

Mathematics Section A

Section Id :	864351155
Section Number :	5
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	20
Number of Questions to be attempted :	20
Section Marks :	80
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	864351155
Question Shuffling Allowed :	Yes

Question Number : 61 Question Id : 8643512311 Question Type : MCQ Option Shuffling : Yes Is
Question Mandatory : No
Correct Marks : 4 Wrong Marks : 1

If the foot of the perpendicular from point $(4, 3, 8)$ on the line $L_1 : \frac{x - a}{l} = \frac{y - 2}{3} = \frac{z - b}{4}$,

$l \neq 0$ is $(3, 5, 7)$, then the shortest distance between the line L_1 and line

$L_2 : \frac{x - 2}{3} = \frac{y - 4}{4} = \frac{z - 5}{5}$ is equal to :

Options :

8643516931. $\frac{1}{\sqrt{6}}$

8643516932. $\frac{1}{2}$

8643516933. $\frac{1}{\sqrt{3}}$

8643516934. $\sqrt{\frac{2}{3}}$

Question Number : 62 Question Id : 8643512312 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let the lengths of intercepts on x -axis and y -axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, ($a < 0$) be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to :

Options :

8643516935. $\sqrt{10}$

8643516936. $\sqrt{11}$

8643516937. $\sqrt{7}$

8643516938. $\sqrt{6}$

Question Number : 63 Question Id : 8643512313 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$

and $\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$, $\alpha \in \mathbb{R}$, then the value of $\alpha + |\vec{r}|^2$ is equal to :

Options :

8643516939. 9

8643516940. 11

8643516941. 13

8643516942. 15

Question Number : 64 Question Id : 8643512314 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let f be a real valued function, defined on $\mathbb{R} - \{-1, 1\}$ and given by

$$f(x) = 3 \log_e \left| \frac{x-1}{x+1} \right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function $f(x)$ is increasing ?

Options :

8643516943. $(-\infty, \infty) - \{-1, 1\}$

8643516944. $(-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty \right) - \{1\} \right)$

8643516945. $(-\infty, \frac{1}{2}] - \{-1\}$

8643516946. $(-1, \frac{1}{2}]$

Question Number : 65 Question Id : 8643512315 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If the points of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie

on the curve $y^2 = 3x^2$, then b is equal to :

Options :

8643516947. 5

8643516948. 6

8643516949. 10

8643516950. 12

Question Number : 66 Question Id : 8643512316 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let C be the locus of the mirror image of a point on the parabola $y^2=4x$ with respect to the line $y=x$. Then the equation of tangent to C at P(2, 1) is :

Options :

8643516951. $x+3y=5$

8643516952. $2x+y=5$

8643516953. $x-y=1$

8643516954. $x+2y=4$

Question Number : 67 Question Id : 8643512317 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

Options :

8643516955. $\frac{4}{9}$

8643516956. $\frac{3}{7}$

8643516957. $\frac{11}{27}$

8643516958. $\frac{9}{56}$

Question Number : 68 Question Id : 8643512318 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x) y = \sin x$, $0 \leq x \leq \frac{\pi}{3}$, with

$y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

Options :

8643516959. $\left(\frac{1}{2\sqrt{2}}\right) \log_e 2$

8643516960. $\frac{1}{2} \log_e 2$

8643516961. $\log_e 2$

8643516962. $\frac{1}{4} \log_e 2$

Question Number : 69 Question Id : 8643512319 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $\alpha \in \mathbb{R}$ be such that the function $f(x) = \begin{cases} \frac{\cos^{-1}(1 - \{x\}^2) \sin^{-1}(1 - \{x\})}{\{x\} - \{x\}^3}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$ is

continuous at $x=0$, where $\{x\} = x - [x]$, $[x]$ is the greatest integer less than or equal to x .

Then :

Options :

8643516963. $\alpha = 0$

8643516964. no such α exists

8643516965. $\alpha = \frac{\pi}{\sqrt{2}}$

8643516966. $\alpha = \frac{\pi}{4}$

Question Number : 70 Question Id : 8643512320 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

If (x, y, z) be an arbitrary point lying on a plane P which passes through the points $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of the expression

$$3 + \frac{x-11}{(y-19)^2 (z-12)^2} + \frac{y-19}{(x-11)^2 (z-12)^2} + \frac{z-12}{(x-11)^2 (y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

is equal to :

Options :

8643516967. -45

8643516968. 39

8643516969. 0

8643516970. 3

Question Number : 71 Question Id : 8643512321 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Correct Marks : 4 Wrong Marks : 1

Let $A = \{2, 3, 4, 5, \dots, 30\}$ and ' \simeq ' be an equivalence relation on $A \times A$, defined by $(a, b) \simeq (c, d)$, if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to :

Options :

8643516971. 5

8643516972. 6

8643516973. 7

8643516974. 8

Question Number : 72 Question Id : 8643512322 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x) dx = 1$ and $P(x)$ leaves remainder 5 when it is divided by $(x - 2)$. Then the value of $9(b + c)$ is equal to :

Options :

8643516975. 7

8643516976. 9

8643516977. 11

8643516978. 15

Question Number : 73 Question Id : 8643512323 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

Options :

8643516979. 1173

8643516980. 1890

8643516981. 717

8643516982. 795

Question Number : 74 Question Id : 8643512324 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Consider the integral

$$I = \int_0^{10} \frac{[x] e^{[x]}}{e^{x-1}} dx,$$

where $[x]$ denotes the greatest integer less than or equal to x . Then the value of I is equal

to :

Options :

8643516983. 45 (e + 1)

8643516984. 9 (e + 1)

8643516985. 45 (e - 1)

8643516986. 9 (e - 1)

Question Number : 75 Question Id : 8643512325 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $A(-1, 1)$, $B(3, 4)$ and $C(2, 0)$ be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

Options :

8643516987. 1

8643516988. $\frac{4}{15}$

8643516989. 2

8643516990. 3

Question Number : 76 Question Id : 8643512326 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

The least value of $|z|$ where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z| + 3)(|z| - 1)}{|z| + 1} \log_e 2\right) \geq \log_{\sqrt{2}} |5\sqrt{7} + 9i|, i = \sqrt{-1}, \text{ is equal to :}$$

Options :

8643516991. 2

8643516992. $\sqrt{5}$

8643516993. 3

8643516994. 8

Question Number : 77 Question Id : 8643512327 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

$$\text{The maximum value of } f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in \mathbf{R} \text{ is :}$$

Options :

8643516995. $\sqrt{5}$

8643516996. 5

8643516997. $\sqrt{7}$

8643516998. $\frac{3}{4}$

Question Number : 78 Question Id : 8643512328 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1}x$ is equal to :

Options :

8643516999. 0

8643517000. 1

8643517001. 2

8643517002. 3

Question Number : 79 Question Id : 8643512329 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let $f: S \rightarrow S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g: S \rightarrow \mathbb{R}$ be defined as $g(x) = \log_e f(x)$, then the value of $|g'(5) - g'(1)|$ is equal to :

Options :

8643517003. $\frac{205}{144}$

8643517004. $\frac{197}{144}$

8643517005. $\frac{187}{144}$

8643517006. 1

Question Number : 80 Question Id : 8643512330 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No

Correct Marks : 4 Wrong Marks : 1

Let C_1 be the curve obtained by the solution of differential equation $2xy \frac{dy}{dx} = y^2 - x^2, x > 0$.

Let the curve C_2 be the solution of $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$. If both the curves pass through (1, 1), then

the area enclosed by the curves C_1 and C_2 is equal to :

Options :

8643517007. $\frac{\pi}{4} + 1$

8643517008. $\pi - 1$

8643517009. $\frac{\pi}{2} - 1$

8643517010. $\pi + 1$

Mathematics Section B

Section Id :	864351156
Section Number :	6
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	10
Number of Questions to be attempted :	5
Section Marks :	20
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	864351156
Question Shuffling Allowed :	Yes

Question Number : 81 Question Id : 8643512331 Question Type : SA
Correct Marks : 4 Wrong Marks : 0

For real numbers α, β, γ and δ , if

$$\int \frac{(x^2-1) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1) \tan^{-1}\left(\frac{x^2+1}{x}\right)} dx$$
$$= \alpha \log_e \left(\tan^{-1} \left(\frac{x^2+1}{x} \right) \right) + \beta \tan^{-1} \left(\frac{\gamma(x^2-1)}{x} \right) + \delta \tan^{-1} \left(\frac{x^2+1}{x} \right) + C$$

where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 82 **Question Id :** 8643512332 **Question Type :** SA

Correct Marks : 4 **Wrong Marks :** 0

In ΔABC , the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC , then the value of $2R + r$ (in cm) is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 83 **Question Id :** 8643512333 **Question Type :** SA

Correct Marks : 4 **Wrong Marks :** 0

If the distance of the point $(1, -2, 3)$ from the plane $x + 2y - 3z + 10 = 0$ measured parallel to

the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 84 Question Id : 8643512334 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let \vec{c} be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. If

$\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$ then the value of $\vec{c} \cdot (\vec{a} \times \vec{b})$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 85 Question Id : 8643512335 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x + a, & x < 0 \\ |x - 1|, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x + 1, & x < 0 \\ (x - 1)^2 + b, & x \geq 0 \end{cases}$$

where a, b are non-negative real numbers. If $(g \circ f)(x)$ is continuous for all $x \in \mathbb{R}$, then $a + b$ is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 86 Question Id : 8643512336 Question Type : SA

Correct Marks : 4 Wrong Marks : 0

Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 87 **Question Id :** 8643512337 **Question Type :** SA

Correct Marks : 4 **Wrong Marks :** 0

Let n be a positive integer. Let $A = \sum_{k=0}^n (-1)^k nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 88 **Question Id :** 8643512338 **Question Type :** SA

Correct Marks : 4 **Wrong Marks :** 0

Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries such that $A = XB$, where

$X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in \mathbb{R}$. If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2 + 1)b_2^2 \neq -2b_1b_2$, then the

value of k is _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 89 **Question Id :** 8643512339 **Question Type :** SA

Correct Marks : 4 **Wrong Marks :** 0

Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}$, $\frac{1}{b}$, 6 be in A.P., where $a, b > 0$. Then $72(a + b)$ is equal to

_____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

100

Question Number : 90 **Question Id :** 8643512340 **Question Type :** SA

Correct Marks : 4 **Wrong Marks :** 0

Let

$S_n(x) = \log_{a/2} x + \log_{a/3} x + \log_{a/6} x + \log_{a/11} x + \log_{a/18} x + \log_{a/27} x + \dots$ up to n -terms,

where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _____.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

