta & -2\sin \theta \cos \theta \\ -2\sin \theta \cos \theta & 1 - 2\sin^2 \theta \end{bmatrix}
\]

Now \( Ax = \lambda x \) for some non-zero real matrix \( x \)

\[
\Rightarrow (A - \lambda I)x = 0
\]

\[
\Rightarrow |A - \lambda I| = 0
\]

\[
\Rightarrow \begin{vmatrix} 1 - 2\cos^2 \theta & -2\sin \theta \cos \theta \\ -2\sin \theta \cos \theta & 1 - 2\sin^2 \theta \end{vmatrix} = 0
\]

\[
\Rightarrow -(\cos^2 \theta + \lambda)(\cos \theta - \lambda) - \sin^2 \theta \sin \theta = 0
\]

\[
\Rightarrow \cos^2 \theta - \lambda^2 + \sin^2 \theta \sin \theta = 0
\]

\[
\Rightarrow \lambda^2 = 1
\]

\[
\Rightarrow \lambda = \pm 1
\]

Sum of the square of values of \( \lambda \) is \( 1^2 + (-1)^2 = 2 \)

28. Let \( \vec{a} = \hat{i} + \hat{j} + \hat{k} \), \( \vec{b} = \hat{i} - 8\hat{j} + 2\hat{k} \) and \( \vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k} \) be three vectors such that \( \vec{b} \times \vec{a} = \vec{c} \times \vec{a} \). If the angle between the vector \( \vec{c} \) and the vector \( 3\hat{i} + 4\hat{j} + \hat{k} \) is, \( \theta \) then the greatest integer less than or equal to \( \tan^2 \theta \) is

NTA (38)

Reso. (38)

Sol. \( \vec{b} \times \vec{a} = \vec{c} \times \vec{a} \)

\[
\Rightarrow (\vec{c} - \vec{b}) \times \vec{a} = 0
\]

\[
\Rightarrow \vec{c} = \vec{b} = \lambda \vec{a}
\]

\[
\Rightarrow \vec{c} = \vec{b} + \lambda \vec{a}
\]

\[
\Rightarrow \vec{c} =((-1+\lambda)\hat{i}+(-8+\lambda)\hat{j})+(2+\lambda)\hat{k}
\]

but \( \Rightarrow \vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k} \)
29. If three successive terms of a G.P. with common ratio \( r \) \((r > 1)\) are the lengths of the sides of a triangle and \([r]\) denotes the greatest integer less than or equal to \( r \), then \( 3[r] + \lfloor -r \rfloor \) is equal to_____.

**NTA (1) Reso. (1)**

**Sol.** Let three terms of GP are \( \frac{a}{r}, a, ar \) \((r > 1)\)

Sum of two smaller sides > third side

\[
\Rightarrow \frac{a}{r} + a > ar \Rightarrow 1 + r > r^2
\]

\[
\Rightarrow r^2 - r - 1 < 0 \Rightarrow \frac{1 - \sqrt{5}}{2} < r < \frac{1 + \sqrt{5}}{2}
\]

but \( r > 1 \) \( \Rightarrow r \in \left\{ 1, \frac{1 + \sqrt{5}}{2} \right\} \)

\( \Rightarrow [r] = 1 \) and \([r] = -2 \)

So \( 3[r] + \lfloor -r \rfloor = 3 - 2 = 1 \)

30. The lines \( L_1, L_2, \ldots \ldots, L_{20} \) are distinct. For \( n=1, 2, 3, \ldots, 10 \) all the lines \( L_{2n-1} \) are parallel to each other and all the lines \( L_{2n} \) pass through a given point P. The maximum number of points of intersection of pairs of lines from the set \( \{L_1, L_2, \ldots \ldots, L_{20}\} \) is equal to_____.

**NTA (101) Reso. (101)**

**Sol.** Since \( L_1, L_3, L_5, \ldots, L_{19} \) all 10 lines are parallel to each other so does not intersect each other again \( L_2, L_4, L_6, \ldots, L_{20} \) all passes through a fixed point P.

So they intersect at only point P.

Now Each member of \( L_1, L_3, L_5, \ldots, L_{19} \) intersects each member of \( L_2, L_4, L_6, \ldots, L_{20} \) at 10 distinct points (for maximum point of intersection)

\( \Rightarrow \) Number of such point of Intersection = \( 10 \times 10 = 100 \)

So Maximum Number of Point of Intersection = \( 100 + 1 = 101 \)
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