Gravitation JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Gravitation

1. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force is exerts on a point mass $'m'$ at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by: **(+4, -1)**

[12 Jan. 2019 I]

a. $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$

b. $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

c. $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$

d. $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$

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2. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius $R(R << L)$. A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then : **(+4, -1)**

[Online•April•10,•2015]

a. $T^2 \propto r^3$

b. $T \propto r^2$

c. $T \propto r$

d. $T \propto \sqrt{r}$

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3. Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the $semimajor$ axis, and ca as the $semiminor$ axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then : **(+4, -1)**

[Online April 9, 2016]

a. $t_1 = t_2$

b. $t_1 = 2t_2$

c. $t_1 = 3t_2$

d. $t_1 = 4t_2$

4. Four identical particles of mass M are located at the corners of a square of side $'a'$. What should be their speed if each of them revolves under the influence of other?s gravitational field in a circular orbit circumscribing the square? **(+4, -1)**

[8•April•2019•I]

a. $1.21\sqrt{\frac{GM}{a}}$

b. $1.41\sqrt{\frac{GM}{a}}$

c. $1.16\sqrt{\frac{GM}{a}}$

d. $1.35\sqrt{\frac{GM}{a}}$

5. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction, the speed of each particle is **(+4, -1)**

[2014]

a. $\sqrt{\frac{GM}{R}}$

b. $\sqrt{2\sqrt{2}\frac{GM}{R}}$

c. $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

d. $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

6. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V=0$ at $r=\infty$, the potential at the centre of the cavity thus formed is ($G=$ gravitational constant) **(+4, -1)**

[2015]

a. $-\frac{2GM}{3R}$

b. $\frac{-2GM}{R}$

c. $\frac{-GM}{2R}$

d. $\frac{-GM}{R}$

7. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is : **(+4, -1)**

[9 Jan. 2019 I]

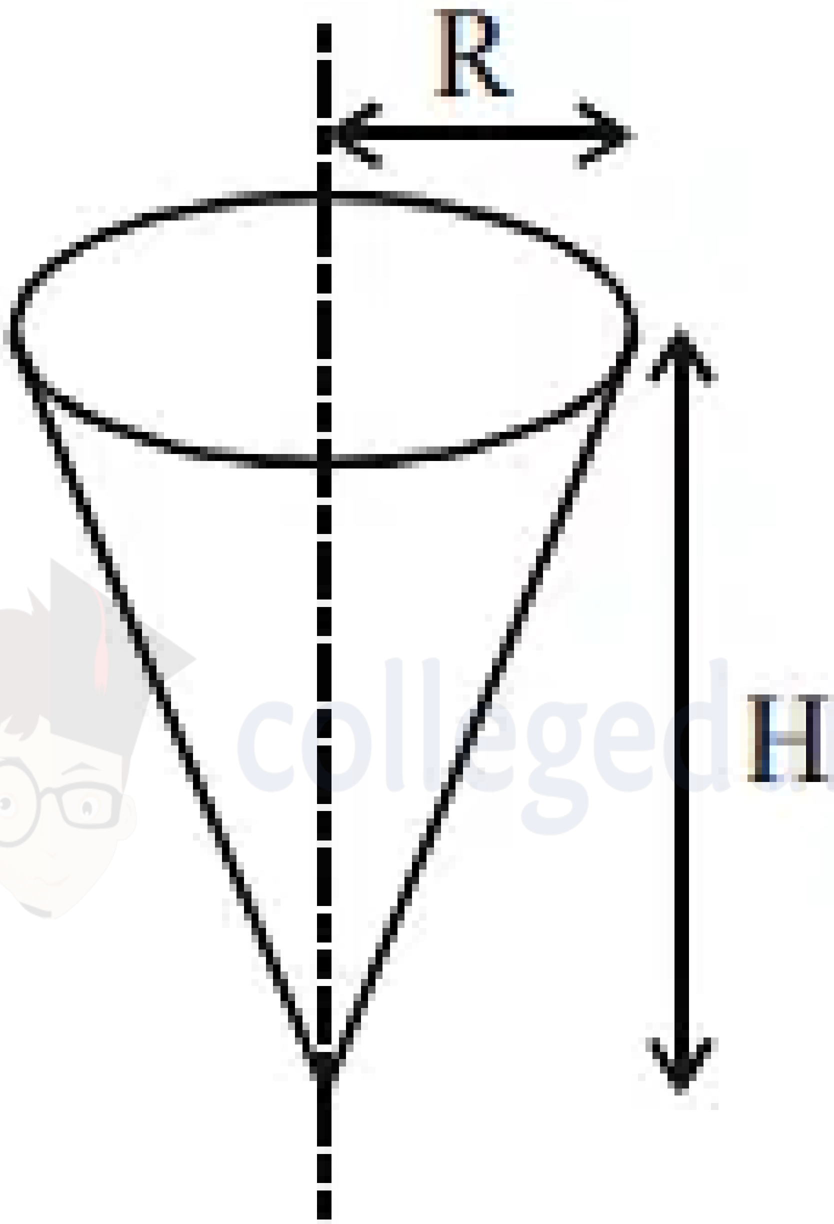
a. $\frac{4L}{m}$

b. $\frac{L}{m}$

c. $\frac{L}{2m}$

d. $\frac{2L}{m}$

8. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is: **(+4, -1)**



a. $\frac{MR^2}{2}$

b. $\frac{MH^2}{3}$

c. $\frac{MR^2}{3}$

d. $\frac{M\left(R^2+H^2\right)}{4}$

9. Separation between earth and sun is given by 1.5×10^6 km. Time period of another planet is 2.83 year. Find distance of another planet from sun? **(+4, -1)**

a. 3×10^6 km

b. 2×10^7 km

c. 3×10^7 km

d. 2×10^6 km

10. Statement-I : If we move upward and downward from the surface of earth surface acceleration due to gravity decreases in both upward and downward direction. **(+4, -1)**
Statement-II : Acceleration due to gravity changes by same amount when we go up to height h and depth d when h = d.
Choose the correct options based on above statements.

a. Both statement-I and Statement-II are true.

b. Statement-I is true and Statement-II is false.

c. Statement-I is false and Statement-II are true.

d. Both statement-I and Statement-II are false.

Answers

1. Answer: b

Explanation:

$$dm = \left(A + Bx^2\right) dx$$
$$dF = \frac{GMdm}{x^2}$$
$$= F = \int_a^{a+L} \frac{GM}{x^2}\left(A + Bx^2\right) dx$$
$$= GM\left[-\frac{A}{x} + Bx\right]_a^{a+L}$$
$$= GM\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

Concepts:

1. Gravitation:

In mechanics, the universal force of attraction acting between all matter is known as **Gravity**, also called **gravitation**, . It is the weakest known force in nature.

Newton's Law of Gravitation

According to Newton's law of gravitation, "Every particle in the universe attracts every other particle with a force whose magnitude is,

- $F \propto (M_1M_2)$ . . . . (1)
- $(F \propto 1/r^2)$ . . . . (2)

On combining equations (1) and (2) we get,

$F \propto M_1M_2/r^2$

$F = G \times [M_1M_2]/r^2$ . . . . (7)

Or, $f(r) = GM_1M_2/r^2$

The dimension formula of G is $[M^{-1}L^3T^{-2}]$.

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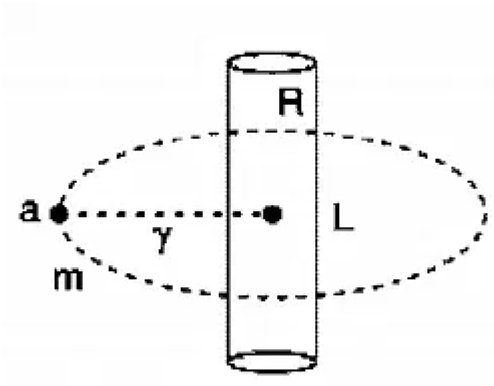
2. Answer: c

Explanation:



Let the linear mass density of the cylindrical galaxy be $\lambda\, kg/m$.
Gravitational field $= \frac{2G\lambda}{r} = E_r$
Therefore, gravitational force $F = mE_g = m\omega^2 r$
Hence, $m\left(\frac{2G\lambda}{r}\right) = m \cdot \left(\frac{2\pi}{T}\right)^2 r$
$\Rightarrow T^2 \propto r^2 \Rightarrow T \propto r$

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3. **Answer: c**

Explanation:

Area abca = x
Area SABCS $= x + \frac{1}{2}x$
Area SADCS $= x - \frac{1}{2}x = \frac{1}{2}$
$\frac{1+\frac{1}{2}}{1-\frac{1}{2}} = \frac{t_1}{t_2}$
$\frac{\frac{3}{2}}{\frac{1}{2}} = \frac{t_1}{t_2} t_1 = 3t_2$
$\frac{3}{1} = \frac{t_1}{t_2}$

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- F ∝ (M_1M_2) . . . . (1)
- (F ∝ $1/r^2$) . . . . (2)

On combining equations (1) and (2) we get,

F ∝ M_1M_2/r^2

F = G × $[M_1M_2]/r^2$ . . . . (7)

Or, f(r) = GM_1M_2/r^2

The dimension formula of G is $[M^{-1}L^3T^{-2}]$.

4. **Answer: c**

Explanation:

Net force on particle towards centre of circle is $F_C = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2}$
$= \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$
This force will act as centripetal force. Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$
$r = \frac{a}{\sqrt{2}}, F_C = \frac{mv^2}{r}$
$\frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$
$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}} + 1\right)$
$v^2 = \frac{GM}{a}(1.35)$
$v = 1.16\sqrt{\frac{GM}{a}}$

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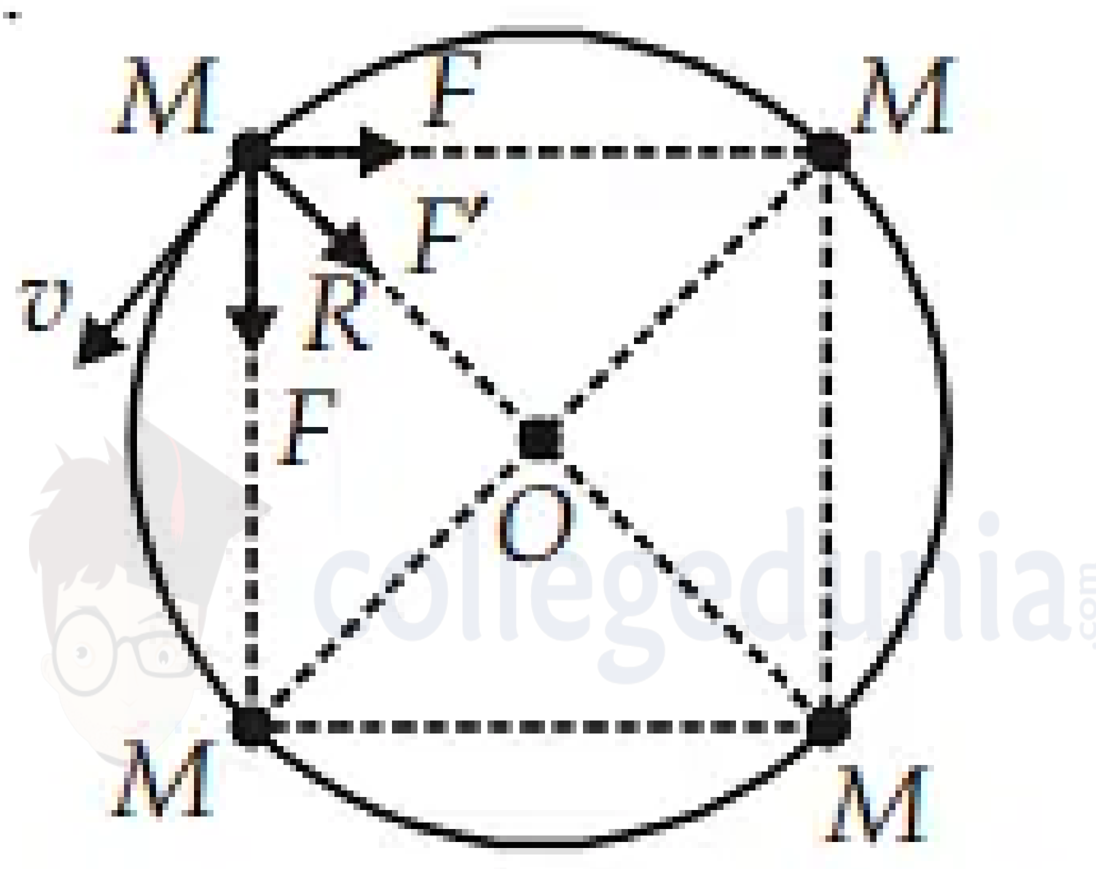
5. **Answer: d**

Explanation:



$\frac{F}{\sqrt{2}} + \frac{F}{\sqrt{2}} + F' = \frac{Mv^2}{R}$

$\frac{2 \times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$

$\frac{GM^2}{R}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = Mv^2$

$v = \sqrt{\frac{Gm}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{Gm}{R}(1+2\sqrt{2})}$

The Correct Option is (D): $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

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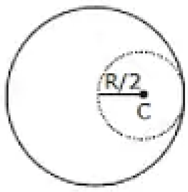
6. Answer: d

Explanation:

Solid sphere is of mass M, radius R.
Spherical portion removed have radius $R/2$, therefore its mass is $M/8$.
Potential at the centre of cavity $= V_{\text{solid sphere}} + V_{\text{removed part}}$



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7. **Answer: c**

Explanation:

$\frac{dA}{dt} = \frac{L}{2m}$

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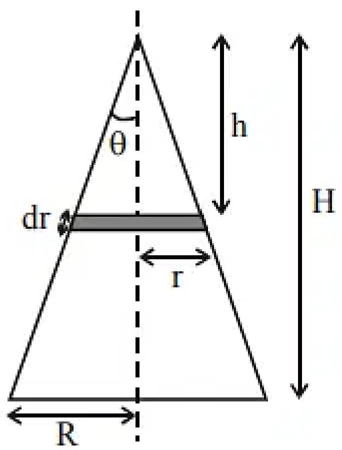
8. **Answer: a**

Explanation:



Area $= \pi R\ell = \pi R\left(\sqrt{H^2 + R^2}\right)$ Area of element $dA = 2\pi r d\ell = 2\pi r \frac{dh}{\cos\theta}$ mass of element $dm = \frac{M}{\pi R\sqrt{H^2+R^2}} \times \frac{2\pi r dh}{\cos\theta}$ $dm = \frac{2Mh\tan\theta dh}{R\sqrt{H^2+R^2}\cos\theta}$ (here $r = h\tan\theta$) $I = \int (dm)r^2 =$ $\int \frac{h^2\tan^2\theta}{\cos\theta}\left(\frac{2m}{R}\frac{h\tan\theta}{\sqrt{R^2+H^2}}\right)dh = \frac{2M}{\cos\theta R}\frac{\tan^3\theta}{\sqrt{R^2+H^2}}\int_0^H h^3 dh = \frac{MR^2H^4}{2RH^3\sqrt{R^2+H^2}\cos\theta} = \frac{MR^2H\sqrt{R^2+H^2}}{2\sqrt{R^2+H^2}\times H} = \frac{MR^2}{2}$

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9. **Answer: a**

Explanation:

$T2 \propto R3$

$(T_1/T_2)^2 = (R_1/R_2)^3$

$(\frac{1}{2.83})^2 = \{(1.5 \times 10^6)/R_2\}^3$

$R^2 = (1.5 \times 10^6)(2.83)\frac{2}{3}$ km

$= (1.5 \times 10^6)(8)\frac{1}{3}$

$= 3 \times 10^6$ km

The correct option is (A): 3×10^6 km

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10. **Answer: b**

Explanation:

The correct option is B

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