

Mathematics Section A

Section Id :	864351828
Section Number :	5
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	20
Number of Questions to be attempted :	20
Section Marks :	80
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Sub-Section Number :	1
Sub-Section Id :	8643511055
Question Shuffling Allowed :	Yes

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let P and Q be two distinct points on a circle which has center at $C(2, 3)$ and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set $\{P, Q\}$ is equal to :

Options :

1. $\{(2 + 2\sqrt{2}, 3 + \sqrt{5}), (2 - 2\sqrt{2}, 3 - \sqrt{5})\}$

2. $\{(2 + 2\sqrt{2}, 3 - \sqrt{5}), (2 - 2\sqrt{2}, 3 + \sqrt{5})\}$

3. $\{(-1, 5), (5, 1)\}$

4. $\{(4, 0), (0, 6)\}$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product

$(\vec{a} + \vec{b}) \times \left(\left(\vec{a} \times \left((\vec{a} - \vec{b}) \times \vec{b} \right) \right) \times \vec{b} \right)$ is equal to :

Options :

1. $5(30\hat{i} - 5\hat{j} + 7\hat{k})$

2. $7(30\hat{i} - 5\hat{j} + 7\hat{k})$

3. $5(34\hat{i} - 5\hat{j} + 3\hat{k})$

4. $7(34\hat{i} - 5\hat{j} + 3\hat{k})$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If the coefficients of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x - \frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the

value of b is equal to :

Options :

1. -1
2. 2
3. -2
4. 1

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If the area of the bounded region

$$R = \left\{ (x, y) : \max\{0, \log_e x\} \leq y \leq 2^x, \frac{1}{2} \leq x \leq 2 \right\}$$

is, $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of $(\alpha + \beta - 2\gamma)^2$ is equal to :

Options :

1. 1
2. 2
3. 4
4. 8

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in \mathbf{R}$, I is a 2×2 identity matrix, then $4(\alpha - \beta)$ is

equal to :

Options :

1. 2
2. 4
3. 5
4. $\frac{8}{3}$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Two tangents are drawn from the point $P(-1, 1)$ to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B , and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to :

Options :

1. 2
2. 4
3. $(3\sqrt{2} + 2)$

4. $3(\sqrt{2} - 1)$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let \mathbf{C} be the set of all complex numbers. Let

$$S_1 = \{z \in \mathbf{C} \mid |z - 3 - 2i|^2 = 8\},$$

$$S_2 = \{z \in \mathbf{C} \mid \operatorname{Re}(z) \geq 5\} \text{ and}$$

$$S_3 = \{z \in \mathbf{C} \mid |z - \bar{z}| \geq 8\}.$$

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to :

Options :

1. 0
2. 1
3. 2
4. Infinite

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let the plane passing through the point $(-1, 0, -2)$ and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of $a + b + c$ is equal to :

Options :

1. 5
2. 3

3. 4

4. 8

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let α, β be two roots of the equation $x^2 + (20)^{\frac{1}{4}}x + (5)^{\frac{1}{2}} = 0$. Then $\alpha^8 + \beta^8$ is equal to :

Options :

1. 100

2. 10

3. 50

4. 160

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $f : \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \rightarrow \mathbf{R}$ be defined as

$$f(x) = \begin{cases} (1 + |\sin x|)^{\frac{3a}{|\sin x|}}, & -\frac{\pi}{4} < x < 0 \\ b, & x = 0 \\ e^{\cot 4x / \cot 2x}, & 0 < x < \frac{\pi}{4} \end{cases}$$

If f is continuous at $x=0$, then the value of $6a + b^2$ is equal to :

Options :

1. $1 + e$
2. $1 - e$
3. e
4. $e - 1$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let

$$A = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 2x^2 + 2y^2 - 2x - 2y = 1\},$$

$$B = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 4x^2 + 4y^2 - 16y + 7 = 0\} \text{ and}$$

$$C = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x^2 + y^2 - 4x - 2y + 5 \leq r^2\}.$$

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to :

Options :

1. $\frac{3 + \sqrt{10}}{2}$

2. $1 + \sqrt{5}$

3. $\frac{2 + \sqrt{10}}{2}$

4. $\frac{3 + 2\sqrt{5}}{2}$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If the mean and variance of the following data :

6, 10, 7, 13, a, 12, b, 12

are 9 and $\frac{37}{4}$ respectively, then $(a - b)^2$ is equal to :

Options :

1. 16

2. 12

3. 24

4. 32

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $y=y(x)$ be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with $y(0)=0$.

If $y\left(-\frac{2}{3}\log_e 2\right) = \alpha \log_e 2$, then the value of α is equal to :

Options :

1. $-\frac{1}{4}$

2. $\frac{1}{4}$

3. 2

4. $-\frac{1}{2}$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to :

Options :

1. 23

2. -23

3. 27

4. -27

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The probability that a randomly selected 2-digit number belongs to the set $\{n \in \mathbb{N} : (2^n - 2) \text{ is a multiple of } 3\}$ is equal to :

Options :

1. $\frac{1}{2}$

2. $\frac{1}{3}$

3. $\frac{2}{3}$

4. $\frac{1}{6}$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

A ray of light through $(2, 1)$ is reflected at a point P on the y -axis and then passes through the point $(5, 3)$. If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the

distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix

can be :

Options :

1. $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$
2. $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$
3. $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$
4. $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to :

Options :

1. $\sim(P \Rightarrow Q)$
2. $P \wedge \sim Q$
3. $\sim(P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$
4. $P \vee Q$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(2) = 4$ and $f'(2) = 1$. Then, the value of

$\lim_{x \rightarrow 2} \frac{x^2 f(2) - 4f(x)}{x - 2}$ is equal to :

Options :

1. 4
2. 8
3. 12
4. 16

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The value of $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \frac{(2j - 1) + 8n}{(2j - 1) + 4n}$ is equal to :

Options :

1. $5 + \log_e \left(\frac{3}{2} \right)$
2. $1 + 2 \log_e \left(\frac{3}{2} \right)$

3. $2 - \log_e \left(\frac{2}{3} \right)$

4. $3 + 2 \log_e \left(\frac{2}{3} \right)$

Question Type : MCQ Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The value of the definite integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1 + e^{x \cos x})(\sin^4 x + \cos^4 x)}$$

is equal to :

Options :

1. $\frac{\pi}{2\sqrt{2}}$

2. $-\frac{\pi}{4}$

3. $-\frac{\pi}{2}$

4. $\frac{\pi}{\sqrt{2}}$

Mathematics Section B

Section Id :	864351829
Section Number :	6
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	10
Number of Questions to be attempted :	5
Section Marks :	20
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Sub-Section Number :	1
Sub-Section Id :	8643511056
Question Shuffling Allowed :	Yes

Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let a plane P pass through the point $(3, 7, -7)$ and contain the line,

$\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to

_____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 Wrong Marks : 0

$$\text{Let } f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}, x \in [0, \pi].$$

Then the maximum value of $f(x)$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let $F : [3, 5] \rightarrow \mathbb{R}$ be a twice differentiable function on $(3, 5)$ such that

$$F(x) = e^{-x} \int_3^x (3t^2 + 2t + 4F'(t)) dt.$$

If $F'(4) = \frac{\alpha e^\beta - 224}{(e^\beta - 4)^2}$, then $\alpha + \beta$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, \vec{b} and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and

$\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector \vec{b} on the vector $\vec{a} \times \vec{c}$ is l , then the value of $3l^2$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let the domain of the function

$$f(x) = \log_4 \left(\log_5 \left(\log_3 \left(18x - x^2 - 77 \right) \right) \right) \text{ be } (a, b).$$

Then the value of the integral

$$\int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a + b - x))} dx$$

is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 Wrong Marks : 0

If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 Wrong Marks : 0

For real numbers α and β , consider the following system of linear equations :

$$x + y - z = 2, \quad x + 2y + \alpha z = 1, \quad 2x - y + z = \beta.$$

If the system has infinite solutions, then $\alpha + \beta$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 **Wrong Marks :** 0

Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f: S \rightarrow S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 **Wrong Marks :** 0

If $y = y(x)$, $y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation

$\sec y \frac{dy}{dx} - \sin(x + y) - \sin(x - y) = 0$, with $y(0) = 0$, then $5y'\left(\frac{\pi}{2}\right)$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let $f: [0, 3] \rightarrow \mathbf{R}$ be defined by

$$f(x) = \min\{x - [x], 1 + [x] - x\}$$

where $[x]$ is the greatest integer less than or equal to x .

Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1