Mathematics Section A

Section Id :	864351918
Section Number :	5
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	20
Number of Questions to be attempted :	20
Section Marks :	80
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Sub-Section Number :	1
Sub-Section Id :	8643511145
Question Shuffling Allowed :	Yes

Question Number : 61 Question Id : 86435120230 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The domain of the function $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

Options :

86435167271. $\left[-\frac{1}{2}, \infty\right) - \{0\}$

86435167272. $\left(-\frac{1}{2}, \infty\right) - \{0\}$

86435167273. $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

86435167274. $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$

Question Number : 62 Question Id : 86435120231 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

If $(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation :

Options :

86435167275. $x^2 + (\sqrt{3}-1)x - \sqrt{3} = 0$

86435167276. $x^2 - (\sqrt{3}-1)x - \sqrt{3} = 0$

86435167277. $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

86435167278. $x^2 + (\sqrt{3}+1)x + \sqrt{3} = 0$

Question Number : 63 Question Id : 86435120232 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$$x + y + z = 5$$
$$x + 2y + 3z = \mu$$
$$x + 3y + \lambda z = 1$$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then :

Options :

86435167279. $p = \frac{1}{6}$ and $q = \frac{1}{36}$

86435167280. $p = \frac{5}{6}$ and $q = \frac{1}{36}$

86435167281. $p = \frac{1}{6}$ and $q = \frac{5}{36}$

86435167282. $p = \frac{5}{6}$ and $q = \frac{5}{36}$

Question Number : 64 Question Id : 86435120233 Question Type : MCQ Option Shuffling : Yes Is Correct Marks : 4 Wrong Marks : 1

Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to :

Options :

86435167283. A^6

86435167284. $A^5 - A$

86435167285. $A^6 - A$

86435167286. A^5

Question Number : 65 Question Id : 86435120234 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

$\lim\limits_{x \to 2} \left(\sum\limits_{n=1}^{9} \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4} \right)$ is equal to :

Options :

86435167287. $\frac{7}{36}$

86435167288. $\frac{1}{5}$

86435167289. $\frac{9}{44}$

86435167290. $\frac{5}{24}$

Question Number : 66 Question Id : 86435120235 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The local maximum value of the function

$$f(x) = \left(\frac{2}{x}\right)^{x^2}, x > 0, \text{ is :}$$

Options :

86435167291. $(2\sqrt{e})^{\frac{1}{e}}$

86435167292. $(e)^{\frac{2}{e}}$

86435167293. $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

86435167294. 1

Question Number : 67 Question Id : 86435120236 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right) dx$ is :

Options :

86435167295. $\frac{3\pi}{2}$

86435167296. $\frac{3\pi}{4}$

86435167297. $\frac{\pi}{2}$

86435167298. $\frac{5\pi}{4}$

Question Number : 68 Question Id : 86435120237 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

Let $y(x)$ be the solution of the differential equation $2x^2\,dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$, then $y(1)$ is equal to :

Options :

86435167299. 2

86435167300. $\log_e(2e)$

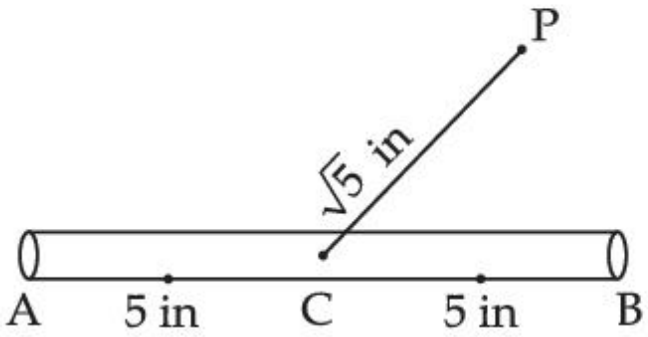
86435167301. $\log_e 2$

86435167302. 0

Question Number : 69 Question Id : 86435120238 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$.

The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :



Options :

86435167303. $\tan^{-1}\left(\frac{4}{3}\right)$

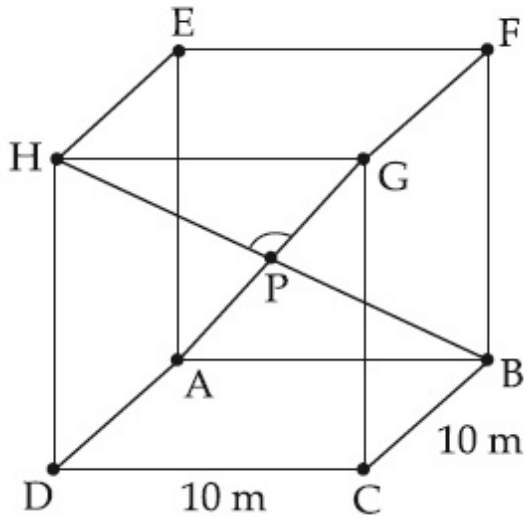
86435167304. $\tan^{-1}\left(\frac{3}{4}\right)$

86435167305. $\tan^{-1}(1)$

86435167306. $\tan^{-1}\left(\frac{1}{2}\right)$

Question Number : 70 Question Id : 86435120239 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

A hall has a square floor of dimension 10 m × 10 m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :



Options :

86435167307. $5\sqrt{2}$

86435167308. $5\sqrt{3}$

86435167309. 5

86435167310. $2\sqrt{10}$

Question Number : 71 Question Id : 86435120240 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

A circle C touches the line $x=2y$ at the point (2, 1) and intersects the circle $C_1 : x^2+y^2+2y-5=0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

Options :

86435167311. 15

86435167312. $4\sqrt{15}$

86435167313. $\sqrt{285}$

86435167314. $7\sqrt{5}$

Question Number : 72 Question Id : 86435120241 Question Type : MCQ Option Shuffling : Yes Is

Correct Marks : 4 Wrong Marks : 1

The point $P(-2\sqrt{6}, \sqrt{3})$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the points Q and R respectively, then QR is equal to :

Options :

86435167315. $4\sqrt{3}$

86435167316. $3\sqrt{6}$

86435167317. 6

86435167318. $6\sqrt{3}$

Question Number : 73 Question Id : 86435120242 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is :

Options :

86435167319. $x^3(x-2) = y^2$

86435167320. $x^2(x-2) = y^3$

86435167321. $y^2(x-2)=x^3$

86435167322. $y^3(x-2)=x^2$

Question Number : 74 Question Id : 86435120243 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}\,dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in \mathbf{R}$, $5\alpha+6\beta=0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha+\beta)^2$ is equal to :

Options :

86435167323. 25

86435167324. 36

86435167325. 100

86435167326. 16

Question Number : 75 Question Id : 86435120244 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

Consider the two statements :

(S1) : $(p \rightarrow q) \vee (\sim q \rightarrow p)$ is a tautology.

(S2) : $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

Then :

Options :

86435167327. only (S1) is true.

86435167328. only (S2) is true.

86435167329. both (S1) and (S2) are false.

86435167330. both (S1) and (S2) are true.

Question Number : 76 Question Id : 86435120245 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

Let [t] denote the greatest integer less than or equal to t.

Let $f(x) = x - [x]$, $g(x) = 1 - x + [x]$, and $h(x) = \min\{f(x), g(x)\}$, $x \in [-2, 2]$.

Then h is :

Options :

86435167331. not continuous at exactly four points in $[-2, 2]$

86435167332. not continuous at exactly three points in $[-2, 2]$

86435167333. continuous in $[-2, 2]$ but not differentiable at more than four points in $(-2, 2)$

86435167334. Continous in $[-2, 2]$ but not differentiable at exactly three poionts in $(-2, 2)$

Question Number : 77 Question Id : 86435120246 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

If $\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$, then the value of tan p is :

Options :

86435167335. $\frac{50}{51}$

86435167336. $\frac{51}{50}$

86435167337. $\frac{101}{102}$

86435167338. 100

Question Number : 78 Question Id : 86435120247 Question Type : MCQ Option Shuffling : Yes Is Correct Marks : 4 Wrong Marks : 1

A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5 \mid X > 2)$ is :

Options :

86435167339. $\frac{11}{36}$

86435167340. $\frac{25}{36}$

86435167341. $\frac{5}{6}$

86435167342. $\frac{125}{216}$

Question Number : 79 Question Id : 86435120248 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes $\vec{r} \cdot \left(\hat{i} + \hat{j} + 4\hat{k}\right) = 16$ and $\vec{r} \cdot \left(-\hat{i} + \hat{j} + \hat{k}\right) = 6$.

Then which of the following points does **NOT** lie on P ?

Options :

86435167343. $(-8, 8, 6)$

86435167344. $(6, -6, 2)$

86435167345. $(4, 2, 2)$

86435167346. $(3, 3, 2)$

Question Number : 80 Question Id : 86435120249 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right) \text{ is :}$$

Options :

86435167347. $\frac{1}{8}$

86435167348. $\frac{1}{8\sqrt{2}}$

86435167349. $\frac{1}{4}$

86435167350. $\frac{1}{4\sqrt{2}}$

Mathematics Section B

Section Id :	864351919
Section Number :	6
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	10
Number of Questions to be attempted :	5
Section Marks :	20
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Sub-Section Number :	1
Sub-Section Id :	8643511146
Question Shuffling Allowed :	Yes

Question Number : 81 Question Id : 86435120250 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let $\lambda \neq 0$ be in **R**. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :
1

Question Number : 82 Question Id : 86435120251 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let A be a 3×3 real matrix. If $\det(2\ \text{Adj}(2\ \text{Adj}(\text{Adj}\ (2A))))=2^{41}$, then the value of $\det(A^2)$ equals _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :
1

Question Number : 83 Question Id : 86435120252 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let $\binom{n}{k}$ denote ${}^{n}C_{k}$ and

$$\left[\begin{matrix}n\\k\end{matrix}\right]=\begin{cases}\binom{n}{k}, & \text{if } 0\le k\le n\\ 0, & \text{otherwise.}\end{cases}$$

If $A_k=\sum_{i=0}^{9}\binom{9}{i}\left[\begin{matrix}12\\12-k+i\end{matrix}\right]+\sum_{i=0}^{8}\binom{8}{i}\left[\begin{matrix}13\\13-k+i\end{matrix}\right]$ and $A_4-A_3=190\ p$, then p is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :
1

Question Number : 84 Question Id : 86435120253 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :
1

Question Number : 85 Question Id : 86435120254 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let a and b respectively be the points of local maximum and local minimum of the function

$$f(x) = 2x^3 - 3x^2 - 12x.$$

If A is the total area of the region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, then 4 A is equal to _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText

Possible Answers :

1

Question Number : 86 Question Id : 86435120255 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let the mean and variance of four numbers 3, 7, x and y $(x > y)$ be 5 and 10 respectively. Then the mean of four numbers $3+2x$, $7+2y$, $x+y$ and $x-y$ is _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Number : 87 Question Id : 86435120256 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Number : 88 Question Id : 86435120257 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}$, $i = \sqrt{-1}$, is a positive integer, is _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :
1

Question Number : 89 Question Id : 86435120258 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let Q be the foot of the perpendicular from the point P(7, −2, 13) on the plane containing the lines $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$.

Then $(PQ)^2$, is equal to _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :
1

Question Number : 90 Question Id : 86435120259 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let $a_1, a_2, ..., a_{10}$ be an AP with common difference -3 and $b_1, b_2, ..., b_{10}$ be a GP with common ratio 2. Let $c_k = a_k + b_k$, $k = 1, 2, ..., 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :
1