Mathematics Section A

Section Id :	708191950
Section Number :	5
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	20
Number of Questions to be attempted :	20
Section Marks :	80
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	7081911230
Question Shuffling Allowed :	Yes

Question Number : 61 Question Id : 70819121154 Question Type : MCQ Option Shuffling : Yes

Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If the mirror image of the point $(1, 3, 5)$ with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals :

Options :

70819168611. 39

70819168612. 41

70819168613. 43

70819168614. 47

Question Number : 62 Question Id : 70819121155 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let A = {1, 2, 3, ..., 10} and $f: A \to A$ be defined as

$$f(k) = \begin{cases} k+1 & \text{if k is odd} \\ k & \text{if k is even} \end{cases}$$

Then the number of possible functions $g: A \to A$ such that $gof = f$ is :

Options :

70819168615. $5!$

70819168616. ${}^{10}C_5$

70819168617. 5^5

70819168618. 10^5

Question Number : 63 Question Id : 70819121156 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let A_1 be the area of the region bounded by the curves $y=\sin x$, $y=\cos x$ and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y=\sin x$, $y=\cos x$, x-axis and $x=\frac{\pi}{2}$ in the first quadrant. Then,

Options :

70819168619. $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

70819168620. $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

70819168621. $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

70819168622. $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

Question Number : 64 Question Id : 70819121157 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If $0 < a, b < 1$, and $\tan^{-1}a + \tan^{-1} b = \frac{\pi}{4}$, then the value of

$$(a + b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots \text{ is :}$$

Options :

70819168623. e

70819168624. $e^2 - 1$

70819168625. $\log_e 2$

70819168626. $\log_e\left(\frac{e}{2}\right)$

Question Number : 65 Question Id : 70819121158 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let slope of the tangent line to a curve at any point $P(x, y)$ be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point $(3, y)$ lies on the curve, is :

Options :

70819168627. $-\frac{4}{3}$

70819168628. $-\frac{18}{19}$

70819168629. $\frac{18}{35}$

70819168630. $-\frac{18}{11}$

Question Number : 66 Question Id : 70819121159 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to :

Options :

70819168631. $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

70819168632. $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

70819168633. $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

70819168634. $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$

Question Number : 67 Question Id : 70819121160 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $f(x)=\int_0^x e^t f(t)dt+e^x$ be a differentiable function for all $x \in R$. Then $f(x)$ equals :

Options :

70819168635. $2e^{(e^x-1)}-1$

70819168636. $e^{(e^x-1)}$

70819168637. $e^{e^x}-1$

70819168638. $2e^{e^x}-1$

Question Number : 68 Question Id : 70819121161 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $f(x)$ be a differentiable function at $x=a$ with $f'(a)=2$ and $f(a)=4$. Then $\lim\limits_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals :

Options :

70819168639. $2a-4$

70819168640. $4-2a$

70819168641. $2a+4$

70819168642. $a+4$

Question Number : 69 Question Id : 70819121162 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $f(x)=\sin^{-1}x$ and $g(x)=\frac{x^2-x-2}{2x^2-x-6}$. If $g(2)=\lim\limits_{x\to 2} g(x)$, then the domain of the function fog is :

Options :

70819168643. $(-\infty,-2]\cup\left[-\frac{3}{2},\infty\right)$

70819168644. $(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$

70819168645. $(-\infty,-1]\cup[2,\infty)$

70819168646. $(-\infty,-2]\cup[-1,\infty)$

Question Number : 70 Question Id : 70819121163 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let A(1, 4) and B(1, −5) be two points. Let P be a point on the circle $(x-1)^2+(y-1)^2=1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points, P, A and B lie on :

Options :

70819168647. an ellipse

70819168648. a hyperbola

70819168649. a parabola

70819168650. a straight line

Question Number : 71 Question Id : 70819121164 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If vectors $\vec{a_1} = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a_2} = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

Options :

70819168651. $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$

70819168652. $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

70819168653. $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$

70819168654. $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$

Question Number : 72 Question Id : 70819121165 Question Type : MCQ Option Shuffling : Yes

Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then :

Options :

70819168655. F_1 and F_2 both are tautologies

70819168656. F_1 is a tautology but F_2 is not a tautology

70819168657. F_1 is not a tautology but F_2 is a tautology

70819168658. Both F_1 and F_2 are not tautologies

Question Number : 73 Question Id : 70819121166 Question Type : MCQ Option Shuffling : Yes

Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

Options :

70819168659. $\frac{3}{7}$

70819168660. $\frac{6}{7}$

70819168661. $\frac{1}{7}$

70819168662. $\frac{4}{7}$

Question Number : 74 Question Id : 70819121167 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Consider the following system of equations :

$$x + 2y - 3z = a$$
$$2x + 6y - 11z = b$$
$$x - 2y + 7z = c,$$

where a, b and c are real constants. Then the system of equations :

Options :

70819168663. has a unique solution for all a, b and c

70819168664. has a unique solution when $5a = 2b + c$

70819168665. has infinite number of solutions when $5a = 2b + c$

70819168666. has no solution for all a, b and c

Question Number : 75 Question Id : 70819121168 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The triangle of maximum area that can be inscribed in a given circle of radius 'r' is :

Options :

70819168667. An isosceles triangle with base equal to 2r.

70819168668. A right angle triangle having two of its sides of length 2r and r.

70819168669. An equilateral triangle of height $\frac{2r}{3}$.

70819168670. An equilateral triangle having each of its side of length $\sqrt{3}\,r$.

Question Number : 76 Question Id : 70819121169 Question Type : MCQ Option Shuffling : Yes

Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let L be a line obtained from the intersection of two planes $x+2y+z=6$ and $y+2z=4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from $(3, 2, 1)$ on L, then the value of $21(\alpha+\beta+\gamma)$ equals :

Options :

70819168671. 68

70819168672. 102

70819168673. 136

70819168674. 142

Question Number : 77 Question Id : 70819121170 Question Type : MCQ Option Shuffling : Yes

Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x), & \text{if } x > 1 \end{cases}$$

If $f(x)$ is continuous on $\mathbf{R}$, then $a+b$ equals :

Options :

70819168675. -3

70819168676. -1

70819168677. 1

70819168678. 3

Question Number : 78 Question Id : 70819121171 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2+y^2=1$ is a circle of radius r, then r is equal to :

Options :

70819168679. $\frac{1}{4}$

70819168680. $\frac{1}{3}$

70819168681. $\frac{1}{2}$

70819168682. 1

Question Number : 79 Question Id : 70819121172 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

A natural number has prime factorization given by $n=2^x3^y5^z$, where y and z are such that $y+z=5$ and $y^{-1}+z^{-1}=\frac{5}{6}$, $y>z$. Then the number of odd divisors of n, including 1, is :

Options :

70819168683. 6

70819168684. 11

70819168685. 12

70819168686. $6x$

Question Number : 80 Question Id : 70819121173 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to :

Options :

70819168687. 0

70819168688. 1

70819168689. $\frac{1}{2}$

70819168690. -1

Mathematics Section B

Section Id :	708191951
Section Number :	6
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	10

Number of Questions to be attempted : 5

Section Marks : 20

Mark As Answered Required? : Yes

Sub-Section Number : 1

Sub-Section Id : 7081911231

Question Shuffling Allowed : Yes

Question Number : 81 Question Id : 70819121174 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, for $m, n \geq 1$, and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx = \alpha\, I_{m,n}$, $\alpha \in R$, then α equals _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 82 Question Id : 70819121175 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let z be those complex numbers which satisfy

$|z+5| \leq 4$ and $z(1+i) + \bar{z}(1-i) \geq -10, i = \sqrt{-1}$.

If the maximum value of $|z+1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 83 Question Id : 70819121176 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through $(3, -3)$ and $(4, -2\sqrt{2})$, and given that $a - 2\sqrt{2}\,b = 3$, then $(a^2 + b^2 + ab)$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 84 Question Id : 70819121177 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let a be an integer such that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval $(a, a+1)$.
Then, $|a|$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 85 Question Id : 70819121178 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let $X_1, X_2, \ldots, X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i - \alpha) = 36$ and $\sum_{i=1}^{18}(X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 86 Question Id : 70819121179 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 87 Question Id : 70819121180 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of p_n^2 is _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 88 Question Id : 70819121181 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 89 Question Id : 70819121182 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence $-16, 8, -4, 2, \ldots$ satisfy the equation $4x^2-9x+5=0$, then $p+q$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001

Question Number : 90 Question Id : 70819121183 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let L be a common tangent line to the curves $4x^2+9y^2=36$ and $(2x)^2+(2y)^2=31$. Then the square of the slope of the line L is _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

5 to 5.001