

Mathematics Section A

Section Id :	86435111
Section Number :	5
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	20
Number of Questions to be attempted :	20
Section Marks :	80
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	86435111
Question Shuffling Allowed :	Yes

Question Number : 61 Question Id : 864351151 Question Type : MCQ Option Shuffling : Yes Is
Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$. Then, the system of linear equations $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

Options :

864351451. No solution

864351452. A unique solution

864351453. Infinitely many solutions

864351454. Exactly two solutions

Question Number : 62 Question Id : 864351152 Question Type : MCQ Option Shuffling : Yes Is
Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

Let the functions $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} x + 2, & x < 0 \\ x^2, & x \geq 0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x^3, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}$$

Then, the number of points in \mathbb{R} where $(f \circ g)(x)$ is NOT differentiable is equal to :

Options :

864351455. 0

864351456. 1

864351457. 2

864351458. 3

Question Number : 63 Question Id : 864351153 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(-3, -6, 1) and B(2, 4, -3) in ratio k : 1 then the value of k is equal to :

Options :

864351459. 2

864351460. 1.5

864351461. 3

864351462. 4

Question Number : 64 Question Id : 864351154 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to :

Options :

864351463. $\sqrt{31}$

864351464. $\sqrt{66}$

864351465. $\sqrt{41}$

864351466. $\sqrt{55}$

Question Number : 65 Question Id : 864351155 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Consider three observations a, b and c such that $b = a + c$. If the standard deviation of $a + 2, b + 2, c + 2$ is d , then which of the following is true ?

Options :

864351467. $b^2 = 3(a^2 + c^2) - 9d^2$

864351468. $b^2 = 3(a^2 + c^2) + 9d^2$

864351469. $b^2 = a^2 + c^2 + 3d^2$

864351470. $b^2 = 3(a^2 + c^2 + d^2)$

Question Number : 66 Question Id : 864351156 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are $(4, -1, 2)$ and $(-2, 1, -2)$, respectively. Let lines PR and QS intersect at T. If the vector \vec{TA} is perpendicular to both \vec{PR} and \vec{QS} and the length of vector \vec{TA} is $\sqrt{5}$ units, then the modulus of a position vector of A is :

Options :

864351471. $\sqrt{5}$

864351472. $\sqrt{171}$

864351473. $\sqrt{227}$

864351474. $\sqrt{482}$

Question Number : 67 Question Id : 864351157 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β) , $(0, \beta)$ and $(0, 0)$ is equal to :

Options :

864351475. $\frac{1}{2}$

864351476. 1

864351477. $2\sqrt{2}$

864351478. $\frac{1}{\sqrt{2}}$

Question Number : 68 Question Id : 864351158 Question Type : MCQ Option Shuffling : Yes Is
Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

The number of roots of the equation,

$$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$$

in the interval $[0, \pi]$ is equal to :

Options :

864351479. 2

864351480. 3

864351481. 4

864351482. 8

Question Number : 69 Question Id : 864351159 Question Type : MCQ Option Shuffling : Yes Is
Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

Options :

864351483. $\frac{22}{425}$

864351484. $\frac{52}{867}$

864351485. $\frac{39}{50}$

864351486. $\frac{3}{4}$

Question Number : 70 Question Id : 864351160 Question Type : MCQ Option Sh
Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The range of $a \in \mathbb{R}$ for which the function

$$f(x) = (4a - 3)(x + \log_e 5) + 2(a - 7) \cot\left(\frac{x}{2}\right) \sin^2\left(\frac{x}{2}\right), \quad x \neq 2n\pi, n \in \mathbb{N}$$

is :

Options :

864351487. $[1, \infty)$

864351488. $(-\infty, -1]$

864351489. $\left[-\frac{4}{3}, 2\right]$

864351490. $(-3, 1)$

Question Number : 71 Question Id : 864351161 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then $(n - 1)$ is divisible

by :

Options :

864351491. 30

864351492. 8

864351493. 26

864351494. 7

Question Number : 72 Question Id : 864351162 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $[x]$ denote greatest integer less than or equal to x . If for $n \in \mathbb{N}$,

$$(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j, \text{ then}$$

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} \text{ is equal to :}$$

Options :

864351495. 2^{n-1}

864351496. n

864351497. 2

864351498. 1

Question Number : 73 Question Id : 864351163 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Which of the following Boolean expression is a tautology ?

Options :

864351499. $(p \wedge q) \vee (p \vee q)$

864351500. $(p \wedge q) \vee (p \rightarrow q)$

864351501. $(p \wedge q) \wedge (p \rightarrow q)$

864351502. $(p \wedge q) \rightarrow (p \rightarrow q)$

Question Number : 74 Question Id : 864351164 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $S_k = \sum_{r=1}^k \tan^{-1} \left(\frac{6^r}{2^{2r+1} + 3^{2r+1}} \right)$. Then $\lim_{k \rightarrow \infty} S_k$ is equal to :

Options :

864351503. $\frac{\pi}{2}$

864351504. $\cot^{-1} \left(\frac{3}{2} \right)$

864351505. $\tan^{-1} \left(\frac{3}{2} \right)$

864351506. $\tan^{-1} (3)$

Question Number : 75 Question Id : 864351165 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The number of elements in the set $\{x \in \mathbb{R} : (|x| - 3) |x + 4| = 6\}$ is equal to :

Options :

864351507. 1

864351508. 2

864351509. 3

864351510. 4

Question Number : 76 Question Id : 864351166 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If for $x \in \left(0, \frac{\pi}{2} \right)$, $\log_{10} \sin x + \log_{10} \cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2} (\log_{10} n - 1)$, $n > 0$,

then the value of n is equal to :

Options :

864351511. 9

864351512. 12

864351513. 16

864351514. 20

Question Number : 77 Question Id : 864351167 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If $y = y(x)$ is the solution of the differential equation, $\frac{dy}{dx} + 2y \tan x = \sin x$, $y\left(\frac{\pi}{3}\right) = 0$, then

the maximum value of the function $y(x)$ over \mathbb{R} is equal to :

Options :

864351515. 8

864351516. $\frac{1}{2}$

864351517. $-\frac{15}{4}$

864351518. $\frac{1}{8}$

Question Number : 78 Question Id : 864351168 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the

hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

Options :

864351519. $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

864351520. $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

864351521. $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

864351522. $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

Question Number : 79 Question Id : 864351169 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0)$ $a \neq 0$, then 'a' must be greater than :

Options :

864351523. 1

864351524. -1

864351525. $\frac{1}{2}$

864351526. $-\frac{1}{2}$

Question Number : 80 Question Id : 864351170 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let a complex number z , $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}} \left(\frac{|z| + 11}{(|z| - 1)^2} \right) \leq 2$. Then, the largest value of

$|z|$ is equal to _____.

Options :

864351527. 5

864351528. 6

864351529. 7

864351530. 8

Mathematics Section B

Section Id :	86435112
Section Number :	6
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	10
Number of Questions to be attempted :	5
Section Marks :	20
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	86435112
Question Shuffling Allowed :	Yes

Question Number : 81 Question Id : 864351171 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left| \frac{z+i}{z-3i} \right| = 1$ and $\text{Re}(w)$

has minimum value. Then, the minimum value of $n \in \mathbb{N}$ for which w^n is real, is equal to

_____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 82 Question Id : 864351172 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function such that $f(x) + f(x+1) = 2$, for all $x \in \mathbb{R}$. If $I_1 = \int_0^8 f(x) dx$

and $I_2 = \int_{-1}^3 f(x) dx$, then the value of $I_1 + 2I_2$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 83 Question Id : 864351173 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

If the normal to the curve $y(x) = \int_0^x (2t^2 - 15t + 10) dt$ at a point (a, b) is parallel to the line

$x + 3y = -5$, $a > 1$, then the value of $|a + 6b|$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 84 Question Id : 864351174 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

If $\lim_{x \rightarrow 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2$, then $a + b + c$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 85 Question Id : 864351175 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

Question Number : 86 Question Id : 864351176 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$ where $\omega = \frac{-1 + i\sqrt{3}}{2}$, and I_3 be the

identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 87 Question Id : 864351177 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let $f: (0, 2) \rightarrow \mathbb{R}$ be defined as $f(x) = \log_2 \left(1 + \tan \left(\frac{\pi x}{4} \right) \right)$.

Then, $\lim_{n \rightarrow \infty} \frac{2}{n} \left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f(1) \right)$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 88 Question Id : 864351178 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

The total number of 3×3 matrices A having entries from the set $\{0, 1, 2, 3\}$ such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 89 Question Id : 864351179 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3} \beta$, where α, β are integers, then $\alpha + \beta$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 90 Question Id : 864351180 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let the curve $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} = 2(x + 1)$. If the numerical value of area bounded by the curve $y = y(x)$ and x -axis is $\frac{4\sqrt{8}}{3}$, then the value of $y(1)$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100