SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Considering only the principal values of the inverse trigonometric functions, the value of

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$$

is

(A) $\frac{7}{24}$

(B) $\frac{-7}{24}$

(C) $\frac{-5}{24}$

(D) $\frac{5}{24}$

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Let $S=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}:\ x\geq 0, y\geq 0,\ y^2\leq 4x,\ y^2\leq 12-2x \text{ and } 3y+\sqrt{8}\,x\leq 5\sqrt{8}\right\}$. If the area of the region S is $\alpha\sqrt{2}$, then α is equal to

(A) $\frac{17}{2}$

(B) $\frac{17}{3}$

(C) $\frac{17}{4}$

(D) $\frac{17}{5}$

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Let $k \in \mathbb{R}$. If $\lim_{x \to 0+} \left(\sin(\sin kx) + \cos x + x\right)^{\frac{2}{x}} = e^6$, then the value of k is

(A) 1 (B) 2 (C) 3 (D) 4

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Negative Marks : −1 In all other cases.

Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by

$$f(x)=\begin{cases} x^2\sin\left(\dfrac{\pi}{x^2}\right), & \text{if } x\neq 0,\\ 0, & \text{if } x=0.\end{cases}$$

Then which of the following statements is TRUE?

(A) $f(x)=0$ has infinitely many solutions in the interval $\left[\dfrac{1}{10^{10}},\infty\right)$.

(B) $f(x)=0$ has no solutions in the interval $\left[\dfrac{1}{\pi},\infty\right)$.

(C) The set of solutions of $f(x)=0$ in the interval $\left(0,\dfrac{1}{10^{10}}\right)$ is finite.

(D) $f(x)=0$ has more than 25 solutions in the interval $\left(\dfrac{1}{\pi^2},\dfrac{1}{\pi}\right)$.

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).

Let S be the set of all $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$ such that

$$\lim_{x \to \infty} \frac{\sin(x^2)(\log_e x)^\alpha \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^\beta} = 0.$$

Then which of the following is (are) correct?

(A) $(-1,\ 3) \in S$

(B) $(-1,\ 1) \in S$

(C) $(1,\ -1) \in S$

(D) $(1,\ -2) \in S$

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- For each question, choose the option(s) corresponding to (all) the correct answer(s).

A straight line drawn from the point $P(1,3,2)$, parallel to the line $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-6}{1}$, intersects the plane $L_1: x-y+3z=6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2: 2x-y+z=-4$ at the point R. Then which of the following statements is (are) TRUE?

(A) The length of the line segment PQ is $\sqrt{6}$

(B) The coordinates of R are $(1,6,3)$

(C) The centroid of the triangle PQR is $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$

(D) The perimeter of the triangle PQR is $\sqrt{2}+\sqrt{6}+\sqrt{11}$

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- For each question, choose the option(s) corresponding to (all) the correct answer(s).

Let A_1, B_1, C_1 be three points in the xy-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1, respectively. If $O = (0,0)$ and $C_1 = (-4,0)$, then which of the following statements is (are) TRUE?

(A) The length of the line segment OA_1 is $4\sqrt{3}$

(B) The length of the line segment A_1B_1 is 16

(C) The orthocenter of the triangle $A_1B_1C_1$ is $(0,0)$

(D) The orthocenter of the triangle $A_1B_1C_1$ is $(1,0)$

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases.

Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $f(x+y)=f(x)+f(y)$ for all $x,\ y\in\mathbb{R}$, and $g:\mathbb{R}\to(0,\infty)$ be a function such that $g(x+y)=g(x)g(y)$ for all $x,y\in\mathbb{R}$. If $f\left(\frac{-3}{5}\right)=12$ and $g\left(\frac{-1}{3}\right)=2$, then the value of $\left(f\left(\frac{1}{4}\right)+g(-2)-8\right)g(0)$ is ________.

Ans. 51

SECTION 3 (Maximum Marks: 24)

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- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For $i = 1, 2, 3$, let $W_i, G_i,$ and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball, and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 | W_1 \cap G_2) = \frac{2}{9}$, then N equals ______.

Ans. 11

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  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

Let the function $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\frac{\sin x}{e^{\pi x}}\frac{\left(x^{2023}+2024x+2025\right)}{\left(x^{2}-x+3\right)}+\frac{2}{e^{\pi x}}\frac{\left(x^{2023}+2024x+2025\right)}{\left(x^{2}-x+3\right)}.$$

Then the number of solutions of $f(x)=0$ in $\mathbb{R}$ is ______.

Ans. 1

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  Zero Marks : 0 In all other cases.

Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers $\alpha, \beta,$ and $\gamma,$ we have

$$15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha\left(2\vec{p} + \vec{q}\right) + \beta\left(\vec{p} - 2\vec{q}\right) + \gamma\left(\vec{p} \times \vec{q}\right),$$

then the value of γ is _______.

Ans. 2

SECTION 3 (Maximum Marks: 24)

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  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases.

A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0,-\alpha)$ to the parabola $x^2=-4ay$, where $a>0$. Let L be the line passing through $(0,-\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If $r:s=1:16$, then the value of $24a$ is __________.

Ans. 12

SECTION 3 (Maximum Marks: 24)

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- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

Let the function $f:[1,\infty)\to\mathbb{R}$ be defined by

$$f(t)=\begin{cases}(-1)^{n+1}2, & \text{if } t=2n-1,\ n\in\mathbb{N},\\ \dfrac{(2n+1-t)}{2}f(2n-1)+\dfrac{(t-(2n-1))}{2}f(2n+1), & \text{if } 2n-1<t<2n+1,\ n\in\mathbb{N}.\end{cases}$$

Define $g(x)=\int_1^x f(t)\,dt,\ x\in(1,\infty)$. Let α denote the number of solutions of the equation $g(x)=0$ in the interval $(1,8]$ and $\beta=\lim_{x\to1+}\dfrac{g(x)}{x-1}$. Then the value of $\alpha+\beta$ is equal to _____.

Ans. 5

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**

PARAGRAPH "I"

Let $S=\{1,2,3,4,5,6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a,b)\in R$, we have $|a-b|\geq 2$.

Let $Y=\{R\in X:$ The range of R has exactly one element$\}$ and

$Z=\{R\in X: R$ is a function from S to $S\}$.

Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

If $n(X)={}^{m}C_{6}$, then the value of m is __________.

Ans. 20

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$Z=\{R\in X: R$ is a function from S to $S\}$.

Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

If the value of $n(Y)+n(Z)$ is k^2, then $|k|$ is ____________.

Ans. 36

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**

PARAGRAPH "II"

Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be the function defined by $f(x)=\sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\to[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

The value of $2\int_0^{\frac{\pi}{2}} f(x)g(x)dx-\int_0^{\frac{\pi}{2}} g(x)dx$ is __________.

Ans. 0

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- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**

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Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be the function defined by $f(x)=\sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\to[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

The value of $\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx$ is ________.

Ans. 0.25