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JEE (MAIN) 2024

QUESTIONS & SOLUTIONS

SHIFT-2

DATE & DAY: 01st February 2024 & Thursday

PAPER-1

Duration: 3 Hrs.
Time: 03:00 PM - 06:00 PM

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. If the domain of the function $f(x)=\frac{\sqrt{x^2-25}}{\left(4-x^2\right)}+\log_{10}\left(x^2+2x-15\right)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to :

(1) 140 (2) 175 (3) 125 (4) 150

NTA **(4)**

Reso. **(4)**

Sol. $x^2 - 25 \geq 0$

$x \in (-\infty,-5] \cup [5,\infty)$ ....(i)

$4 - x^2 \neq 0$

$x \neq \pm 2$ ....(ii)

$x^2 + 2x - 15 > 0$

$(x-3)(x+5) > 0$

$x \in (-\infty,-5) \cup (3,\infty]$ ....(iii)

$x \in$ (i) $\cap$ (ii) $\cap$ (iii)

$x \in (-\infty,-5) \cup [5,\infty)$

$\Rightarrow \alpha = -5, \beta = 5$

$\alpha^2 + \beta^3 = 25 + 125 = 150$

2. If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z+\frac{1}{2}(3+4i)\right|$ is :

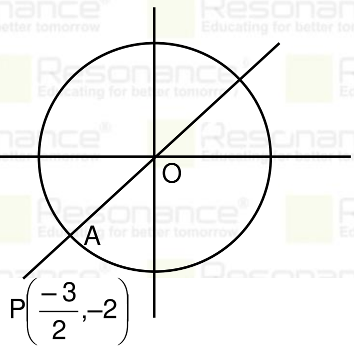
(1) 2 (2) $\frac{5}{2}$ (3) $\frac{3}{2}$ (4) 3

NTA **(3)**

Reso. **Bonus**

(There is a correction $|z| \leq 1$ then Ans. (3))

Sol.



$$\left|z-\left(\frac{-3}{2}-2i\right)\right|_{min} = PA = OP - r$$

$$= \sqrt{\frac{9}{4}+4} - 1 = \frac{5}{2} - 1 = \frac{3}{2}$$

3. consider a ΔABC where A(1,3, 2), B(–2, 8, 0) and C(3, 6, 7) . If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is :

(1) $\frac{37}{2\sqrt{38}}$ (2) $\sqrt{19}$ (3) $\frac{39}{2\sqrt{38}}$ (4) $\frac{\sqrt{38}}{2}$

NTA **(1)**

Reso. **(1)**

Sol. $\overrightarrow{AB} = -3\hat{i}+5\hat{j}-2\hat{k} \Rightarrow |\overrightarrow{AB}| = \sqrt{38}$

$\overrightarrow{AC} = 2\hat{i}+3\hat{j}+5\hat{k} \Rightarrow |\overrightarrow{AC}| = \sqrt{38}$

Hence AB = AC

So median and angle bisector of angle A is same

coordinate of D are $\left(\frac{1}{2}, 7, \frac{7}{2}\right)$

$\Rightarrow \overrightarrow{AD} = -\frac{1}{2}\hat{i}+4\hat{j}+\frac{3}{2}\hat{k}$

Projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$ $\Rightarrow \overrightarrow{AC} = \frac{\overrightarrow{AD}.\overrightarrow{AC}}{|\overrightarrow{AC}|} = \frac{-1+12+\frac{15}{2}}{\sqrt{38}} = \frac{37}{2\sqrt{38}}$

4. Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in R$ and $(a, b) R_2(c, d) \Leftrightarrow a+d = b+c$ for all $(a, b), (c, d) \in N \times N$

(1) R_1 and R_2 both are equivalence relations

(2) Only R_1 is an equivalence relation

(3) Only R_2 is an equivalence relation

(4) Neither R_1 nor R_2 is an equivalence relation

NTA **(3)**

Reso. **(3)**

Sol. $R_1 = \{(a, b) \in R \times R : a^2 + b^2 = 1\}$

R_1 is not reflexive

$\therefore R_1$ is not equivalence

$R_2 : (a, b) R (c, d) \Rightarrow a + d = b + c$

Reflexive : $(a, b) R (a, b) \Rightarrow a + b = b + a$ True

Symmetric : (a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ d + a = c + b

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c,d) R (a, b) True

Transitive (a, b) R (c, d) $\Rightarrow$ a + d = b + c ……(1

(c, d) R (e, f) $\Rightarrow$ c + f = d + e …..(2)

$\Rightarrow$ a + f = b + e by (1) and (2)

$\Rightarrow$ (a, b) R (e, f) transitive

$\therefore$ R_2 is equivalence

5. Let the system of equations x+2y+3z=5, 2x+3y+z=9, 4x + 3y + λz = μ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to:

(1) 22 (2) 17 (3) 15 (4) 28

NTA (2)

Reso. (2)

Sol. System of equation's are

x + 2y + 3z = 5

2x + 3y + z = 9

4x + 3y + λz = μ

have infinite many solutions only if $\Delta = 0$ and $\Delta_1 = 0$, $\Delta_2 = 0$ & $\Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 3\lambda + 18 + 8 - 36 - 3 - 4\lambda = 0$

$\Rightarrow \lambda = -13$

$$\text{Now } \Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix}$$

$= 5(-42) - 9(-35) + \mu(-7)$

$= -210 + 315 - 7\mu$

$= 105 - 7\mu = 7(15 - \mu)$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 9 & 1 \\ 4 & \mu & -13 \end{vmatrix}$$

$= 4(-22) - \mu(-5) - 13(-1)$

$= -88 + 5\mu + 13$

$= 5\mu - 75$

$= 5(\mu - 15)$

$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & \mu \end{vmatrix}$

$= 4(3) - 3(-1) + \mu(-1)$

$= (15 - \mu)$

since for $\mu = 15$, all $\Delta_1 = \Delta_2 = \Delta_3 = 0$

So equations have infinite many solutions for $\lambda = -13$ & $\mu = 15$

now $\lambda + 2\mu = -13 + 30 = 17$

6. If $\int_0^{\frac{\pi}{3}} \cos^4 x dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then 9a + 8b is equal to :

(1) 2 (2) 1 (3) 3 (4) $\frac{3}{2}$

NTA (1)

Reso. (1)

Sol. $\int_0^{\pi/3} \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int_0^{\pi/3}\left(1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right)dx$

$= \frac{1}{8}\int_0^{\pi/3}(3 + 4\cos 2x + \cos 4x)dx$

$= \frac{1}{8}\left(3x + 2\sin 2x + \frac{1}{4}\sin 4x\right)_0^{\frac{\pi}{3}}$

$= \frac{1}{8}\left(\pi + 2\sin\frac{2\pi}{3} + \frac{1}{4}\sin\frac{4\pi}{3}\right)$

$= \frac{1}{8}\left(\pi + \sqrt{3} - \frac{\sqrt{3}}{8}\right)$

$= \frac{1}{8}\left(\pi + \frac{7\sqrt{3}}{8}\right)$

$\Rightarrow a = \frac{1}{8}, b = \frac{7}{64}$

$9a + 8b = \frac{9}{8} + \frac{7}{8} = 2$

7. Let α and β be the roots of the equation $px^2+qx-r=0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non constant G.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$, then the value of $(\alpha-\beta)^2$ is :

(1) 8 (2) 9 (3) $\frac{20}{3}$ (4) $\frac{80}{9}$

NTA **(4)**

Reso. **(4)**

Sol. $\alpha+\beta=\frac{-q}{p}$ and $\alpha\beta=\frac{-r}{p}$

$\because \frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4} \Rightarrow \frac{\alpha+\beta}{\alpha\beta}=\frac{3}{4}$

$\frac{q}{r}=\frac{3}{4}$ hence common ratio = $\frac{4}{3}$

$$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=\left(\frac{q}{p}\right)^2+4\left(\frac{r}{p}\right)=\frac{16}{9}+4.\left(\frac{4}{3}\right)^2=\frac{80}{9}$$

8. Let Ajay will not appear in JEE exam with probability p = $\frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability q = $\frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

(1) $\frac{9}{35}$ (2) $\frac{3}{35}$ (3) $\frac{24}{35}$ (4) $\frac{18}{35}$

NTA **(4)**

Reso. **(4)**

Sol. A : – Event that Ajay appear in Exam

B : – Event that Vijay appear in Exam

Given $p=P(\overline{A})=\frac{2}{7}$ and $q=P(\overline{A}\cap\overline{B})=P(A).\,P(B)=\frac{1}{5}$

$\Rightarrow P(A)=1-p=\frac{5}{7}$

$\Rightarrow P(B)=\frac{7}{25}$

Hence $P(A\cap\overline{B})=P(A).\,P(\overline{B})$

$=\frac{5}{7}.\frac{18}{25}=\frac{18}{35}$

9. Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through P and parallel to y-axis meet the circle $x^2+y^2=9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4: 3 as P moves on the ellipse, is:

(1) $\frac{13}{21}$ (2) $\frac{\sqrt{139}}{23}$ (3) $\frac{\sqrt{13}}{7}$ (4) $\frac{11}{19}$

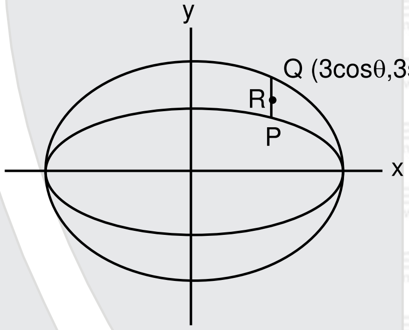
NTA (3)

Reso. (3)

Sol. $P \equiv (3\cos\theta, 2\sin\theta)$

$Q \equiv (3\cos\theta, 3\sin\theta)$

$R \equiv (h, k)$ $h = 3\cos\theta$, $k = \frac{12+6}{7}\sin\theta$



locus of R is

$$\frac{x^2}{9}+\frac{49y^2}{(18)^2}=1$$

$$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\left(\frac{18}{3\times 7}\right)^2}=\sqrt{\frac{49-36}{49}}=\frac{\sqrt{13}}{7}$$

10. Consider 10 observations x_1, x_2, x_{10} such that $\sum_{i=1}^{10}(x_i-\alpha)=2$ and $\sum_{i=1}^{10}(x_i-\beta)^2=40$ where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. Then $\frac{\beta}{\alpha}$ is equal to

(1) 2 (2) 1 (3) $\frac{5}{2}$ (4) $\frac{3}{2}$

NTA (1)

Reso. (1)

Sol. Mean $\Rightarrow$ $\alpha+\frac{\sum_{i=1}^{10}(x_i-\alpha)}{10}=\frac{6}{5}$

$\Rightarrow$ $\alpha+\frac{2}{10}=\frac{6}{5}$

$\Rightarrow$ $\alpha = 1$

Also variance $\Rightarrow \frac{\sum(x_i-\beta)^2}{10}-\left(\frac{\sum(x_i-\beta)}{10}\right)^2=\frac{84}{25}$

$$\Rightarrow \quad \frac{40}{10} - \left(\frac{\sum(x_i - \beta)}{10}\right)^2 = \frac{84}{25}$$

$$\Rightarrow \quad \left(\frac{\sum(x_i - \beta)}{10}\right)^2 = 4 - \frac{84}{25} = \frac{16}{25}$$

$$\Rightarrow \quad \frac{\sum(x_i - \beta)}{10} = \pm\frac{4}{5}$$

$$\Rightarrow \quad \sum(x_i - \beta) = \pm 8$$

Case-I When $\sum(x_i - \beta) = 8$

by mean $\frac{6}{5} = \beta + \frac{\sum(x_i - \beta)}{10}$

$\Rightarrow \quad \frac{6}{5} = \beta + \frac{8}{10} \Rightarrow \beta = \frac{2}{5}$ (Not integer)

Case-II When $\sum(x_i - \beta) = -8$

By mean $\frac{6}{5} = \beta - \frac{8}{10} \Rightarrow \beta = 2$

Hence $\frac{\beta}{\alpha} = 2$ Ans.

11. Let f(x) = $|2x^2 + 5|x| - 3|$, $x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to:

(1) 5 (2) 3 (3) 2 (4) 0

NTA **(2)**

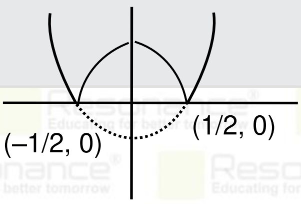
Reso. **(2)**

Sol. $f(x) = |2x^2 + 5|x| - 3|$

$2x^2 + 5x - 3 = (2x-1)(x + 3)$

f(x) is continuous for $x \in R$

and non-differentiable at



$x = \pm\frac{1}{2}, 0$

$\Rightarrow$ m = 0, n = 3

m + n = 3

12. The number of solutions of the equation $4 \sin^2 x - 4 \cos^3 x + 9 - 4 \cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

(1) 0 (2) 3 (3) 1 (4) 2

NTA **(1)**

Reso. **(1)**

Sol. $4 - 4\cos^2 x - 4\cos^3 x - 4\cos x + 9 = 0$

$4\cos^3 x + 4\cos^2 x + 4\cos x - 13 = 0$

$(\cos^2 x + \frac{1}{2})^2 + \frac{3}{4} = \frac{13}{4} \sec x$

L.H.S $\in [1, 3]$

R.H.S $\in \left[-\infty, -\frac{13}{4}\right] \cup \left[\frac{13}{4}, \infty\right]$

Number of solution = 0

13. Let the locus of the midpoint of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q. Then, the length of PQ is :

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

NTA **(3)**

Reso. **(3)**

Sol. Let mid point is (x_1, y_1)

$x^2 + y^2 - 2y = 0$

$xx_1 + yy_1 - (y + y_1) = x_1^2 + y_1^2 - 2y_1$

It is passing through origin

So, $0 + 0 - (0 + y_1) = x_1^2 + y_1^2 - 2y_1$

$\Rightarrow -y_1 = x_1^2 + y_1^2 - 2y_1$

$\Rightarrow x_1^2 + y_1^2 - y_1 = 0$

Locus is $x^2 + y^2 - y = 0$ ____(1)

$\because$ it intersects the line $x + y = 1$

so put $x = (1 - y)$ is equation (1)

$(1 - y)^2 + y^2 - y = 0$

$2y^2 - 3y + 1 = 0$

$(y - 1)(2y - 1) = 0$

$y = 1, \frac{1}{2}$

$P(0, 1)$ & $Q\left(\frac{1}{2}, \frac{1}{2}\right)$

So, $PQ = \sqrt{\left(\frac{1}{2}-0\right)^2 + \left(\frac{1}{2}-1\right)^2} \Rightarrow PQ = \frac{1}{\sqrt{2}}$

14. Let α be a non-zero real number. Suppose $f : R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim\limits_{x \to -\infty} f(x) = 1$. if $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to ________

(1) 7 (2) 9 (3) 3 (4) 5

NTA **(2)**

Reso. **(2) or Bonus**

Sol. $f'(x) = \alpha f(x) + 3$

$\Rightarrow \frac{f'(x)}{\alpha f(x)+3} = 1$

$\Rightarrow \frac{1}{\alpha}\log_e|\alpha f(x)+3| = x + c$

given $f(0) = 2$

$\Rightarrow \frac{1}{\alpha}\log_e|2\alpha+3| = c$

$\Rightarrow \frac{1}{\alpha}\log_e|\alpha f(x)+3| = x + \frac{1}{\alpha}\log_e|2\alpha+3|$

$\Rightarrow \frac{1}{\alpha}\log_e\left|\frac{\alpha f(x)+3}{2\alpha+3}\right| = x$

$\Rightarrow \left|\frac{\alpha f(x)+3}{2\alpha+3}\right| = e^{\alpha x}$ ______(1)

Since $\lim\limits_{x \to -\infty} f(x) = 1$

Case-I $\alpha > 0$ then by equation (1)

$\Rightarrow 3 + \alpha = 0 \quad \Rightarrow \alpha = -3$ (ambiguity)

$\Rightarrow |f(x)-1| = e^{-3x}$

$\Rightarrow f(x) = 1 \pm e^{-3x}$

$\Rightarrow f(-\log_e 2) = 1 \pm e^{3\log_e 2}$

$= 1 \pm 8 = 9$ or -7

Case-II $\alpha < 0$ then by (1)

$\left|\frac{\alpha+3}{2\alpha+3}\right| = \infty$ not difined

15. Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point R (1, 2, 3). If the centroid of the triangle PQR is (α , β , γ), then $\alpha^2 + \beta^2 + \gamma^2$ is :

(1) 18 (2) 24 (3) 26 (4) 36

NTA (1)

Reso. (1)

Sol. Let $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2} = \lambda$

$\Rightarrow$ coordinated of any point on it is P $(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$

Its distance from R(1, 2, 3) is 6 unit

$\Rightarrow$ $(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$

$\Rightarrow$ $(64 + 4 + 4)\lambda^2 + (16 + 4 + 16) - (64 - 8 + 16)\lambda = 36$

$\Rightarrow$ $72\lambda^2 - 72\lambda = 0$

$\Rightarrow$ $\lambda = 0$ or 1

Hence P (–3, 4, –1) & Q(5, 6, 1)

Centroid of ΔPQR is $(\alpha, \beta, \gamma) = (1, 4, 1)$

$\Rightarrow$ $\alpha^2 + \beta^2 + \gamma^2 = 18$

16. The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{1/3} dx$ is equal to :

(1) –1 (2) 2 (3) 0 (4) 1

NTA (3)

Reso. (3)

Sol. $I = \int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$

$= \int_0^1 ((2x-1)(x^2 - x - 1))^{\frac{1}{3}} dx$

$= \int_0^1 [(2(1-x) - 1)((1-x)^2 - (1-x) - 1)]^{\frac{1}{3}} dx$

$= \int_0^1 ((1-2x)(x^2 - x - 1))^{\frac{1}{3}} dx$

$= -\int_0^1 \left((2x-1)(x^2-x-1)\right)^{\frac{1}{3}} dx$

$I = -I$

$2I = 0$

$I = 0$

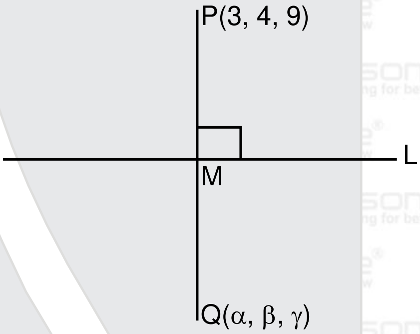
17. If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ) then $14(\alpha, \beta, \gamma)$ is :

(1) 102 (2) 138 (3) 132 (4) 108

NTA **(4)**

Reso. **(4)**

Sol. Let $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda$



Let foot of perpendicular from P(3, 4, 9) on line is M

$M(1 + 3\lambda, -1 + 2\lambda, 2 + \lambda)$

direction ratio of PM : $3\lambda - 2, 2\lambda - 5, \lambda - 7$

direction ratio of line L : 3, 2, 1

now $PM \perp L \Rightarrow 3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0$

$9\lambda - 6 + 4\lambda - 10 + \lambda - 7 = 0$

$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$

Now M is mid point of PQ

$\frac{\alpha+3}{2} = 1 + 3\lambda$

$\frac{\beta+4}{2} = -1 + 2\lambda$

$\frac{\gamma+9}{2} = 2 + \lambda$

$\therefore \frac{\alpha+\beta+\gamma+16}{2} = 2 + 6\lambda = \frac{166}{14}$

$\alpha + \beta + \gamma = \frac{166}{7} - 16 = \frac{54}{7}$

$\therefore 14(\alpha + \beta + \gamma) = 108$

18. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to :

(1) 800 (2) 890 (3) 790 (4) 690

NTA **(3)**

Reso. **(3)**

Sol. $S_n = \frac{n}{2}(2a + (n-1)d)$

and $T_n = a + (n-1)d$

Now $\frac{T_5}{T_{10}} = \frac{7}{15} \Rightarrow \frac{a+4d}{a+9d} = \frac{7}{15} \Rightarrow 8a = 3d$ ______(1)

also $S_{10} = 5(2a + 9d) = 390$

$\Rightarrow 2a + 9d = 78$

$\Rightarrow 2a + 24a = 78$

$\Rightarrow a = 3$ & $d = 8$

Hence $S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8)$

$= 45 + 15\times14\times4 - 15 - 5\times16$

$= 45 + 840 - 15 - 80$

$= 885 - 95$

$= 790$

19. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{3}}}\right)$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is

(1) $\frac{1}{9}$ (2) $\frac{1}{4}$ (3) $\frac{4}{9}$ (4) $\frac{9}{4}$

NTA **(4)**

Reso. **(4)**

Sol. $m = {}^{18}C_6\left(\frac{1}{3}\right)^{12}\left(\frac{1}{2}\right)^{6}$

$n = {}^{18}C_{12}\left(\frac{1}{3}\right)^{6}\left(\frac{1}{2}\right)^{12} = \frac{{}^{18}C_{12}}{(12)^6}$

$\frac{n}{m} = \left(\frac{18}{12}\right)^6 \frac{{}^{18}C_{12}}{{}^{18}C_6} = \frac{3^6}{2^6}$

$\left(\frac{n}{m}\right)^{\frac{1}{3}} = \frac{9}{4}$

20. Let $f(x) = \begin{cases} x-1 & x \text{ is even} \\ 2x, & x \text{ is odd} \end{cases}$ $x \in N$. If for some $a \in N$, $f(f(f(a))) = 21$ then $\lim_{x \to a^-} \left\{ \frac{|x|^3}{a} - \left[\frac{x}{a}\right] \right\}$ where[t] denotes the greatest integer less than or equal to t, is equal to :

(1) 169 (2) 121 (3) 225 (4) 144

NTA (4)

Reso. (4)

Sol. Let a is even then f(a) = a – 1 (odd)

$f(f(a)) = f(a-1) = 2a - 2$ (even)

$f(f(f(a))) = (2a - 2) - 1 = 2a - 3$

$\Rightarrow 2a - 3 = 21 \Rightarrow a = 12$

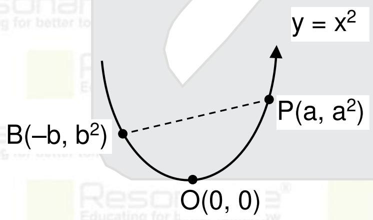
Now $\lim_{x \to a^-} \left(\frac{|x|^3}{12} - \left[\frac{x}{12}\right] \right)$ $x < 12$

$= 144 - 0 = 144$

If a is odd then $f(f(f(a))) = 21 \Rightarrow a \notin N$

21. Three points O(0, 0), P(a, a²), Q(–b, b²), a > 0, b > 0, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, gcd(m, n) = 1 , then m + n is equal to _______.

NTA (7)
Reso. (7)
Sol.



Equation of line PQ is

$y - a^2 = \frac{b^2 - a^2}{-b - a}(x - a)$

$\Rightarrow y - a^2 = (a - b)(x - a)$

$\Rightarrow y = (a - b)x + ab$

$S_1 = \int_{-b}^{a} \{(a - b)x + ab - x^2\} dx$

$$= \left|\left\{(a-b)\frac{x^2}{2} + abx - \frac{x^3}{3}\right\}\right|_{-b}^{a}$$

$$= \left\{\frac{1}{2}\left(a^3 - a^2b\right) + a^2b - \frac{a^3}{3}\right\} - \left\{\frac{ab^2}{2} - \frac{b^3}{2} - ab^2 + \frac{b^3}{3}\right\}$$

$$= \frac{1}{6}a^3 + \frac{1}{2}a^2b + \frac{1}{2}ab^2 + \frac{1}{6}b^3 = \frac{1}{6}\left(a^3+b^3\right) + \frac{1}{2}ab(a+b) = \frac{1}{6}(a+b)^3$$

Also area $S_2 = \frac{1}{2}\left(a^2b + ab^2\right) = \frac{1}{2}ab(a+b)$

$$\Rightarrow \frac{S_1}{S_2} = \frac{1}{3}\frac{(a+b)^2}{ab} = \frac{1}{3}\left(\frac{a}{b} + \frac{b}{a} + 2\right)$$

$$\Rightarrow \left(\frac{S_1}{S_2}\right)_{min} = \frac{1}{3}(2+2) = \frac{4}{3} \Rightarrow m+n = 7$$

22. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y-x)$ is maximum, is equal to__________

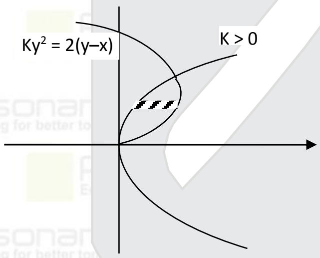
NTA (8)

Reso. (8)

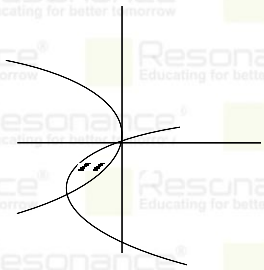
Sol. On solving $2y^2 = kx$ and $ky^2 = 2(y-x)$ we have $ky^2 = 2y - 2\left(\frac{2y^2}{k}\right)$

$(k^2 + 4)\, y^2 - 2ky = 0$

$\Rightarrow y = 0$ and $y = \frac{2k}{k^2+4}$



or



Required area $= \int_0^{\frac{2k}{k^2+4}} \left\{\left(\frac{2y - ky^2}{2}\right) - \left(\frac{2y^2}{k}\right)\right\} dy = \left(\frac{y^2}{2} - \frac{ky^3}{6} - \frac{2y^3}{3k}\right)\Bigg|_0^{\frac{2k}{k^2+4}}$

$= \left.\frac{y^2}{2} - \frac{(k^2+4)y^3}{6k}\right|_0^{\frac{2k}{k^2+4}} = \frac{2k^2}{(k^2+4)^2} - \frac{4k^2}{3(k^2+4)^2} = \frac{2k^2}{3(k^2+4)^2}$

Area = $\frac{2k^2}{3(k^2+4)^2}$ is maximum (at k = ± 2 by A.M.- G.M.)

So sum of square of values of k is $(2)^2 + (-2)^2 = 8$

23. $y = \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$, then $96y'\left(\frac{\pi}{6}\right)$ is equal to _______.

NTA **(105)**

Reso. **(105)**

Sol. $y = \frac{(\sqrt{x}+1)(\sqrt{x}-1)\left((\sqrt{x})^2+\sqrt{x}+1\right)}{(x+\sqrt{x}+1)} + \frac{1}{15}(3\cos^5x - 5\cos^3x)$

$y = (x-1) + \frac{1}{15}(3\cos^5x - 5\cos^3x)$

$y' = 1 + (\cos^4x\,(-\sin x) + \cos^2x \sin x)$

$= 1 + \cos^2x\sin x(1 - \cos^2x)$

$y' = 1 + \cos^2x\text{six}^3x$

$y'(\pi/6) = 1 + \frac{3}{4}.\frac{1}{8} = \frac{32+3}{32} = \frac{35}{32}$

$96y'(\pi/6) = 105$

24. If $\frac{dx}{dy} = \frac{1+x-y^2}{y}$, x(1) = 1, then 5x(2) is equal to _________.

NTA **(5)**

Reso. **(5)**

Sol. $\frac{dx}{dy} = \frac{1+x-y^2}{y} \Rightarrow \frac{dx}{dy} - \frac{1}{y}x = \frac{1-y^2}{y}$ linear differential equation

I.F. = $e^{-\int\frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$

solution is

$x.\frac{1}{y} = \int \frac{1-y^2}{y}.\frac{1}{y}dy + C$

$\frac{x}{y} = \int\left(\frac{1}{y^2} - 1\right)dy + C$

$\frac{x}{y} = -\frac{1}{y} - y + C$

$x = -1 - y^2 + Cy$

$\therefore x(1) = 1 \Rightarrow 1 = -1 - 1 + C$

$C = 3$

$\therefore x = -1 - y^2 + 3y$

$\therefore x(2) = -1 - 4 + 6 = 1$

25. Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x tf(t)dt$. If $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to ________.

NTA **(219)**

Reso. **(219)**

Sol. $F(x) = \int_0^x t\,f(t)dt$ and $F(x)^2 = x^4 + x^5$

$F'(x) = xf(x)$ and $2xF'(x^2) = 4x^3 + 5x^4$

$2F'(x^2) = 4x^2 + 5x^3$

$2F'(x) = 4x + 5x^{\frac{3}{2}}$

$\Rightarrow 2x + \frac{5}{2}x^{\frac{3}{2}} = xf(x)$

$\Rightarrow f(x) = 2 + \frac{5}{2}x^{\frac{1}{2}}$

$\Rightarrow f(x^2) = 2 + \frac{5}{2}x$

$\sum_{r=1}^{12} f(r^2) = \sum_{r=1}^{12}(2 + \frac{5}{2}r)$

$= 24 + \frac{5}{2}\sum_{r=1}^{12} r$

$= 24 + \frac{5}{2} \times \frac{12 \times 13}{2}$

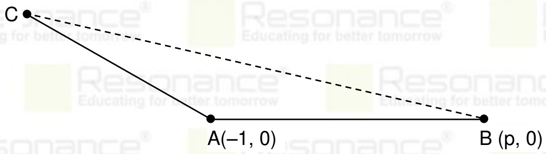
$24 + 195 = 219$

26. Let ABC be an isosceles triangle in which A is at $(-1, 0)$, $\angle A = \frac{2\pi}{3}$, AB=AC and B is on the positive x-axis. If BC = $4\sqrt{3}$ and the line BC intersects the line y=x+3 at (α, β), then $\frac{\beta^4}{\alpha^2}$ is_____

NTA **(36)**

Reso. **(36)**

Sol.



Let B(p, 0)

$\Rightarrow$ AB = P + 1, P > 1

Hence coordinate of C are $\left(-\frac{1}{2}p-\frac{3}{2},\frac{\sqrt{3}}{2}(p+1)\right)$

$$\Rightarrow BC^2=\left(-\frac{3}{2}(p+1)\right)^2+\left(\frac{\sqrt{3}}{2}(p+1)\right)^2$$

$$\Rightarrow BC=\sqrt{3}(p+1)=4\sqrt{3}\ \Rightarrow p=3$$

Thus equation of BC is

$$y=-\frac{1}{\sqrt{3}}(x-3)$$

Solve with line y = x + 3 we have $x+3=-\frac{1}{\sqrt{3}}(x-3)$

$$(\sqrt{3}+1)x=3(1-\sqrt{3})$$

$$\Rightarrow x=-3\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)=\frac{-3}{2}(4-2\sqrt{3})$$

$$\Rightarrow \alpha=-3(2-\sqrt{3})$$

$$\&\ \beta=-3+3\sqrt{3}$$

$$\Rightarrow \frac{\beta^4}{\alpha^2}=\frac{81(\sqrt{3}-1)^4}{9(2-\sqrt{3})^2}=\frac{9(4-2\sqrt{3})^2}{(2-\sqrt{3})^2}=36$$

27. Let A=I_2 – 2 MM^T, where M is a real matrix of order 2 × 1 such that the relation $M^TM=I_1$ holds. If λ is a real number such that the relation AX = λX holds for some non-zero real matrix X of order 2 × 1, then the sum of squares of all possible values of λ is equal to

NTA **(2)**

Reso. **(2)**

Sol. Let $M=\begin{bmatrix}a\\b\end{bmatrix}$

$$\Rightarrow M^TM=[a\ \ b]\begin{bmatrix}a\\b\end{bmatrix}=[a^2+b^2]=I_1\ \text{(given)}$$

Hence $a^2 + b^2 = 1$

Let $a = \cos\theta$, $b = \sin\theta$

then $A = I_2 - 2\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\end{bmatrix}$

$$= \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} - \begin{bmatrix}2\cos^2\theta & 2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & 2\sin^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}1-2\cos^2\theta & -2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 1-2\sin^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}-\cos2\theta & -\sin2\theta \\ -\sin2\theta & \cos2\theta\end{bmatrix}$$

Now $Ax = \lambda x$ for some non-zero real matrix x

$\Rightarrow (A - \lambda I)x = 0$

$\Rightarrow |A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix}-\cos2\theta - \lambda & -\sin2\theta \\ -\sin2\theta & \cos2\theta - \lambda\end{vmatrix} = 0$$

$\Rightarrow -(\cos2\theta + \lambda)(\cos2\theta - \lambda) - \sin^2 2\theta = 0$

$\Rightarrow \cos^2 2\theta - \lambda^2 + \sin^2 2\theta = 0$

$\Rightarrow \lambda^2 = 1$

$\Rightarrow \lambda = \pm 1$

Sum of the square of values of λ is $= (1)^2 + (-1)^2 = 2$

28. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is, θ then the greatest integer less than or equal to $\tan^2\theta$ is

NTA **(38)**

Reso. **(38)**

Sol. $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$

$\Rightarrow (\vec{c} - \vec{b}) \times \vec{a} = 0$

$\Rightarrow \vec{c} - \vec{b} = \lambda\vec{a}$

$\Rightarrow \vec{c} = \vec{b} + \lambda\vec{a}$

$\Rightarrow \vec{c} = (-1+\lambda)\hat{i} + (-8+\lambda)\hat{j} + (2+\lambda)\hat{k}$

but $\Rightarrow \vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\Rightarrow \lambda = 5$, $c_2 = -3$, $c_3 = 7$

Now angle between $\Rightarrow \vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ and $\Rightarrow 3\hat{i} + 4\hat{j} + \hat{k}$ is θ

$$\Rightarrow \cos\theta = \frac{12 - 12 + 7}{\sqrt{16+9+49}\sqrt{9+16+1}} = \frac{7}{\sqrt{74}\sqrt{26}}$$

$$\Rightarrow \sec^2\theta = \frac{74 \times 26}{49} = 39.26$$

$$\Rightarrow \tan^2\theta = \sec^2\theta - 1 = 38.26$$

$\Rightarrow$ $[\tan^2\theta] = 38$

29. If three successive terms of a G.P. with common ratio r (r > 1) are the lengths of the sides of a triangle and [r] denotes the greatest integer less than or equal to r, then 3[r] + [– r] is equal to______.

NTA (1)

Reso. (1)

Sol. Let three terms of GP are $\frac{a}{r}$, a, ar (r > 1)

Sum of two smaller sides > third side

$\Rightarrow \frac{a}{r} + a > ar$ $\Rightarrow$ $1 + r > r^2$

$\Rightarrow r^2 - r - 1 < 0$ $\Rightarrow$ $\frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2}$

but $r > 1 \Rightarrow r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$\Rightarrow [r] = 1$ and $[-r] = -2$

So $3[r] + [-r] = 3 - 2 = 1$

30. The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For n=1, 2, 3, ..., 10 all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is equal to________

NTA (101)

Reso. (101)

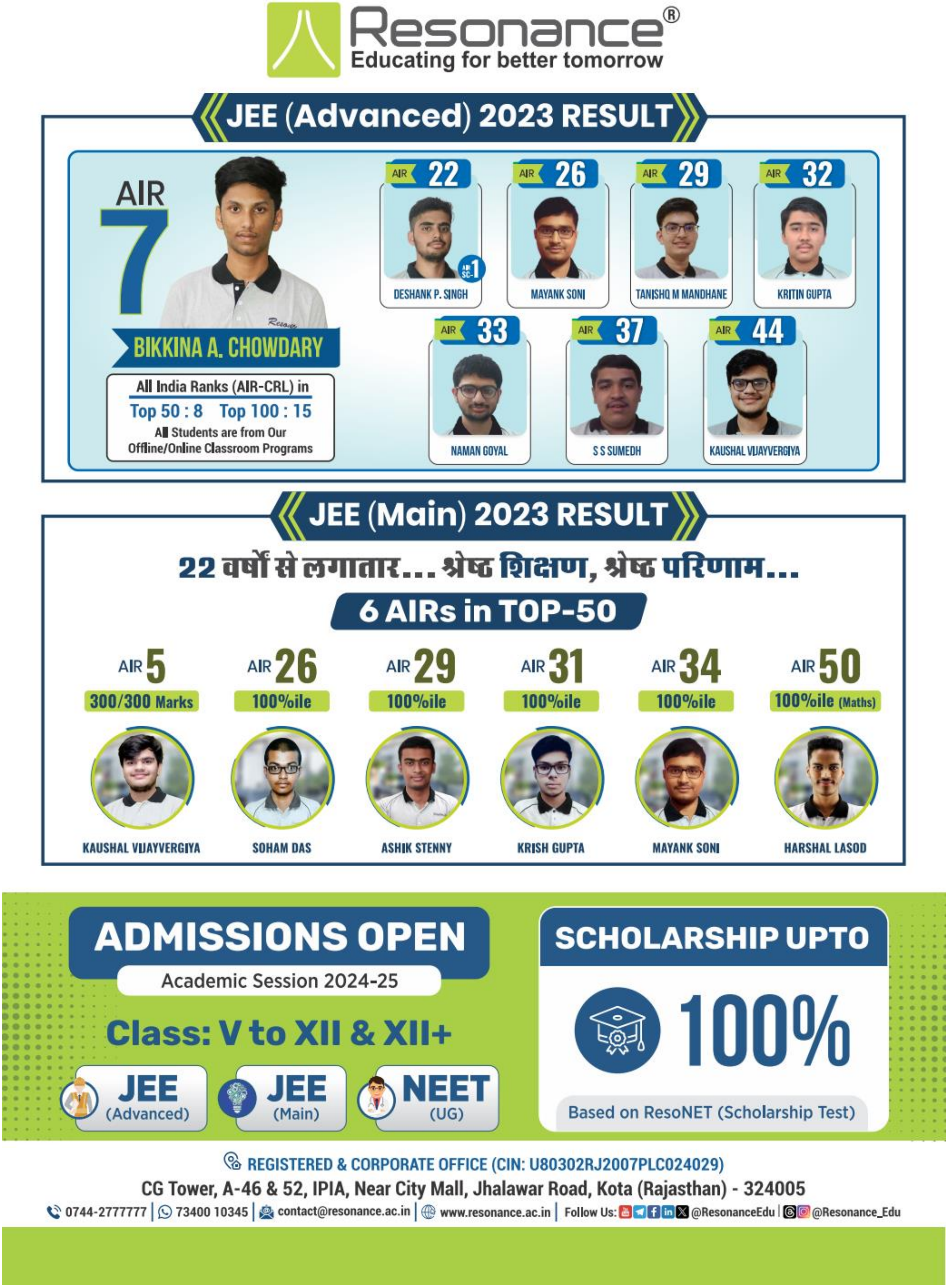
Sol. Since $L_1, L_3, L_5, \ldots, L_{19}$ all 10 lines are parallel to each other so does not intersect each other again $L_2, L_4, L_6, \ldots, L_{20}$ all passes through a fixed point P.

So they intersect at only point P.

Now Each member of $L_1, L_3, L_5, \ldots, L_{19}$ intersects each member of $L_2, L_4, L_6, \ldots, L_{20}$ at 10 distinct points (for maximum point of intersection)

$\Rightarrow$ Number of such point of Intersection = 10 × 10 = 100

So Maximum Number of Point of Intersection = 100 +1 = 101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