Mathematics Section A

Section Id :	708191632
Section Number :	5
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	20
Number of Questions to be attempted :	20
Section Marks :	80
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	708191912
Question Shuffling Allowed :	Yes

Question Number : 61 Question Id : 70819116384 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

Let $a, b \in \mathbf{R}$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to :

Options :

70819154301. 84

70819154302. 86

70819154303. 88

70819154304. 90

Question Number : 62 Question Id : 70819116385 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), $(2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is :

Options :

70819154305. $-\frac{69}{256}$

70819154306. $-\frac{71}{256}$

70819154307. $\frac{69}{256}$

70819154308. $\frac{71}{256}$

Question Number : 63 Question Id : 70819116386 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

If P is a point on the parabola $y=x^2+4$ which is closest to the straight line $y=4x-1$, then the co-ordinates of P are :

Options :

70819154309. $(3, 13)$

70819154310. $(2, 8)$

70819154311. $(-2, 8)$

70819154312. $(1, 5)$

Question Number : 64 Question Id : 70819116387 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

For which of the following curves, the line $x+\sqrt{3}y=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

Options :

70819154313. $x^2+y^2=7$

70819154314. $x^2+9y^2=9$

70819154315. $y^2=\frac{1}{6\sqrt{3}}x$

70819154316. $2x^2-18y^2=9$

Question Number : 65 Question Id : 70819116388 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The value of the integral, $\int_1^3[x^2-2x-2]dx$, where $[x]$ denotes the greatest integer less than or equal to x, is :

Options :

70819154317. -5

70819154318. -4

70819154319. $-\sqrt{2}-\sqrt{3}-1$

70819154320. $-\sqrt{2}-\sqrt{3}+1$

Question Number : 66 Question Id : 70819116389 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The negation of the statement

$\sim p\wedge(p\vee q)$ is :

Options :

70819154321. $\sim p \vee q$

70819154322. $\sim p \wedge q$

70819154323. $p \wedge \sim q$

70819154324. $p \vee \sim q$

Question Number : 67 Question Id : 70819116390 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)\,X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

Options :

70819154325. no solution

70819154326. a unique solution

70819154327. exactly two solutions

70819154328. infinitely many solutions

Question Number : 68 Question Id : 70819116391 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

Let $f(x)$ be a differentiable function defined on $[0, 2]$ such that $f'(x) = f'(2-x)$ for all $x \in (0, 2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)\,dx$ is :

Options :

70819154329. $1 - e^2$

70819154330. $1 + e^2$

70819154331. $2(1 + e^2)$

70819154332. $2(1 - e^2)$

Question Number : 69 Question Id : 70819116392 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

Let f be a twice differentiable function defined on $\mathbf{R}$ such that $f(0) = 1$, $f'(0) = 2$ and $f'(x) \neq 0$ for all $x \in \mathbf{R}$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in \mathbf{R}$, then the value of $f(1)$ lies in the interval :

Options :

70819154333. $(0, 3)$

70819154334. $(3, 6)$

70819154335. $(6, 9)$

70819154336. $(9, 12)$

Question Number : 70 Question Id : 70819116393 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

If the curve $y = ax^2 + bx + c,\ x \in \mathbf{R}$, passes through the point (1, 2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are :

Options :

70819154337. $a = 1,\ b = 1,\ c = 0$

70819154338. $a = 1,\ b = 0,\ c = 1$

70819154339. $a = -1,\ b = 1,\ c = 1$

70819154340. $a = \frac{1}{2},\ b = \frac{1}{2},\ c = 1$

Question Number : 71 Question Id : 70819116394 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

If $n \geq 2$ is a positive integer, then the sum of the series

$${}^{n+1}C_2 + 2\left({}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + \ldots + {}^{n}C_2\right) \text{ is :}$$

Options :

70819154341. $\frac{n(n-1)(2n+1)}{6}$

70819154342. $\frac{n(n+1)(2n+1)}{6}$

70819154343. $\frac{n(2n+1)(3n+1)}{6}$

70819154344. $\frac{n(n+1)^2(n+2)}{12}$

Question Number : 72 Question Id : 70819116395 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

For the system of linear equations :

$x-2y=1,\ x-y+kz=-2,\ ky+4z=6,\ k\in \mathbf{R}$,

consider the following statements :

(A) The system has unique solution if $k\neq 2, k\neq -2$.

(B) The system has unique solution if $k=-2$.

(C) The system has unique solution if $k=2$.

(D) The system has no-solution if $k=2$.

(E) The system has infinite number of solutions if $k\neq -2$.

Which of the following statements are correct ?

Options :

70819154345. (A) and (D) only

70819154346. (A) and (E) only

70819154347. (C) and (D) only

70819154348. (B) and (E) only

Question Number : 73 Question Id : 70819116396 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let $A = \{x \in \mathbf{R} : f \text{ is increasing}\}$. Then A is equal to :

Options :

70819154349. $(-\infty, -5) \cup (4, \infty)$

70819154350. $(-5, -4) \cup (4, \infty)$

70819154351. $(-\infty, -5) \cup (-4, \infty)$

70819154352. $(-5, \infty)$

Question Number : 74 Question Id : 70819116397 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

Options :

70819154353. $\frac{1}{\sqrt{7}}$

70819154354. $\frac{1}{2\sqrt{2}}$

70819154355. $\sqrt{7} - 1$

70819154356. $2\sqrt{2} - 1$

Question Number : 75 Question Id : 70819116398 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

Options :

70819154357. $3600\sqrt{3}$ m

70819154358. $2400\sqrt{3}$ m

70819154359. $1800\sqrt{3}$ m

70819154360. $1200\sqrt{3}$ m

Question Number : 76 Question Id : 70819116399 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $\vec{r}\cdot\left(\hat{i} - 2\hat{j}\right) = -2$, and the point (1, 0, 2) is :

Options :

70819154361. $\vec{r}\cdot\left(3\hat{i}+7\hat{j}+3\hat{k}\right)=7$

70819154362. $\vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

70819154363. $\vec{r}\cdot\left(\hat{i}-7\hat{j}+3\hat{k}\right)=\frac{7}{3}$

70819154364. $\vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=\frac{7}{3}$

Question Number : 77 Question Id : 70819116400 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct ?

Options :

70819154365. (a) is a tautology but not (b).

70819154366. (b) is a tautology but not (a).

70819154367. (a) and (b) both are tautologies.

70819154368. (a) and (b) both are not tautologies.

Question Number : 78 Question Id : 70819116401 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

If a curve $y=f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

Options :

70819154369. 10

70819154370. $\frac{31}{5}$

70819154371. 5

70819154372. $\frac{62}{5}$

Question Number : 79 Question Id : 70819116402 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

Options :

70819154373. $\frac{65}{2^7}$

70819154374. $\frac{65}{2^8}$

70819154375. $\frac{135}{2^9}$

70819154376. $\frac{35}{2^7}$

Question Number : 80 Question Id : 70819116403 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

The area of the region : $R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$ is :

Options :

70819154377. $6\sqrt{3}$ square units

70819154378. $9\sqrt{3}$ square units

70819154379. $11\sqrt{3}$ square units

70819154380. $12\sqrt{3}$ square units

Mathematics Section B

Section Id :	708191633
Section Number :	6
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	10
Number of Questions to be attempted :	5

Section Marks :	20
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	708191913
Question Shuffling Allowed :	Yes

Question Number : 81 Question Id : 70819116404 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 82 Question Id : 70819116405 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point $(-5, 0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 83 Question Id : 70819116406 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5, 7) is A, then 24A is equal to _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 84 Question Id : 70819116407 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let $i=\sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}}+\frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}}=k$, and $n=[|k|]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 85 Question Id : 70819116408 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 86 Question Id : 70819116409 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$$ exists, is equal to _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 87 Question Id : 70819116410 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

If $a+\alpha=1$, $b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}$, $x\neq 0$, then the value of the expression

$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is_____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 88 Question Id : 70819116411 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

Let λ be an integer. If the shortest distance between the lines $x-\lambda=2y-1=-2z$ and $x=y+2\lambda=z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 89 Question Id : 70819116412 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001

Question Number : 90 Question Id : 70819116413 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is _____.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
5 to 5.001