JEE(ADVANCED)–2024 (EXAMINATION)

(Held On Sunday 26th MAY, 2024)

MATHEMATICS | **TEST PAPER WITH ANSWER**

PAPER-2

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

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1. Considering only the principal values of the inverse trigonometric functions, the value of

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$$

is

(A) $\frac{7}{24}$ (B) $\frac{-7}{24}$

(C) $\frac{-5}{24}$ (D) $\frac{5}{24}$

Ans. (B)

2. Let $S=\{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12-2x \text{ and } 3y+\sqrt{8}\, x \leq 5\sqrt{8}\}$. If the area of the region S is $\alpha\sqrt{2}$, then α is equal to

(A) $\frac{17}{2}$ (B) $\frac{17}{3}$

(C) $\frac{17}{4}$ (D) $\frac{17}{5}$

Ans. (B)

3. Let $k \in \mathbb{R}$. If $\lim\limits_{x\to 0^+}(\sin(\sin kx)+\cos x+x)^{\frac{2}{x}}=e^6$, then the value of k is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (B)

4. Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Then which of the following statements is TRUE ?

(A) $f(x) = 0$ has infinitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty\right)$.

(B) $f(x) = 0$ has no solutions in the interval $\left[\frac{1}{\pi}, \infty\right)$.

(C) The set of solutions of $f(x) = 0$ in the interval $\left(0, \frac{1}{10^{10}}\right)$ is finite.

(D) $f(x) = 0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$.

Ans. (D)

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

5. Let S be the set of all $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$ such that

$$\lim_{x \to \infty} \frac{\sin(x^2)(\log_e x)^{\alpha} \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^{\beta}} = 0 .$$

Then which of the following is (are) correct ?

(A) $(-1, 3) \in S$ (B) $(-1, 1) \in S$

(C) $(1, -1) \in S$ (D) $(1, -2) \in S$

Ans. **(B,C)**

6. A straight line drawn from the point $P(1, 3, 2)$, parallel to the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$, intersects the plane $L_1 : x - y + 3z = 6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2 : 2x - y + z = -4$ at the point R.

Then which of the following statements is (are) TRUE?

(A) The length of the line segment PQ is $\sqrt{6}$

(B) The coordinates of R are $(1, 6, 3)$

(C) The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(D) The perimeter of the triangle PQR is $\sqrt{2} + \sqrt{6} + \sqrt{11}$

Ans. **(A,C)**

7. Let A_1, B_1, C_1 be three points in the xy-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1, respectively. If $O = (0, 0)$ and $C_1 = (-4, 0)$, then which of the following statements is (are) TRUE?

(A) The length of the line segment OA_1 is $4\sqrt{3}$

(B) The length of the line segment A_1B_1 is 16

(C) The orthocenter of the triangle $A_1B_1C_1$ is $(0, 0)$

(D) The orthocenter of the triangle $A_1B_1C_1$ is $(1, 0)$

Ans. **(A,C)**

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

8. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, and $g: \mathbb{R} \to (0, \infty)$ be a function such that $g(x+y) = g(x)\, g(y)$ for all $x, y \in \mathbb{R}$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right) g(0)$ is _____ .

Ans. **(51)**

9. A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For $i = 1, 2, 3$, let W_i, G_i, and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball, and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 \mid W_1 \cap G_2) = \frac{2}{9}$, then N equals _____ .

Ans. **(11)**

10. Let the function $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{\sin x}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)} + \frac{2}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)}.$$

Then the number of solutions of $f(x) = 0$ in $\mathbb{R}$ is _____.

Ans. **(1)**

11. Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α, β and γ, we have

$$15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q}),$$

then the value of γ is _____ .

Ans. **(2)**

12. A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0, -\alpha)$ to the parabola $x^2 = -4ay$, where $a > 0$. Let L be the line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola.

Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If $r : s = 1 : 16$, then the value of $24a$ is _____.

Ans. **(12)**

13. Let the function $f: [1, \infty) \to \mathbb{R}$ be defined by

$$f(t) = \begin{cases} (-1)^{n+1} 2, & \text{if } t = 2n-1,\ n \in \mathbb{N}, \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1), & \text{if } 2n-1 < t < 2n+1,\ n \in \mathbb{N} \end{cases}$$

Define $g(x) = \int_1^x f(t)dt$, $x \in (1, \infty)$. Let α denote the number of solutions of the equation $g(x) = 0$ in the interval $(1, 8]$ and $\beta = \lim\limits_{x \to 1^+} \frac{g(x)}{x-1}$. Then the value of $\alpha + \beta$ is equal to _____.

Ans. **(5)**

SECTION-4 : (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

"PARAGRAPH I"

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties.

i. R has exactly 6 elements.

ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$ and

$Z = \{R \in X : R \text{ is a function from } S \text{ to } S.\}$

Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

14. If $n(X) = {}^{m}C_{6}$, then the value of m is _____.

Ans. **(20.00)**

"PARAGRAPH I"

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties.

i. R has exactly 6 elements.

ii. For each (a, b) $\in$ R, we have $|a - b| \geq 2$.

Let $Y = \{R \in X : \text{The range of R has exactly one element}\}$ and

$Z = \{R \in X : R \text{ is a function from } S \text{ to } S.\}$

Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

15. If the value of $n(Y) + n(Z)$ is k^2, then $|k|$ is _____.

Ans. **(36.00)**

"PARAGRAPH II"

Let $f: \left[0, \frac{\pi}{2}\right] \to [0,1]$ be the function defined by $f(x) = \sin^2 x$ and let $g: \left[0, \frac{\pi}{2}\right] \to [0,\infty)$ be the function defined by $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

16. The value of $2\int_0^{\frac{\pi}{2}} f(x)g(x)dx - \int_0^{\frac{\pi}{2}} g(x)dx$ is _____.

Ans. (0.00)

"PARAGRAPH II"

Let $f: \left[0, \frac{\pi}{2}\right] \to [0,1]$ be the function defined by $f(x) = \sin^2 x$ and let $g: \left[0, \frac{\pi}{2}\right] \to [0,\infty)$ be the function defined by $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

17. The value of $\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx$ is _____.

Ans. (0.25)

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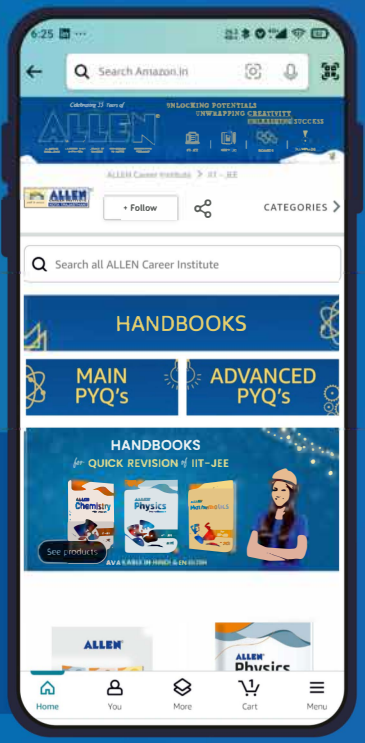

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