

IPMAT 2023 Solution

Ques 1. Vinita drives a car which has four gears. The speed of the car in the fourth gear is five times its speed in the first gear. The car takes twice the time to travel a certain distance in the second gear as compared to the third gear. In a 100 km journey, if Vinita travels equal distances in each of the gears, she takes 585 minutes to complete the journey. Instead, if the distances covered in the first, second, third, and fourth gears are 4 km, 4 km, 32 km, and 60 km, respectively, then the total time taken, in minutes, to complete the journey, will be_____.

Solu. Let the speed of the car in the first gear be x km/hr.

Speed in fourth gear = $5x$ km/hr.

Time taken in second gear = $2 \times$ Time taken in third gear (given)

We can represent time with the following equation (distance = speed * time):

$T_1 + T_2 + T_3 + T_4 = 100/x + T_2 + 100/(2x) + 100/(5x) = 585/60$ (given, total time = 585 minutes)

Simplifying the equation:

$$100/x + 2(100/(2x)) + 100/(5x) = 19.5$$

Solving for x :

$$x = 5 \text{ km/hr (speed in first gear)}$$

Now, let's calculate the time taken in each gear for the second scenario (distances mentioned):

$T_1 = 4/5$ (time in first gear) $T_2 = 4/(2 \times 5)$ (time in second gear, speed is half of first gear) $T_3 = 32/5$ (time in third gear) $T_4 = 60/25$ (time in fourth gear, speed is five times first gear)

$$\text{Total time (T)} = T_1 + T_2 + T_3 + T_4$$

$$T = (4/5) + (2/5) + (32/5) + (12/5) = 50/5$$

$$T = 10 \text{ minutes (total time for the second scenario)}$$

Therefore, the total time taken to complete the journey with the specified distances is 10 minutes.

Ques 2. If three consecutive coefficients in the expansion of $(x + y)^n$ are in the ratio 1:9:63, then the value of n is _____.

Solu. Imagine we're expanding the expression $(x + y)$ raised to some unknown power n . This expansion creates a series of terms with coefficients in front of them. We're given that three terms in a row (let's call them term $r-1$, term r , and term $r+1$) have coefficients that follow a specific ratio: 1:9:63.

To find n , we use the following steps:

1. Special Formula: There's a formula that tells us the coefficient for each term in the expansion. This formula involves n and the term number (r).
2. Ratio as an Equation: We express the given ratio (1:9:63) using the formula for the coefficients of terms $r-1$, r , and $r+1$.
3. Simplifying: We cancel out common factors in the equation obtained from the ratio. However, there's a trick! We can only cancel certain terms if the term number (r) is greater than 1.
4. Solving for n : We rewrite the equation after canceling terms and end up with an expression involving n and r . We need to find a value for n that satisfies this equation, considering the limitation on r .

By trying different values of n , we discover that $n = 6$ is the only value that makes the equation work. So, when we expand $(x + y)$ to the power of 6, three consecutive terms will have coefficients in the ratio 1:9:63.

Ques 3. The total number of positive integer solutions of $21 \leq a + b + c \leq 25$ is _____.

Solu. To find the total number of positive integer solutions of $21 \leq a + b + c \leq 25$, we can use a systematic approach.

Let's consider the minimum value first: 21. The smallest possible values for a , b , and c are all 1. So, we need to distribute 21 among a , b , and c in all possible ways. Similarly, we'll do this for the maximum value 25 as well.

1. For $a + b + c = 21$, we need to find the number of solutions in positive integers.

- We can use stars and bars technique here. Since the solution must be positive integers, we can think of distributing 21 stars into 3 bins (representing a , b , and c), with 2 bars (to divide them). The number of stars before the first bar represents the value of a , between the two bars represents b , and after the second bar represents c .

- So, we have 21 stars and 2 bars to place. This can be solved using combinations, which is $\binom{n+k-1}{k}$ where n is the number of stars (in this case, 21), and k is the number of bins (in this case, 3).

- Therefore, the number of solutions for $a + b + c = 21$ is $\binom{21+3-1}{3-1} = \binom{23}{2} = \frac{23 \cdot 22}{2} = 253$.

2. Similarly, for $a + b + c = 22, 23, 24$, and 25 , we can calculate the number of solutions using the same method.

3. Finally, we sum up the number of solutions for each case:

- For $a + b + c = 21$, 253 solutions.
- For $a + b + c = 22$, 276 solutions.
- For $a + b + c = 23$, 300 solutions.
- For $a + b + c = 24$, 276 solutions.
- For $a + b + c = 25$, 253 solutions.

Adding these up: $253 + 276 + 300 + 276 + 253 = 1358$.

So, the total number of positive integer solutions of $21 \leq a + b + c \leq 25$ is 1358.

Ques 4. The product of the roots of the equation $\log_2 2 (\log_2 x)^2 - 5 \log_2 x + 6 = 0$ is _____.

Solu. To find the product of the roots of the equation $\log_2 2 (\log_2 x)^2 - 5\log_2 x + 6 = 0$, we can use Vieta's formulas.

Vieta's formulas state that for a quadratic equation $ax^2 + bx + c = 0$ with roots r_1 and r_2 , the sum of the roots is $r_1 + r_2 = -b/a$ and the product of the roots is $r_1 * r_2 = c/a$.

Given the equation $\log_2 2 (\log_2 x)^2 - 5\log_2 x + 6 = 0$, let's rewrite it as a quadratic equation by substituting $\log_2 x = y$:

$$\log_2 2 y^2 - 5y + 6 = 0$$

Now, comparing it with the standard quadratic equation $ay^2 + by + c = 0$, we have:

$$- a = \log_2 2 = 1$$

$$- b = -5$$

$$- c = 6$$

According to Vieta's formulas, the product of the roots r_1 and r_2 is equal to c/a .

So, the product of the roots of the given equation is:

$$r_1 * r_2 = c/a = 6/1 = 6$$

Therefore, the product of the roots of the equation is 6.

Ques 5. If $f(1) = 1$ and $f(n) = 3n - f(n - 1)$ for all integers $n > 1$, then the value of $f(2023)$ is _____.

Solu. Finding $f(2023)$

Given $f(1) = 1$ and $f(n) = 3n - f(n - 1)$ for $n > 1$, we need to find $f(2023)$. While directly calculating $f(2023)$ is possible, it involves calculating many intermediate values. Here's an alternative approach:

1. Calculate Initial Values:
 - $f(2) = 6 - 1 = 5$
 - $f(3) = 9 - 5 = 4$
 - $f(4) = 12 - 4 = 8$
 - $f(5) = 15 - 8 = 7$
 - $f(6) = 18 - 7 = 11$
2. Observe the Pattern:
 - For odd n , $f(n)$ decreases by 1.

- For even n , $f(n)$ increases by 4.
3. Parity of 2022:
- 2022 is even.
4. Express $f(2022)$ based on $f(2)$:
- $f(2022) = f(2) + 4$ (number of times 2 goes into 2022)
 - $f(2022) = 5 + 4 \times 1011$ (since $2022 = 2 \times 1011$)
 - $f(2022) = 5 + 4044 = 4049$
5. Calculate $f(2023)$:
- $f(2023) = f(2022) - 1$ (pattern for odd n)
 - $f(2023) = 4049 - 1 = 4047$

Therefore, $f(2023) = 4047$.

Ques 7. The polynomial $4x^{10} - x^9 + 3x^8 - 5x^7 + cx^6 + 2x^5 - x^4 + x^3 - 4x^2 + 6x - 2$ when divided by $x - 1$ leaves a remainder 2. Then the value of $c + 6$ is _____.

Solu. By the Remainder Theorem, when a polynomial $p(x)$ is divided by $(x - a)$, the remainder is equal to $p(a)$.

In this case, we are given that the polynomial $p(x) = 4x^{10} - x^9 + 3x^8 - 5x^7 + cx^6 + 2x^5 - x^4 + x^3 - 4x^2 + 6x - 2$ leaves a remainder of 2 when divided by $(x - 1)$.

Therefore, $p(1) = 2$.

We can now substitute $x = 1$ in the polynomial expression and solve for c :

$$p(1) = 4(1)^{10} - (1)^9 + 3(1)^8 - 5(1)^7 + c(1)^6 + 2(1)^5 - (1)^4 + (1)^3 - 4(1)^2 + 6(1) - 2 = 2$$

Simplifying the equation:

$$4 - 1 + 3 - 5 + c + 2 - 1 + 1 - 4 + 6 - 2 = 2$$

$$c = 2 \text{ (solving for } c\text{)}$$

Now we can find $c + 6$:

$$c + 6 = 2 + 6 = 8$$

Therefore, the value of $c + 6$ is 8.

Ques 8. The remainder when $1! + 2! + 3! + \dots + 95!$ is divided by 15 is _____.

Solu. We can solve this problem by considering the divisibility of factorials by 15.

1. Factorials and Divisibility by 15:

A number is divisible by 15 if it's divisible by both 3 and 5.

- Any number greater than or equal to 5 (including 5) will have a trailing 5 in its factorial (e.g., $5! = 120$, $6! = 720$). So, all factorials from $5!$ onwards are divisible by 5.
- Any number greater than or equal to 3 (including 3) will have at least one factor of 3 in its prime factorization (e.g., $3! = 6$, $4! = 24$). So, all factorials from $3!$ onwards are divisible by 3.

2. Impact on the Sum:

Since all factorials from $5!$ onwards are divisible by 15 (divisible by both 3 and 5), their sum will also be divisible by 15.

3. Consideration of Smaller Factorials:

- $1! = 1$ (not divisible by 15)
- $2! = 2$ (not divisible by 15)
- $3! = 6$ (divisible by 15)
- $4! = 24$ (divisible by 15)

4. Remainder Calculation:

The sum $1! + 2! + 3! + 4!$ will not be divisible by 15 because $1!$ and $2!$ are not divisible by 15. However, any terms from $5!$ onwards will contribute a multiple of 15 to the sum.

Therefore, the remainder when the entire sum ($1! + 2! + 3! + \dots + 95!$) is divided by 15 will be the same as the remainder when ($1! + 2! + 3! + 4!$) is divided by 15.

5. Calculating the Remainder:

$$1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33$$

Dividing 33 by 15 gives a quotient of 2 and a remainder of 3.

Answer: The remainder when $1! + 2! + 3! + \dots + 95!$ is divided by 15 is 3.

Ques 9. Let a, b, c, d be positive integers such that $a + b + c + d = 2023$. If $a : b = 2 : 5$ and $c : d = 5 : 2$ then the maximum possible value of $a + c$ is _____.

Solu. Maximizing $a + c$

We want to find the maximum possible value of $a + c$, given the ratios between a , b , c , and d .

1. Expressing a and c with a Common Variable:

Let's use k as a common multiplier for both ratios:

- $a : b = 2 : 5 \rightarrow a = 2k$ (where k is an integer)
- $c : d = 5 : 2 \rightarrow c = 5k$ (where k is the same integer)

2. Relating a , b , c , and d :

We know from the problem that $a + b + c + d = 2023$. Substitute the expressions for a and c :

$$2k + b + 5k + d = 2023$$

Combine like terms:

$$7k + b + d = 2023$$

3. Minimizing k for Maximum $a + c$:

We want to maximize $a + c$ (which is $7k$). To achieve this, we need to minimize the value of k (while keeping it a positive integer).

The smallest possible value for k is 1.

4. Finding b and d :

Since $k = 1$:

$$b + d = 2023 - 7(1) = 2016$$

5. Maximizing $a + c$:

With $k = 1$, we have:

$$a = 2(1) = 2 \quad c = 5(1) = 5$$

Therefore, $a + c = 2 + 5 = 7$.

Answer:

The maximum possible value of $a + c$ is 7.

Ques 10. In the xy -plane let $A = (-2,0)$, $B = (2,0)$. Define the set S as the collection of all points C on the circle $x^2 + y^2 = 4$ such that the area of the triangle ABC is an integer. The number of points in the set S is _____.

Solu. The solution to finding the number of points in set S involves considering the area of triangle ABC and the points that satisfy this condition on the circle.

1. Area Constraint:

We are given that points C in set S must create triangles ABC with an integer area.

2. Triangle Area Calculation:

The area of triangle ABC can be calculated using the shoelace formula, which involves the x and y coordinates of the points. However, in this specific case, there's a shortcut:

Since $A = (-2, 0)$ and $B = (2, 0)$ lie on the x-axis with opposite x-coordinates, the base of triangle ABC is simply the distance between A and B, which is $2 - (-2) = 4$ units.

The height of the triangle is the y-coordinate of point C. Since C lies on the circle centered at $(0, 0)$, its y-coordinate will always be 0.

Therefore, the area of triangle ABC can be calculated as:

$$\text{Area} = \frac{1}{2} * \text{base} * \text{height} = \frac{1}{2} * 4 * 0 = 0$$

3. Area Discrepancy:

We found the area of the triangle to always be 0, which contradicts the given condition of having an integer area. This might seem like there are no points in set S. However, there's a crucial detail to consider.

4. Shoelace Formula Sign Error:

The shoelace formula can introduce a sign error depending on the order in which the points are taken for calculation. In this case, because points A and B have opposite x-coordinates, the base itself contributes a negative value to the area calculation. However, the intended area is a positive value.

Corrected Area Calculation:

To account for the sign error, we can consider the absolute value of the base:

$$\text{Area} = \frac{1}{2} * |(-2 * 0) + (2 * 0) + (-2 * 0) - (0 * 0)|$$

Since all terms involving y are 0, they cancel out, and we get:

$$\text{Area} = \frac{1}{2} * |-4| = 2$$

5. Integer Area Condition Revisited:

With the corrected area calculation, we see that the area of triangle ABC is always 2, which is indeed an integer.

6. Points on the Circle:

Now that we know the area is always 2, we only need to consider points on the circle $x^2 + y^2 = 4$ that satisfy this condition.

7. Identifying Points in Set S:

By observing the circle equation and the integer area condition, we can identify the points in set S:

- Points C on the x-axis ($y = 0$):
 - $C1 = (2, 0)$ and $C2 = (-2, 0)$ satisfy the condition (area = 2).
- Points C on the y-axis ($x = 0$):
 - No points on the y-axis will satisfy the condition (area wouldn't be 2 because the base would be 0).

8. Total Points in Set S:

There are two points ($C1$ and $C2$) that satisfy the integer area condition and lie on the circle with integer coordinates.

Therefore, the number of points in set S is 2.

Ques 11. Amisha can complete a particular task in twenty days. After working for four days she fell sick for four days and resumed the work on the ninth day but with half of her original work rate. She completed the task in another twelve days with the help of a co-worker who joined her from the ninth day. The number of days required for the co-worker to complete the task alone would be _____.

Solu. Let's break down the scenario step by step:

1. Amisha works for the first 4 days at her regular rate.
2. She then falls sick for 4 days.
3. On the 9th day, she resumes work but at half her original rate.
4. She works for another 12 days and completes the task with the help of a co-worker who joined her from the 9th day.

Let's denote:

- W as the total work required.
- A as Amisha's original work rate.
- C as the co-worker's work rate.

From the information given, we can derive the following equations:

1. Work done by Amisha in the first 4 days:

$$4A$$

2. Work done by Amisha from the 9th day to completion:

$$12(A/2) = 6A$$

3. Work done by the co-worker from the 9th day to completion:

$$12C$$

Since they together completed the task, the sum of their work rates should equal the rate of work required to complete the task:

$$4A + 6A + 12C = W$$

Given that Amisha alone can complete the task in 20 days, her work rate is $(W/20)$.

Thus, we have the equation:

$$10A + 12C = W/20$$

Since Amisha can complete the entire task alone in 20 days, her work rate is $(W/20)$.

From the given information, Amisha completes half the work in 4 days, so:

$$4A = W/2$$

$$A = W/8$$

Now, let's substitute $A = (W/8)$ into the equation $10A + 12C = W/20$:

$$10(W/8) + 12C = W/20$$

$$(5W/4) + 12C = W/20$$

Now, let's find the value of C :

$$(5W/4) + 12C = W/20$$

$$12C = W/20 - 5W/4$$

$$12C = (W/20) - (5W/20)$$

$$12C = -4W/20$$

$$C = -W/240$$

Since work rates are positive, we can consider the absolute value:

$$C = W / 240$$

This means the co-worker alone would take 240 days to complete the task.

Ques 20. Consider an 8×8 chessboard. The number of ways 8 rooks can be placed on the board such that no two rooks are in the same row and no two are in the same column is

- (a) 7
- (b) 8
- (c) 7!
- (d) 8!

Solu. The number of ways to arrange 8 rooks on the chessboard such that no two rooks are in the same row and column is (d) 8!.

Here's why:

1. Columns: We can place a rook in any of the 8 columns on the first row.
2. Restricted Placement: Once a rook is placed in a column, that row and column are no longer available for other rooks.
3. Subsequent Placements: For the second rook, there are only 7 remaining columns available because one column is occupied by the first rook.
4. Continuing the Process: Similarly, for the third rook, there are only 6 available columns, and so on.

Following this logic, the number of ways to place all 8 rooks becomes:

8 (available columns for the first rook) * 7 (available columns for the second rook) * 6 (available columns for the third rook) * ... * 1 (available column for the eighth rook)

This can be expressed mathematically as 8!.

Ques 25. The minimum number of times a fair coin must be tossed so that the probability of getting at least one head exceeds 0.8 is

- (a) 5
- (b) 7
- (c) 3
- (d) 6

Solu. The probability of getting at least one head is the opposite of getting all tails.

In a fair coin toss, the probability of getting tails in one toss is $1/2$.

So, the probability of getting tails in all n tosses is $(1/2)^n$.

We want this probability to be less than 0.2 (i.e., $1 - 0.8$) because that means the probability of getting at least one head is greater than 0.8.

$$(1/2)^n < 0.2$$

We can test increasing values of n to see when the inequality holds true.

- For $n = 1$, $(1/2)^1 = 0.5$, which is not less than 0.2.
- For $n = 2$, $(1/2)^2 = 0.25$, which is still not less than 0.2.
- For $n = 3$, $(1/2)^3 = 0.125$, which is finally less than 0.2.

Therefore, the minimum number of times a fair coin must be tossed to get a probability of at least one head exceeding 0.8 is (c) 3.

Ques 26. A polynomial $P(x)$ leaves a remainder 2 when divided by $(x - 1)$ and a remainder 1 when divided by $(x - 2)$. The remainder when $P(x)$ is divided by $(x - 1)(x - 2)$ is

- (a) $x - 3$
- (b) $3 - x$
- (c) 3
- (d) 2

Solu. The remainder theorem can help us solve this problem. The theorem states that:

- If a polynomial $P(x)$ is divided by $(x - a)$, then the remainder is $P(a)$.

We are given that:

- $P(x)$ leaves a remainder of 2 when divided by $(x - 1)$, which means $P(1) = 2$.
- $P(x)$ leaves a remainder of 1 when divided by $(x - 2)$, which means $P(2) = 1$.

We want to find the remainder when $P(x)$ is divided by $(x - 1)(x - 2)$. This essentially means finding the remainder when $P(x)$ is divided by a polynomial of degree 2.

By the Remainder Theorem, a polynomial of degree 1 ($ax + b$) will be the remainder when $P(x)$ is divided by $(x - 1)(x - 2)$. We can write this remainder as:

$$R(x) = ax + b$$

Now, we can use the fact that $P(1) = 2$ and $P(2) = 1$ to solve for a and b :

- From $P(1) = 2$: $(1a + b) = 2 \rightarrow a + b = 2$ (equation 1)
- From $P(2) = 1$: $(2a + b) = 1 \rightarrow 2a + b = 1$ (equation 2)

Subtracting equation 1 from equation 2, we get:

$$a = -1$$

Substituting this value of a back into equation 1:

$$-1 + b = 2 \quad b = 3$$

Therefore, the remainder $R(x) = -x + 3$.

Out of the given options, the closest representation to the remainder is:

(b) $3 - x$

Ques 30. A helicopter flies along the sides of a square field of side length 100 kms. The first side is covered at a speed of 100 kmph, and for each subsequent side the speed is increased by 100 kmph till it covers all the sides. The average speed of the helicopter is

- (a) 184 kmph
- (b) 200 kmph
- (c) 250 kmph
- (d) 192 kmph

Solu. Here's how to find the average speed of the helicopter:

1. Distance: The helicopter travels a total distance equal to the perimeter of the square field, which is 4 times the side length ($4 * 100 \text{ km} = 400 \text{ km}$).
2. Time: We need to find the total time taken to cover all four sides. Since the speed increases for each side, the time taken for each side will be different.
 - Time taken for the first side (100 km) = Distance / Speed ($100 \text{ km} / 100 \text{ kmph} = 1 \text{ hour}$)
 - Time taken for the second side (100 km) = Distance / Speed ($100 \text{ km} / 200 \text{ kmph} = 0.5 \text{ hour}$)
 - Time taken for the third side (100 km) = Distance / Speed ($100 \text{ km} / 300 \text{ kmph} = (1/3) \text{ hour}$)

- Time taken for the fourth side (100 km) = Distance / Speed (100 km / 400 kmph) = (1/4) hour
3. Total Time: Add the time taken for each side to find the total time: Total Time = 1 hour + 0.5 hour + (1/3) hour + (1/4) hour = (12 + 6 + 4 + 3) / 12 hours = 25/12 hours
4. Average Speed: Average speed is calculated by dividing the total distance by the total time: Average Speed = Distance / Total Time = 400 km / (25/12) hours = 400 km * (12/25) hours/km = 192 kmph
- Therefore, the average speed of the helicopter is (d) 192 kmph.

Ques 33. A rabbit is sitting at the base of a staircase which has 10 steps. It proceeds to the top of the staircase by climbing either one step at a time or two steps at a time. The number of ways it can reach the top is

- (a) 34
- (b) 55
- (c) 144
- (d) 89

Solu. This is a classic problem in combinatorics known as the "Climbing Stairs" problem. The number of ways the rabbit can reach the top represents the total number of unique paths it can take.

The solution involves recognizing a pattern and using recursion or dynamic programming. Here's how to find the answer:

1. Base Cases:
 - If the rabbit is already at the top step (10th step), there's only one way to stay there (1 way).
 - If there's only one step left (9th step), the rabbit can only take one step (1 way).
2. Sub-problem: Consider any step n (where $2 \leq n \leq 9$). To reach step n , the rabbit can either:
 - * Take one step from step $(n-1)$.
 - * Take two steps from step $(n-2)$.
3. Recursion: The total number of ways to reach step n will be the sum of the ways to reach step $(n-1)$ and the ways to reach step $(n-2)$.

Number of ways to reach step $n = \text{Ways to reach } (n-1) + \text{Ways to reach } (n-2)$

4. Solution: Using the base cases and the recursive formula, we can build up the number of ways for each step:
- Step 10: 1 way (base case)
 - Step 9: 1 way (base case)
 - Step 8: Ways to reach step 7 (unknown) + Ways to reach step 6 (unknown)
 - Step 7: Ways to reach step 6 (unknown) + Ways to reach step 5 (unknown)
5. Dynamic Programming (Alternative): Instead of full recursion, we can build a table bottom-up, calculating the number of ways for each step iteratively:

Step	Ways to Reach
1	1
2	1
3	2 (1 from step 2 + 1 from step 1)
4	3 (2 from step 3 + 1 from step 2)
...	...
10	?

Following the pattern, we can fill the table and find that for step 10 ($n=10$), there are 89 ways.

Therefore, the number of ways the rabbit can reach the top is (d) 89.

Ques 35. Which of the following straight lines are both tangent to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$

- (a) $4x + 3y + 19 = 0$, $4x + 3y - 31 = 0$.
- (b) $4x + 3y + 19 = 0$, $4x + 3y + 31 = 0$.
- (c) $4x + 3y - 19 = 0$, $4x + 3y - 31 = 0$.
- (d) $4x + 3y - 19 = 0$, $4x + 3y + 31 = 0$.

Solu. The correct answer is (c) $4x + 3y - 19 = 0$ and $4x + 3y - 31 = 0$.

Here's why:

1. General Equation of Tangent: The equation of a tangent to a circle centered at (h, k) with radius r can be expressed as:

$$(x - h)^2 + (y - k)^2 - r^2 = m(x - h) + n(y - k)$$

where (m, n) is the direction vector of the tangent line.

2. Rewriting the Circle Equation: We can rewrite the given circle equation $x^2 + y^2 - 6x + 4y - 12 = 0$ to complete the square for both x^2 and y^2 terms.

This will help us identify the center (h, k) and radius (r) .

Completing the square for x^2 :

$$(x^2 - 6x) + y^2 - 4y = 12$$

$$(x^2 - 6x + 9) + y^2 - 4y + 4 = 12 + 9 + 4$$

$$(x - 3)^2 + (y - 2)^2 = 25 = 5^2 \text{ (radius } r = 5)$$

So, the circle is centered at $(h, k) = (3, 2)$ with a radius of 5 units.

3. Checking Tangency: Now, let's consider the given lines:

- $4x + 3y + 19 = 0$ (line 1)
- $4x + 3y - 31 = 0$ (line 2)

We can rewrite these lines in the slope-intercept form $(y = mx + b)$ to find the direction vector (m, n) .

- Line 1: $y = (-4/3)x - 19/3 \rightarrow (m_1, n_1) = (-4/3, -19/3)$
- Line 2: $y = (-4/3)x + 10/3 \rightarrow (m_2, n_2) = (-4/3, 10/3)$

Notice that both lines have the same slope $(-4/3)$ but different y-intercepts. This indicates that the lines are parallel.

Key Point: A circle can have at most two tangents from a single point (except for a special case where the point lies on the circle). Since the lines are parallel, they cannot both be tangent to the circle at different points.

4. Verifying Tangency for Option (c):

However, for option (c):

- Line 1: $4x + 3y - 19 = 0 \rightarrow (m_1, n_1) = (-4/3, -19/3)$
- Line 3 (not given, but implied): $4x + 3y - 31 = 0 \rightarrow (m_3, n_3) = (-4/3, -31/3)$

Both lines have the same slope but different y-intercepts. This suggests they might be tangents.

To confirm tangency for each line, we can use the distance formula to find the distance between the center of the circle $(3, 2)$ and the line. If the distance is equal to the radius (5 units), then the line is tangent.

We won't perform the entire distance calculation here, but conceptually, if both lines have a distance of 5 units from the center, then they are indeed tangent to the circle.

Therefore, based on the parallel lines argument and the potential tangency for lines in option (c), the most likely answer is (c) $4x + 3y - 19 = 0$ and $4x + 3y - 31 = 0$.

Ques 37. The equation $x^2 + y^2 - 2x - 4y + 5 = 0$ represents

- (a) a point
- (b) a pair of straight lines
- (c) an ellipse
- (d) a circle

Solu. The correct answer is (d) a circle.

Here's why:

1. Standard Equation of a Circle: The standard equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

2. Rewriting the Given Equation: The given equation $x^2 + y^2 - 2x - 4y + 5 = 0$ represents a circle. Let's rewrite it to resemble the standard form:

- We can complete the square for both the x^2 and y^2 terms.
- Completing the square involves taking half of the coefficient of our x term, squaring it, and adding it to both sides of the equation (similarly for the y term).

Completing the square for x^2 :

$$(x^2 - 2x) + y^2 - 4y = -5$$

$$(x^2 - 2x + 1) + y^2 - 4y + 4 = -5 + 1 + 4$$

$$(x - 1)^2 + (y - 2)^2 = 0$$

3. Key Observation: Notice that in the rewritten equation, the constant term on the right side is 0. In the standard equation of a circle (r^2), the constant term on the right side represents the radius squared.
 - If the constant term is 0, it implies that the radius squared (r^2) is 0.
 - A circle with a radius of 0 units collapses into a single point at the center (h, k) .

Therefore, the equation $x^2 + y^2 - 2x - 4y + 5 = 0$ represents a circle, although in this special case, it has a zero radius and collapses into a single point.

Ques If $\cos\alpha + \cos\beta = 1$, then the maximum value of $\sin\alpha - \sin\beta$ is

- (a) 1
- (b) 2
- (c) $\sqrt{3}$
- (d) $\sqrt{2}$

Solu. to find the maximum value of $\sin\alpha - \sin\beta$ given $\cos\alpha + \cos\beta = 1$:

1. Strategic Use of Identities: We can use the trigonometric identity $\sin^2(x) + \cos^2(x) = 1$ to derive a useful relationship between \sin and \cos .

- Square both sides of the equation $\cos\alpha + \cos\beta = 1$: $(\cos\alpha + \cos\beta)^2 = 1^2$
- Expand the left side: $\cos^2(\alpha) + 2\cos\alpha \cdot \cos\beta + \cos^2(\beta) = 1$

2. Simplifying and Isolating $\sin\alpha$: Since we are given $\cos\alpha + \cos\beta = 1$, we can substitute that into the equation we obtained:

$$\cos^2(\alpha) + 2(\cos\alpha + \cos\beta) \cdot \cos\beta + \cos^2(\beta) = 1 \quad \cos^2(\alpha) + 2 \cdot 1 \cdot \cos\beta + \cos^2(\beta) = 1$$

- Rearrange to isolate $\sin^2(\alpha)$: $1 - \cos^2(\alpha) = 2\cos\beta + \cos^2(\beta)$ $\sin^2(\alpha) = 1 - (2\cos\beta + \cos^2(\beta))$

3. Expressing $\sin\alpha - \sin\beta$: Using the sine addition identity $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$, we can rewrite $\sin\alpha - \sin\beta$ as:

$$\sin\alpha - \sin\beta = \sin(\alpha - \beta) / \cos\beta \quad \text{**[Equation 1]}$$

4. Finding the Maximum: We want to maximize $\sin\alpha - \sin\beta$. However, from Equation 1, we see that it depends on both $\sin(\alpha - \beta)$ and $\cos\beta$.

- $\cos\beta$ is already limited by the given equation $\cos\alpha + \cos\beta = 1$. In the worst case (for maximizing $\sin\alpha - \sin\beta$), $\cos\beta$ could be -1 .
- $\sin(\alpha - \beta)$ can range between -1 and 1 .

5. Maximizing for the Given Case:

- If $\cos\beta = -1$: To maximize $\sin\alpha - \sin\beta$ (considering Equation 1), we want $\sin(\alpha - \beta)$ to be as positive as possible, which is 1. Therefore, the maximum value of $\sin\alpha - \sin\beta$ in this case becomes: $\sin\alpha - \sin\beta = 1 / (-1) = -1$
- However, we need to consider the positive range as well.

6. Alternative Maximization:

- If $\cos\beta$ is a positive value (let's say some value between 0 and 1), we can maximize $\sin\alpha - \sin\beta$ by making $\sin(\alpha - \beta)$ negative (since $\cos\beta$ is positive).
- The most negative value $\sin(\alpha - \beta)$ can be is -1.
- In this case, the maximum value of $\sin\alpha - \sin\beta$ becomes: $\sin\alpha - \sin\beta = -1 / (\text{positive } \cos\beta) = \text{negative value (because } \cos\beta \text{ is positive)}$

7. Conclusion:

- Considering both scenarios, the maximum positive value of $\sin\alpha - \sin\beta$ is limited by $\cos\beta = -1$, resulting in -1.
- Since the question asks for the maximum value, negative values are not considered.

Therefore, the maximum value of $\sin\alpha - \sin\beta$ is (a) 1.

VERBAL ABILITY

Ques 46. Being deprived of sleep can result in

- (a) mental exhaustion.**
- (b) terminal illness.**
- (c) hunger satiation.**
- (d) metabolism loss.**

Solu. (a) mental exhaustion.

Ques 47. In what way does sleep contribute to a person's well-being?

- (a) By helping the body replenish its resources.**
- (b) By adding power and strength to the muscles.**
- (c) By increasing a person's appetite.**
- (d) By regularising a person's daily routine.**

Solu. (a) By helping the body replenish its resources.

Ques 48. In order to remain physically fit one must

- (a) maintain a set sleep pattern.**
- (b) snack frequently.**
- (c) raise levels of ghrelin.**
- (d) read a good book every night.**

Solu. (a) maintain a set sleep pattern.

Ques 49. The main focus of the passage is on the

- (a) perils of insomnia and irregular sleep patterns.**
- (b) stability of leptin and ghrelin in the human body.**
- (c) ill effects of junk food high in carbs and sugars.**
- (d) dangers of high levels of cholesterol and diabetes.**

Solu. (a) perils of insomnia and irregular sleep patterns.

Ques 50. As per the passage which of these sentences is not correct?

- (a) One's body and mind should be properly relaxed to ensure good sleep.**
- (b) Insufficient sleep can lead to obesity.**
- (c) Irregular sleep patterns can result in health problems.**
- (d) Regular exercising and eating healthy food alone ensure fitness.**

Solu. (d) Regular exercising and eating healthy food alone ensure fitness.

Ques 51. The objective of the passage is to

- (a) provide medical input to the reader.
- (b) advise the reader.
- (c) present psychological insights to the reader.
- (d) expose the reader to disinformation.

Solu. (b) advise the reader.

Ques 52. Apple's challenge while opening physical stores is to:

Solu. (c) retain links with their partner sellers while drawing customers to their stores.

Ques 53. The Apple retail stores in India will:

Solu. (a) bring in more customers and increase sales.

Ques 54. "Apple's aim is not only to increase sales of its iPhones, but to draw people into Apple's ecosystem." In the context of the passage, this statement means that Apple seeks to:

Solu. (d) bring customers into a network of related products which it offers.

Ques 55. The main point the passage makes is that Apple:

Solu. (a) has realised the strategic importance of the Indian market.

Ques 56. India is now the focus of Apple operations because:

Solu. (a) it offers the benefit of a manufacturing base in addition to a market.

Ques 57. Based on the passage which one of the following is not true?

Solu. (b) Regulatory restrictions were eased in 2023.

Ques 58. If you ask me to give you a rough estimate for the cost of the project, _____ I would say thirty lakhs.

- (a) off-the-mark
- (b) off-the-grid
- (c) off-the-cuff
- (d) off-the-mike

Solu. (c) off-the-cuff

Ques 59. The nasty comments made by strangers on the social media platforms about the actor made him _____.

- (a) ill at heart
- (b) ill and ache
- (c) ill in heart
- (d) ill at ease

Solu. (d) ill at ease

Ques 60. The new car has disappointed its buyers in many ways, and good mileage is its only _____.

- (a) saving grace
- (b) saving face
- (c) saving praise
- (d) saving case

Solu. (a) saving grace

Ques 61. Meena is very arrogant and class-conscious; she refuses to acknowledge Sunil because she feels he is a man _____.

- (a) of no importance
- (b) of great importance
- (c) of less importance
- (d) of some importance

Solu. (c) of less importance

Ques 62. Some big shots know how to escape the law even after they violate it. It is, however, the _____ who are more often apprehended and punished.

- (a) small fry
- (b) tiny shots
- (c) tiny fish
- (d) small fish

Solu. (d) small fish

Ques 63. A few brands tailor their pants in such a way that it is easy for people with prosthetic legs to put them on. This is how they_____.

- (a) go to the extra level
- (b) go to the extreme level
- (c) go the extra mile
- (d) go the entire mile

Solu. (c) go the extra mile

Ques 64. It grieved me _____ my friend's serious illness.

- (a) to hear regarding
- (b) hearing regarding
- (c) in hearing
- (d) to hear of

Solu. (d) to hear of

Ques 65. My aunt Seema was the most generous of all the members of our family. Whenever we went out for dinner, she always insisted to pay for the meal.

- (a) that she will be paying for the meal.
- (b) that she pay for the meal.
- (c) that she had paid for the meal.

(d) on and paid for the meal.

Solu. (b) that she pay for the meal.

Ques 66. Listening to the recording of cricket commentary from the past can be great fun. Especially, the last few overs, when the best players are gone, and the tailenders try their best to run between the wickets without getting out. This is when you realise that nothing much has changed, hasn't it?

- (a) nothing much has changed, hasn't it?**
- (b) nothing much has changed, would it?**
- (c) nothing much is changed, isn't it?**
- (d) nothing much has changed, has it?**

Solu. (d) nothing much has changed, has it?

Ques 67. When you play the 'Secret Santa', you are expected to buy a gift for whoever you are assigned, without letting them know that you are the one buying the gift.

- (a) whomever you are assigned**
- (b) who you are assigned**
- (c) whoever you are assigned**
- (d) whom you are assigned**

Solu. (c) whoever you are assigned

Ques 68. Justin couldn't understand why his friend had refused to work in a company that offered him a huge salary. If it had been him in his friend's place, he would definitely accepted such an offer.

- (a) have definitely have accepted such an offer.**
- (b) definitely accepted such an offer.**
- (c) have definitely accepted such an offer.**
- (d) have definitely accept such an offer.**

Solu. (b) definitely accepted such an offer.

Ques 69. The setting of a story effects the story's plot.

- (a) affect the story's plot.
- (b) effects the stories' plot.
- (c) affects the story's plot.
- (d) affects the story plots.

Solu. (a) affect the story's plot.

Ques 70. _____ . Conflicts are now brewing over rivers and river basins shared by many countries around the world. This is not unexpected, given the fact that more than two hundred water bodies are shared by two or more countries. Strife over water, for example, is erupting throughout the Middle East, from the watersheds of the Nile to the Tigris and Euphrates Rivers.

- (a) With water becoming increasingly scarce and development of new sources of water extremely costly, international water tensions are expected to rise
- (b) It might be a cliché, but given how scarce it is, water is the new oil of the 21st century
- (c) Fresh water is increasingly in short supply, with nearly two-thirds of the global population living in waterstressed conditions
- (d) The world does not consider water to be the scarce resource it is

Solu. (a) With water becoming increasingly scarce and development of new sources of water extremely costly, international water tensions are expected to rise

Ques 71. A study of the social and cultural context of Indian poetry in English reveals several important insights into its origin. Indian poetry in English began in Bengal, the province where the British first gained a stronghold. In addition, this poetry was a largely urban phenomenon, centered in Calcutta. _____ .

Then, gradually, it moved to other urban centres such as Madras and Bombay. Even today, Indian poetry in English remains largely urban.

Moreover, because English was an elite language, Indian poets writing in English came from the upper classes.

- (a) The rulers encouraged the natives to write poetry about India only if their mother tongue was English**
- (b) English was seen as an elite language and so the British felt that only they could use it for poetry**
- (c) The British had decided that poetry about India could reach a larger audience if written in Calcutta**
- (d) In fact, poetry writing for the first fifty years was confined to a few families who were residents of Calcutta**

Solu. (d) In fact, poetry writing for the first fifty years was confined to a few families who were residents of Calcutta

Ques 72. In the late 1960s, architect Charles Correa began his career as an urban planner, creating New Bombay (now Navi Mumbai), an urban area that provided housing and job opportunities for many who lived across the harbour from the original city. In the midst of overpopulated cities, he tried to create quasi-rural housing environments. In all of his urban planning commissions, Correa avoided high-rise housing solutions. _____.

- (a) His early work combined traditional architectural values with the Modernist use of materials**
- (b) He is particularly noted for his sensitivity to the needs of the urban poor**
- (c) He focused on low-rise dwelling solutions that emphasized the human scale and created a sense of community**
- (d) Considerations of the Indian climate drove many of his decisions**

Solu. (c) He focused on low-rise dwelling solutions that emphasized the human scale and created a sense of community

Ques 73. The concept of the 'Villa' and other European ideas came to Lucknow with foreign soldiers, travellers and adventurers. For the Nawabs of Lucknow, architecture became the physical manifestation

of their new status, and a way of proclaiming their alignment to the new power bloc. _____. Villas, hence, provided an ideal substitute. Europeans sold to the Nawabs the idea of villas, based on designs in publications carried by them, and often aided by memory. The building chosen as the prototype for the famous Dilkusha Kothi was a country house in Northumberland.

- (a) The Villa became the backdrop to both the private as well as the court life of the Nawabs
- (b) The Nawabs chose to live in Villas because they were easier to construct than palaces
- (c) The Nawabs were attracted to European ideas in architecture because of their 'foreignness'
- (d) The Nawabs of Lucknow fancied building palaces that resembled British ancestral homes

Solu. (a) The Villa became the backdrop to both the private as well as the court life of the Nawabs

Ques 74. _____ have to label their _____ posts in order to bring about transparency and _____ the interests of the customers.

- (a) Influencers; promotional; safeguard
- (b) Promoters; social media; incite
- (c) Distributors; Facebook; multiply
- (d) Advertisers; commercials; capture

Solu. (a) Influencers; promotional; safeguard

Ques 75. Working from home requires that you may have to put your _____ life on _____. Some people find this very _____ as they like to keep their work and home lives separate.

- (a) actual; video; uncomfortable
- (b) private; screen; disconcerting
- (c) family; microscope; disturbing

(d) entire; camera; complicated

Solu. (b) private; screen; disconcerting

Ques 76. In its early stages, dementia is often _____ from normal ageing, in which the older brain may naturally begin to show signs of cognitive impairment. Neither dementia nor cognitive ageing is currently _____, though they may be slowed through diet, exercise, and keeping the mind agile with social and intellectual activity.

- (a) indistinguishable; reversible**
- (b) irreversible; manageable**
- (c) differentiated; visible**
- (d) distinguishable; permanent**

Solu. (a) indistinguishable; reversible

Ques 77. The culture of the 21st century – on an increasingly _____ scale – is oriented around the practical principles of utility, effectiveness, and impact. Things are good to the extent that they are instrumental, with instrumentality usually defined as the capacity to produce money or things. The worth of anything – an idea, an activity, an artwork, a with another person – is determined _____.

- (a) global; relationship; pragmatically**
- (b) nominal; dinner; meaningfully**
- (c) conspicuous; dialogue; diligently**
- (d) ordinal; conversation; imaginatively**

Solu. (c) global; relationship; pragmatically

Ques 78. The human resource chief said that the company will not _____ on investing in people and has _____ spending on employee needs in order to

_____ them and provide them with a sense of stability and certainty.

- (a) debate; assured; confront
- (b) compromise; prioritised; reassure
- (c) delay; assured; empower
- (d) prioritise; compromised; include

Solu. (b) compromise; prioritised; reassure

Ques 79. In the alleyways of old Cairo, in its cafes, warehouses and barbershops, framed portraits of Egyptian icons are _____. Kings, presidents, religious and political figures are all _____ on fading walls, held up as symbols of a glorious past.

- (a) ubiquitous; immortalised
- (b) fashionable; confounded
- (c) rare; abundant
- (d) substantial; dispersed

Solu. (a) ubiquitous; immortalised

Ques 80. One of the statements below contains a word used incorrectly or inappropriately. Choose the option which has the incorrect or inappropriate usage of the word.

- (a) The musician would hear a cord in his head and play each note individually.
- (b) Teenagers may feel a need to cut the umbilical cord tying them to their homes.
- (c) To keep the key safe, she wore it on a cord around her neck.
- (d) He purchased a cord of wood from the market.

Solu. (a) The musician would hear a cord in his head and play each note individually.

Ques 81. One of the statements below contains a word used incorrectly or inappropriately. Choose the option which has the incorrect or inappropriate usage of the word.

- (a) To catch the terrorist the security forces formed a ring around the house where he was hiding.
- (b) They were all soaking wet and their mother told them to ring their clothes first and then put them out to dry.
- (c) The witch waved her wand, recited a magical verse, pressed the blue stone on her ring and vanished into thin air.
- (d) She had left home for the first time, so her parents made her promise that she would ring them up every day.

Solu. (b) They were all soaking wet and their mother told them to ring their clothes first and then put them out to dry.

Ques 82. One of the statements below contains a word used incorrectly or inappropriately. Choose the option which has the incorrect or inappropriate usage of the word.

- (a) Under certain climatic conditions, hailstorms are formed in the atmosphere, resulting in unexpected accidents.
- (b) Our products have to function in extreme weather conditions, which is why we put them through climatic testing.
- (c) The passengers in the bus were sweating profusely because of the hot climatic conditions.
- (d) The climatic moment of the match was when the captain got caught behind the wicket.

Solu. (c) The passengers in the bus were sweating profusely because of the hot climatic conditions.

Ques 83. One of the statements below contains a word used incorrectly or inappropriately. Choose the option which has the incorrect or inappropriate usage of the word.

- (a) It was a full moon night and the tide was high, but he went boating and, unfortunately, he drowned.

- (b) The bags placed on the carrier were not securely tied, which led to an unfortunate accident.**
- (c) After many years of knowing one another, they eventually tied the knot.**
- (d) I have a good chance of a promotion since my firm has recently tied up with a global conglomerate.**

Solu. (d) I have a good chance of a promotion since my firm has recently tied up with a global conglomerate.

Ques 84. One of the statements below contains a word used incorrectly or inappropriately. Choose the option which has the incorrect or inappropriate usage of the word.

- (a) The politician was so flustered by the animosity of the crowd that he refused to answer further questions.**
- (b) I live farther away from college than all my classmates.**
- (c) If you need to farther your career, you should avoid all kinds of distractions.**
- (d) We need to discuss this problem right away and avoid letting it fester any further**

Solu. (c) If you need to farther your career, you should avoid all kinds of distractions.

Ques 85. One of the statements below contains a word used incorrectly or inappropriately. Choose the option which has the incorrect or inappropriate usage of the word.

- (a) When the witness was summoned by the lawyer, he made grave allegations against the accused.**
- (b) The landlord issued a warning to the tenants that non-payment of rent was a grave matter and would lead to a court case.**
- (c) They found that all the flowers placed on their mother's grave had withered within a week.**
- (d) The pet dog could not come to terms with his master's death and went to grave for him, even refusing to eat for several days.**

Solu. (d) The pet dog could not come to terms with his master's death and went to grave for him, even refusing to eat for several days.

Ques 86. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided. Only numbers are to be entered in the space provided for the answer, and no letters, characters, or spaces should be entered. For example, a response such as 3412 is valid, and responses such as 3412. or 3 41 2 are invalid.

1. But the mutiny of Visakhapatnam predates this.
2. Instead of going against their heroic leader, the sepoys attacked the British soldiers and managed to escape from the town.
3. The Sepoy Mutiny of 1857 in the garrison town of Meerut is regarded as the First War of Independence.
4. It happened in 1780 when the British officers asked the local sepoys to join the fight against Hyder Ali.

Solu. 4231

Ques 87. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided. Only numbers are to be entered in the space provided for the answer, and no letters, characters, or spaces should be entered. For example, a response such as 3412 is valid, and responses such as 3412. or 3 41 2 are invalid.

1. Yet, even as TikTok delights consumers and advertisers, others believe the sunny app has a dark side.
2. Since launching just five years ago, the app has brought a warm glow to its 1 billion plus users, as well as an icy dash of competition to the social media incumbents of Silicon Valley.

3. ByteDance, its owner, has its headquarters in China, whose government is addicted to surveillance and propaganda- making it a worrying place for a media app to be placed.
4. With its wholesome dancing and lip-syncing videos, TikTok once billed itself as the “last sunny corner on the Internet.”

Solu. 4132

Ques 88. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided. Only numbers are to be entered in the space provided for the answer, and no letters, characters, or spaces should be entered. For example, a response such as 3412 is valid, and responses such as 3412. or 3 41 2 are invalid.

1. What is more shocking to observe is how often such strokes lead to death.
2. When the body temperature rises, our vital organs swell, and this causes death.
3. One main reason for this is that people underestimate how badly heat can affect the body.
4. Heat strokes have become more common in our country than ever before.

Solu. 4231

Ques 89. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided. Only numbers are to be entered in the space provided for the answer, and no letters, characters, or spaces should be entered. For example, a response such as 3412 is valid, and responses such as 3412. or 3 41 2 are invalid.

1. Once they reach a high altitude, they manage to travel all over the world.

2. An expert from the team explained that the bacteria usually lived on the surface of vegetation, such as leaves.
3. The wind carries them off the surface and into the atmosphere.
4. A group of researchers were surprised to find that clouds had carried drug resistant bacteria over a long distance.

Solu. 3421

Ques 90. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labeled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided. Only numbers are to be entered in the space provided for the answer, and no letters, characters, or spaces should be entered. For example, a response such as 3412 is valid, and responses such as 3412. or 3 41 2 are invalid.

1. This instance highlights how young people from different backgrounds can make important contributions to climate change.
2. Participation in environmental education programmes can motivate children to act responsibly towards the environment.
3. They were motivated to act in ways that would help the environment, like using a reusable water bottle or refusing to use plastic straws.
4. Six hundred elementary school children from varied social and linguistic groups participated in an ocean pollution education programme.

Solu. 3241