JEE Main 2023 Question Paper Solution

Date & Shift: April 8 Shift 1

Mathematics Section A

Ques 1. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is

(1) 752
(2) 782
(3) 792
(4) 772

Answer 3)

Sol. n(A) = 5 n(B) = 2 n(A × B) = 10
Number of subset having three elements = 10C3
Number of subset having four elements = 10C4
Number of subset having five elements = 10C5
Number of subset having six elements = 10C6 10C3 + 10C4 + 10C5 + 10C6
= 120 + 210 + 252 + 210 = 792

Ques 2. If for z = +i, |z + 2| = z + 4(1 + i), then + and are the roots of the equation

(1) $x^2 + 3x - 4 = 0$
(2) $x^2 + 7x + 12 = 0$
(3) $x^2 + x - 12 = 0$
(4) $x^2 + 2x - 3 = 0$

Answer (2)

. $z = \alpha + c\beta$

$|z + 2| = z + 4(1 + i)$

$\sqrt{(\alpha+2)^2 + \beta^2} = (\alpha + 4) + i(\beta + 4)$

$\sqrt{(\alpha+2)^2 + \beta^2} = \alpha + 4 \quad \ldots(i)$

$\beta + 4 = 0 \quad \ldots(ii)$

(i) $\Rightarrow \sqrt{(\alpha+2)^2 + 16} = \alpha + 4$

$\alpha^2 + 4\alpha + 20 = \alpha^2 + 8\alpha + 16$

$\alpha = 1$

$\alpha + \beta = -3, \alpha\beta = -4$

Equation with roots – 3 and – 4 is

$x^2 + 7x + 12 = 0$

Ques 3. Let α,β,γ be the three roots of the equation $x^3 + bx + c = 0$. If βγ= 1 = –α, then $b^3 + 2c^3 – 3\alpha^3 – 6\beta^3 – 8\gamma^3$ is equal to

(1) 155/8

(2) 21

(3) 169/8

(4) 19

Answer (4)

Sol. Roots of x 3 + bx + c = 0 are α,β,γ

βγ = 1 = – α

α = –1 …(i)

and α+β+γ = 0 …(ii)

αβγ = –c …(iii)

c = 1 …(iv)

β+γ = 1 …(v)

αβ+βγ+γα = b

α (β+γ) + βγ = b

b = 0 …(vi)

$\beta = -\omega,\ \gamma = -\omega^2$ …(vii)

$b^3 + 2c^3 - 3a^3 - 6\beta^3 - 8\gamma^3 = 0 + 2 + 3 + 6 + 8 = 19$

Ques 4. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $\left|adj\left(adj\left(adj2A\right)\right)\right| = (16)^n$ **then n is equal to**

(1) 8
(2) 10
(3) 9
(4) 12

Answer (2)

Sol. $|A| = 2(3) - 1(2) = 4$

Now $|adj(adj(adj(2A)))|$

$= |2A|^{(n-1)3}$

$=|2A|^3$

$=2^{24}.4^8$

$=16^{10}$

Therefore, n= 10

Ques 5. Let P = $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, **A =** $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **and** $Q = PAP^T$ **. If** $P^T Q^{2007}$ **, P=** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **then 2a + b – 3c – 4d equal to**

(1) 2004
(2) 2005
(3) 2007
(4) 2006

Answer (2)

Solution. $P \times P^T \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Similarly $P^T P = I$

Now, $Q^{2007} = (PAP^T)(PAP^T) \ldots$ 2007 times $= PA^{2007}PT$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

.

.

.

$A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$P^T Q^{2007} P = A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

=> a = 1, b = 2007, c = 0, d = 1

=> 2a + b – 3c – 4d = 2005

Ques 6. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is

(1) 16800

(2) 33600

(3) 18000

(4) 14800

Answer (1)

Sol. Vowels : I, E, E, E, E

Consonants : N N N D D P C

I E E E E, 2N, 2D, P, C

Number of required words (8! / 3!x2!) x 5!/4! = = 16800

Ques 7. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is

(1) 720

(2) 126(5!)^2

(3) 7(360)^2

(4) 7(720)^2

Answer (2)

Sol. 6! $^7C_5 \cdot$ 5! = 720*(7*6/2) *5!

(5!)^2 *7*6*6/2

=> 126 × (5!)^2

Ques 8. If the coefficients of three consecutive terms in the expansion of (1 + x) n are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is

(1) 2436

(2) 5481

(3) 1827

(4) 3654

Ans. 4

Solu. $\frac{{}^nC_{r-1}}{1} = \frac{{}^nC_r}{5} = \frac{{}^nC_{r+1}}{20}$

$\because \quad \frac{{}^nC_r}{{}^nC_{r-1}} = 5 \Rightarrow \frac{\frac{\lfloor n}{\lfloor n-r \rfloor r}}{\frac{\lfloor n}{\lfloor n-r+1 \rfloor r-1}} = 5$

(n-r+1)/r = 5 => n=6r-1 …(i)

$^nC_{r+1}/ {}^nC_r$=4 =>(n-r)/(r+1) =4

=> n=5r + 4…(ii)

from (i) and (ii), r = 5, n = 29

Coefficient of fourth term = $^{29}C_3$ = 3654

Ques 9. Let $S_K = \frac{1+2+...+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}(Bn^2 + Cn + D)$,
where A, B, C, D ∈ N and A has least value. Then
(1) A + C + D is not divisible by B
(2) A + B = 5(D – C)
(3) A + B + C + D is divisible by 5
(4) A + B is divisible by D

Answer (4)

Solu.

$$S_K = \frac{K\cdot(K+1)}{2K} = \frac{K+1}{2}$$

$$\sum_{j=1}^{n}(S_j)^2 = \sum_{j=1}^{n}\frac{1}{4}\left(2^2+3^2+4^2...(n+1)^2\right)$$

$$\Rightarrow \frac{1}{4}\left(\frac{(n+1)(n+2)(2n+3)}{6}-1\right)$$

$$= \frac{1}{4}\left(\frac{\left(n^2+3n+2\right)(2n+3)-6}{6}\right)$$

$$= \frac{1}{4}\left(\frac{2n^3+6n^2+4n+3n^2+9n+6-6}{6}\right)$$

$$= \frac{1}{4}\left(\frac{2n^3+9n^2+13n}{6}\right)$$

$$= \frac{n}{24}\left(2n^2+9n+13\right)$$

$$A = 24,\ B = 2,\ C = 9,\ D = 13$$

$$\frac{A+B}{D} = \frac{26}{13} = 2$$

Ques 10 . $\lim_{x\to 0}\left(\left(\frac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ **is equal to _____**

(1) 15

(2) 9

(3) 18

(4) 24

Answer (3)

Solu. $\lim_{x\to 0}\left(\frac{(1-\cos^2 3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{\left(\log_e(2x+1)^5\right)}\right)$

$$\lim_{x\to 0}\frac{(1-\cos 3x)(1+\cos 3x)9x^2}{\left(\cos^3(4x)\right)9x^2}\,\frac{(\sin 4x)^3(64x)^3(2x)^5}{\left(64x^3\right)\left(\log_e(2x+1)\right)^5(2x)^5}$$

= 9x ½ x 2 x 64 x ½^5 =18.

Ques 11. Let $I(x)=\int\frac{(x+1)}{x\left(1+x\,e^x\right)^2}dx$ **, x>0.** $\lim_{x\to\infty} I(x)=0$ **, then I(1)is equal to**

(1) $\frac{e+2}{e+1}-\log_e(e+1)$

(2) $\frac{e+1}{e+2}+\log_e(e+1)$

(3) $\frac{e+1}{e+2}-\log_e(e+1)$

(4) $\frac{e+2}{e+1}+\log_e(e+1)$

Ans. $I(x)=\int\frac{(x+1)e^x}{xe^x\left(1+xe^x\right)^2}dx$

Let $1 + xe^x = t$

$e^x(x + 1)dx = dt$

I(x) = $\int\frac{1}{(t-1)t^2}dt$

$=\int\frac{\left(1-t^2\right)+t^2}{(t-1)t^2}dt$

$=\int\frac{-(t+1)}{t^2}+\frac{1}{t-1}dt$

= $=\int-\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t-1}dt$

$I(x)=-\ln t+\frac{1}{t}+\ln(t-1)+C$

$I(x)=\ln\left(\frac{xe^x}{xe^x+1}\right)+\frac{1}{xe^x+1}+C$

= $\lim_{x\to\infty} I(x)=0\Rightarrow C=0$

$I(1)=\ln\left(\frac{e}{e+1}\right)+\frac{1}{e+1}=1+\frac{1}{e+1}-\ln(e+1)$

$=\frac{e+2}{e+1}-\ln(e+1)$

=

Ques 12. The area of the region $\left\{(x,y):x^2\le y\le 8-x^2, y\le 7\right\}$ **is**

(1) 27

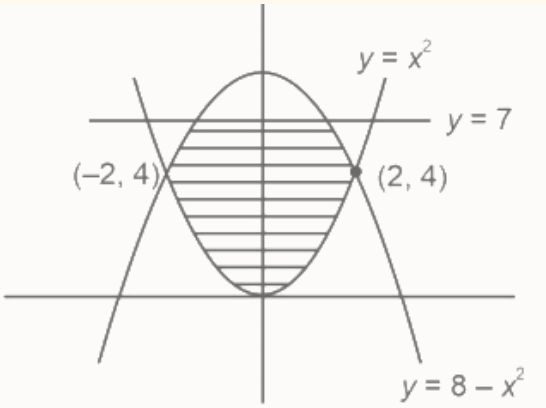
(2) 18

(3) 20

(4) 21

Answer (3)

Solution.



$$\text{Required area} = 2\left[\int_0^4 \sqrt{y}\,dy + \int_4^7 \sqrt{8-y}\,dy\right]$$

$$= 2\left(\left.\frac{y^{\frac{2}{2}}}{\frac{3}{2}}\right|_0^4 - \left.\frac{(8-y)^{\frac{3}{2}}}{\frac{3}{2}}\right|_4^7\right)$$

$$= \frac{4}{3}(8-(1-8))$$

$$= 20 \text{ sq. units}$$

Ques 16. . Let C(α, β) be the circumcentre of the triangle formed by the lines 4x + 3y = 69, 4y – 3x = 17, and x + 7y = 61. Then (α - β) 2 + α +β is equal to

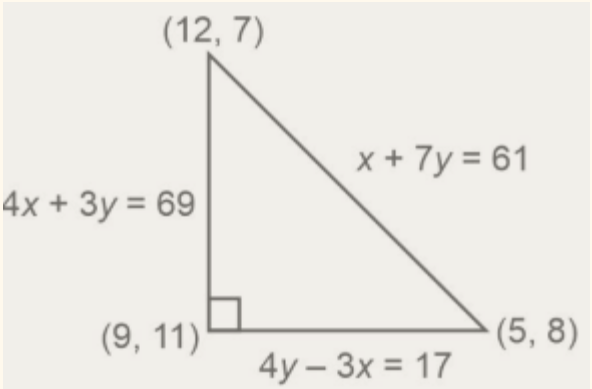
(1) 18

(2) 17

(3) 15

(4) 16

Answer (2)



Sol.

$C\left(\frac{12+5}{2}, \frac{7+8}{2}\right)$

$C\left(\frac{17}{2}, \frac{15}{2}\right) \equiv (\alpha, \beta)$

$(\alpha - \beta)^2 + \alpha + \beta$

$\left(\frac{17}{2} - \frac{15}{2}\right)^2 + \frac{17}{2} + \frac{15}{2}$

= 1+16 = 17

Ques 14. Let R be the focus of the parabola $y^2 = 20x$ and the line y = mx + c intersect the parabola at two points P and Q. Let the points G(10, 10) be the centroid of the triangle PQR. If c – m = 6, then $(PQ)^2$ is

(1) 296

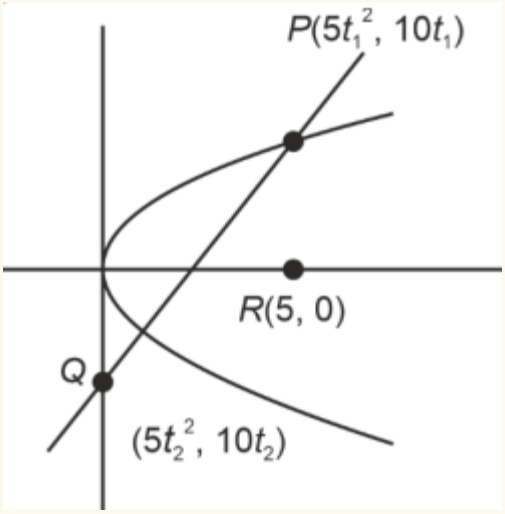
(2) 325

(3) 317

(4) 346

Answer (2)

Sol.



$$\frac{5\left(t_1^2+t_2^2+1\right)}{3}=10$$

$$\therefore \quad t_1^2+t_2^2=5 \qquad \ldots(i)$$

$$\frac{10\left(t_1+t_2\right)}{3}=10$$

$$\therefore \quad t_1+t_2=3 \qquad \ldots(ii)$$

$\therefore \quad t_1 = 1, t_2 = 2$

$\therefore \quad P \equiv (5, 10) \quad Q \equiv (20, 20)$

$$\therefore \quad \text{Equation of } PQ = y-10=\frac{10}{15}(x-5)$$

$$3y - 30 = 2x - 10$$

$$y=\frac{2}{3}x+\frac{20}{3}$$

$\therefore \quad PQ^2 = 225 + 100 = 325$

Ques 15. If the equation of the plane containing the line x + 2y + 3z – 4 = 0 = 2x + y – z + 5 and perpendicular to the plane

$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + k) + \mu(\hat{i} - 2\hat{j} + 3k)$ **is ax + by + cz = 4, then (a – b + c) is equal to**

(1) 18

(2) 22

(3) 20

(4) 24

Answer (2)

Sol. Equation of required plane is P: (x + 2y + 3z – 4) + α(2x + y – z + 5) = 0

(2α + 1) + (α + 2) y + (3 –α)z = 4 – 5α

$\vec{n_1} = (2\alpha + 1)\hat{i} + (\alpha + 2)\hat{j} + (3 - \alpha)k$

Normal of the given plane is

$$\vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = 5\hat{i} - 2\hat{j} - 3k$$

$\vec{n_1} \cdot \vec{n_2} = 0$

=> 5(2α +1)-2(α +2) -3(3-α)=0

27x +30y +25z = 4

Therefore, a=27, b=30, c=25

a-b+c =22

Ques 16. The shortest distance between the lines is

$\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

(1) 6√3

(2) 2√6

(3) 6√2

(4) 3√6

Answer (4)

Solu.

$$\vec{l}_1 \times \vec{l}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$$

$$= -2\hat{i} + \hat{j} + \hat{k}$$

$$d = \left| \frac{(\vec{a} - \vec{b}) \cdot (\vec{l}_1 \times \vec{l}_2)}{|\vec{l}_1 \times \vec{l}_2|} \right|$$

$$= \left| \frac{(3\hat{i} - 5\hat{j} - 7\hat{k}) \cdot (-2\hat{i} + \hat{j} + \hat{k})}{\sqrt{6}} \right|$$

$$= \left| \frac{-6 - 5 - 7}{\sqrt{6}} \right|$$

$= 3\sqrt{6}$

Ques 17. If the points with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$, are collinear, then $(19\alpha - 6\beta)^2$ is equal to

(1) 36
(2) 25
(3) 49
(4) 16

Answer (1)

Sol.

- A(α, 10, 13)
- B(6, 11, 11)
- C(9/2, β, −8)

Since, A, B, C are collinear

11= (- 8k +13)/(k+1)
=> 11k + 11 = -8k +13
=> 19k = 2
k = 2 : 19
(19α + 9)/21 =6
=> 19α -117
(2β+190) /21 =11
=> β= 41/2
Therefore, $(19\alpha - 6\beta)^2 = (117 - 123)^2 = 36$

Ques 18. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolts is drawn at random from the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is
(1) 5/14
(2) 9/28
(3) 3/7
(4) 2/7
Answer (1)
Sol. Using Bayes' Theorem
Required probability = (50 x 2) / (20x3 + 30x4 + 50x2)
=10/(6+12+10) = 10/28 = 5/14

Ques 19. Let $f(x)=\frac{\sin x+\cos-\sqrt{2}}{\sin x-\cos x}, x \in [0,\pi]-\left\{\frac{\pi}{4}\right\}$ **. Then** $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ **is equal to**
(1) 2/9
(2) −2/3
(3) -1/3√3
(4) 2/3√3

Answer (1)

Sol.

$$f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}$$

$$f(x)=\frac{\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)-\sqrt{2}}{\sqrt{2}\left(\sin\left(x-\frac{\pi}{4}\right)\right)}$$

$$f(x)=\frac{\sin\left(x+\frac{\pi}{4}\right)-1}{\sin\left(x-\frac{\pi}{4}\right)}$$

$$f\left(x+\frac{\pi}{4}\right)=\frac{\cos x-1}{\sin x}$$

$$=-\tan\frac{x}{2}$$

$$\Rightarrow\quad f(x)=-\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$f'(x)=\frac{-1}{2}\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$f''(x)=\frac{1}{2}\left(\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right)\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right)$$

$$f\left(\frac{7\pi}{12}\right)=-\tan\left(\frac{7\pi}{24}-\frac{\pi}{8}\right)$$

$$=-\tan\left(\frac{4\pi}{24}\right)$$

$$=-\tan\left(\frac{\pi}{6}\right)=-\frac{1}{\sqrt{3}}$$

Ques 20. Negation of $(p\Rightarrow q)\ (q\Rightarrow p)$ is

(1) $(\sim p)\ \vee\ p$

(2) $q\ \wedge\ (\sim p)$

(3) $(\sim q)\wedge p$

(4) p ∨ (~q)

Answer (2)

Sol. (p→ q) → (q→ p)

$$(p' \vee q)' \vee (q' \vee p)$$

=>

$$(p \wedge q') \vee (q' \vee p)$$

=> p v q'

Now (p v q')'

= p' ∧ q

Mathematics Section B

Ques 21. Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R= {(x ,y) ∈ A x A : x-y is odd positive integer or x – y = 2}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to _____.

Answer (19)

Sol. A = {10, 9, 8, 7, 6, 4, 3, 0}

R = {(10, 9), (10, 8), (10, 7), (10, 3), (9, 8), (9, 7), (9, 6), (9, 4), (9, 0), (8, 7), (8, 6), (8, 3), (7, 6), (7, 4), (7, 0), (6, 4), (6, 3), (4, 3), (3, 0)}

All the elements of R, (a, b) are of type a > b. Hence we need to add total of 19 more elements to R to make in symmetric

Ques 22. Let [t] denote the greatest integer t. If the constant term in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ **is α , then [α] is equal to _____.**

Answer. 1275

Solu. $$\text{General term } (T_{r+1}) = {}^7C_r \left(3x^2\right)^{7-r} \left(-\frac{1}{2x^5}\right)^r$$

$$T_{r+1} = (-1)^r \ {}^7C_r 3^{7-r} \cdot 2^{-r} \ x^{14-7r}$$

$$\text{Constant term } = a = (-1)^2 \cdot {}^7C_2 \ 3^5 \cdot 2^{-2} = \frac{5103}{4}$$

a = 1275.75

[a] = 1275

Ques 23. If a_α is the greatest term in the sequence $a_n = \frac{n^3}{n^4+147}$, n= 1,2,3…., then α is equal to _____.

Answer (5)

Sol. Let $y = x^3/(x^4+147) = f(x)$

For increasing function

dy/dx >0

$$\frac{-x^2\left(x^4-441\right)}{\left(x^4+147\right)^2} > 0$$ => $x^4 < 441$

For maxima/minima dy dx =0

$x^4 = 441$,

x = α, 4<α< 5

Maximum value of f(x) is at x = 4 or x = 5

f(4) = 64/403, f(5) =125/772'

f(5) > f(4)

=> α = 5

Ques 24. Let [t] denote the greatest integer t. Then

$$\frac{2}{\pi}\int_{\pi/6}^{5\pi/6} (8[\text{cosec } x] - 5[\cot x])\, dx$$

is equal to _____.

Answer (14)

Sol. $I = \int_{\pi/6}^{5\pi/6} (8 \cdot [\text{cosec}x] - 5[\cot x])\,dx$

$$= 8 \cdot \int_{\pi/6}^{5\pi/6} [\text{cosec}x]dx - 5 \cdot \int_{\pi/6}^{5\pi/6} [\cot x]dx = 8I_1 - 5I_2$$

$$I_1 = \int_{\pi/6}^{5\pi/6} [\text{cosec}x]dx = \int_{\pi/6}^{5\pi/6} 1 \cdot dx = \frac{2\pi}{3}$$

$$I_2 = \int_{\pi/6}^{5\pi/6} [\cot x]dx = \int_{\pi/6}^{\pi/4} 1 \cdot dx + \int_{\pi/4}^{\pi/2} 0 \cdot dx + \int_{\pi/2}^{3\pi/4} (-1)dx + \int_{3\pi/4}^{5\pi/6} (-2)dx$$

$$= -\frac{\pi}{3}$$

Required value = 2I /π = 2/π [8.2π/3 + 5.π/3] =14

Ques 25. If the solution curve of the differential equation $(y - 2\log_e x)dx + (x\log_e x^2)dy = 0$, $x > 1$ passes through the points $(e, 4/3)$ and (e^4, α), then α is equal to _____.

Answer (03.00)

Sol. $(y - 2\ln x)dx + (2x\ln x)dy = 0$.

$$2x\ln x\frac{dy}{dx}+y=2\ln x$$

$$\frac{dy}{dx}+\frac{y}{2x\ln x}=\frac{1}{x}$$

$\therefore$ I.F. $=e^{\int\frac{1}{2x\ln x}dx}=\sqrt{\ln x}$

$\therefore$ Solution of the equation is:

$$y\cdot\sqrt{\ln x}=\int\frac{\sqrt{\ln x}}{x}dx$$

$$\therefore\quad y\cdot\sqrt{\ln x}=\frac{2}{3}(\ln x)^{\frac{3}{2}}+C \quad \ldots(i)$$

$\therefore$ eq. (i) passes through point $\left(e,\frac{4}{3}\right)$.

$$\therefore\quad C=\frac{2}{3}$$

$$\therefore\quad y\sqrt{\ln x}=\frac{2}{3}(\ln x)^{\frac{3}{2}}+\frac{2}{3} \quad \ldots(ii)$$

This equation passes through point (e^4, α)

$\therefore$ $\alpha = 3$.

Ques 26. The largest natural number n such that 3n divides 66! is _____.

Answer (31.00)

Sol. Exponent of 3 in 66!

$$=\left[\frac{66}{3}\right]+\left[\frac{66}{3^2}\right]+\left[\frac{66}{3^3}\right]+\ldots\ldots$$ = 22+7+2=31

Ques 27. Consider a circle C1 : x 2 + y 2 – 4x – 2y = – 5. Let its mirror image in the line y = 2x + 1 be another circle C2 : 5x 2 + 5y 2 – 10fx – 10gy + 36 = 0. Let r be the radius of C2 . Then + r is equal to _____.

Answer (2)

Solu. C1 : $x^2 + y^2 - 4x - 2y + (5 - \alpha) = 0$

centre $O_1 = (2, 1)$, radius = $\sqrt{\alpha}$

C2 : $x^2 + y^2 - 2fx - 2gy + 36/5 = 0$

Centre $O_2 = (f, g,)$ radius $= r = \sqrt{(f^2 + g^2 - 36/5)}$

O_2 is reflection of O_1 in $2x - y + 1 = 0$

=> $$\frac{f-2}{2} = \frac{g-1}{-1} = -2 \cdot \left(\frac{2 \times 2 - 1 + 1}{2^2 + 1^2} \right) = \frac{-8}{5}$$

=> f= -6/5 and g= 13/5

=> r=1 and α = 1

Ques 28. . Let λ_1 , λ_2 be the values of λ for which the points (5/2, 1, λ) and (–2, 0, 1) are at equal distance from the plane 2x + 3y – 6z + 7 = 0. If 1 > 2 , then the distance of the point (λ1 – λ2 , λ2 , λ1) from the line (x-5)/1 = (y-1)/2 = (z+7)/2 is _____.

Answer (09.00)

Solu.

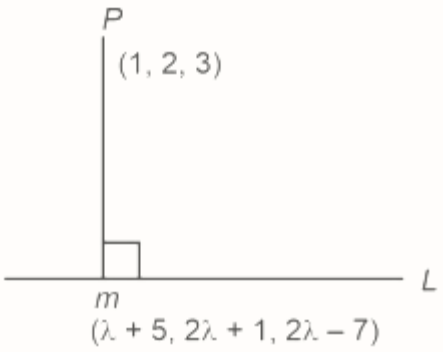
$$\left|\frac{-4+0-6+7}{7}\right| = \left|\frac{15-6\lambda}{7}\right|$$

$$\frac{3}{7} = \left|\frac{15-6\lambda}{7}\right|$$

$\lambda = 2$ or 3

$\lambda_1 = 3, \lambda_2 = 2$

$(\lambda_1 - \lambda_2, \lambda_2, \lambda_1) = (1, 2, 3)$



$\overrightarrow{PM} \cdot \left(\hat{i} + 2\hat{j} + 2k\right) = 0$

$(\lambda + 4) + 2(2\lambda - 1) + 2(2\lambda - 10) = 0$

$\Rightarrow \quad 9\lambda = 18$ or $\lambda = 2$

Distance $= \sqrt{6^2 + 3^2 + 6^2} = 9$

Ques 29. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}$, $\vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$. If $\vec{a} \cdot \vec{c} = -12$, $\vec{c} \cdot \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 5$ then $\vec{c} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right)$ is equal to _____.

Answer (11.00)

Sol.

$\vec{a} \times (\vec{c} - \vec{b}) = 0$

$\lambda \vec{a} = \vec{c} - \vec{b}$

$\Rightarrow \quad \vec{c} = \vec{b} + \lambda \vec{a}$ or $\vec{a} \cdot \vec{c} = \vec{a} \cdot \vec{b} + \lambda |\vec{a}|^2$

$\therefore \quad -12 = (6\alpha + 75) + \lambda(261)$

or $\quad 2\alpha + 87\lambda = -29 \qquad \ldots(i)$

$\vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$

$\vec{c} = \hat{i}(\alpha + 6\lambda) + \hat{j}(11 + 9\lambda) + \hat{k}(-2 + 12\lambda)$

$\Rightarrow \quad (\alpha + 6\lambda) - 2\,(11 + 9\lambda) + (-2 + 12\lambda) = 5$

or $\alpha - 24 = 5$ or $\boxed{\alpha = 29}$

$\Rightarrow \quad \boxed{\lambda = -1}$

$\vec{c} = \hat{i}(23) + \hat{j}(2) + \hat{k}(-14)$

$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 11$

Ques 30. Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If x > y, then 3x – 2y is equal to _____.

Answer (25)

Sol. (x+y+10+12+6+12+4+8)/8 = 9

x + y = 72 – 52 = 20 => x + y = 20 …(1)

$$\frac{(9-x)^2 + (9-y)^2 + (9-10)^2 + (9-12)^2 + (9-6)^2 + (9-12)^2 + (9-4)^2 + (9-8)^2}{8} = 9.25$$

$x^2 + y^2 = 218$ …(2)

From (1) & (2)

x = 13, y = 7 or x = 7, y = 13

x > y

x = 13, y = 7

3x – 2y = 39 – 14 = 25

Physics Section A

Ques 31. The engine of a train moving with speed 10 ms–1 towards a platform sounds a whistle at frequency 400 Hz. The frequency heard by a passenger inside the train is: (neglect air speed. Speed of sound in air = 330 ms–1)
(1) 400 Hz
(2) 200 Hz
(3) 412 Hz
(4) 388 Hz

Ans. (1)

Sol.

$$f' = f_0\left[\frac{v - v_0}{v - v_s}\right]$$

$$= 400\left[\frac{330 - 10}{330 - 10}\right]$$

$$= 400 \text{ Hz}$$

Ques 32. An air bubble of volume 1 cm3 rises from the bottom of a lake 40 m deep to the surface at a temperature of 12°C. The atmospheric pressure is 1 × 105 Pa, the density of water is 1000 kg/m^3 and g = 10 m/s^2. There is no difference of the temperature of water at the depth of 40 m and on the surface. The volume of air bubble when it reaches the surface will be
(1) 2 cm3
(2) 3 cm3
(3) 4 cm3
(4) 5 cm3

Answer (4)
Sol. $P_{bottom} = P0 + \rho gh$
$= 105 + 1000 \times 10 \times 40$
$= 5 \times 10^5$
$P_{top} = 10^5$
PV = constant
$\Rightarrow V_{top} = 5 \cdot V_{bottom} = 5 \text{ cm}^3$

Ques 33. Given below are two statements:
Statement I: If heat is added to a system, its temperature must increase.
Statement II: If positive work is done by a system in a thermodynamic process, its volume must increase.
In the light of the above statements, choose the correct answer from the options given below
(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false
(3) Both Statement I and Statement II are true
(4) Statement I is false but Statement II is true

Answer (4)
Sol. Heat capacity can be negative
⇒ Statement I is wrong.
$W = \int PdV$
⇒ If $W > 0$, volume must increase.

Ques 34. An aluminium rod with Young's modulus Y = 7.0 × 1010 N/m2 undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:
(1) 2800
(2) 11200
(3) 5600
(4) 8400

Answer (3)
Sol. Energy/Volume = 1/2 × stress × strain
$= ½ \times 7 \times 10^{10} \times (0.04/100)^2$
= 5600

Ques 35. Given below are two statements:
Statement I: If E be the total energy of a satellite moving around the earth, then its potential energy will be E/2 .
Statement II: The kinetic energy of a satellite revolving in an orbit is equal to half the magnitude of total energy E.
In the light of the above statements, choose the most appropriate answer from the options given below
(1) Statement I is correct but Statement II is incorrect
(2) Statement I is incorrect but Statement II is correct
(3) Both Statement I and Statement II are correct
(4) Both Statement I and Statement II are incorrect
Answer (4)
Sol. $U = -GMm/r$
$K = GMm/2r$
$T = K+U = -GMm/2r$
⇒ Both statements are false

Ques 36. At any instant the velocity of a particle of mass 500 g is $(2t\hat{i}+3t^2\hat{j})\text{ms}^{-1}$. If the force acting on the particle at t = 1s is $(\hat{i}+x\hat{j})\text{N}$. Then the value of x will be:

(1) 3

(2) 4

(3) 2

(4) 6

Answer (1)

Sol.

$$\vec{v}=2t\hat{i}+3t^2\hat{j}$$

$$\Rightarrow \quad \vec{a}=2\hat{i}+6t\hat{j}$$

$$\Rightarrow \quad \vec{F}=m\vec{a}=\frac{1}{2}\vec{a}=\hat{i}+3t\hat{j}$$

$$\text{At } t=1,\ \vec{F}=\hat{i}+3\hat{j}$$

Ques 37. Two forces having magnitude A and 2 A are perpendicular to each other. The magnitude of their resultant is:

(1) $\sqrt{5}A/4$

(2) $\sqrt{5}A/2$

(3) $5A/2$

(4) $\sqrt{5}A^2/2$

Answer (2)

Sol.

$$A_{\text{net}}=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\theta}$$

$$\Rightarrow \quad F_{\text{net}}=\sqrt{A^2+\frac{A^2}{4}}$$

=>$\sqrt{5}A/2$

Ques 38. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be:

(1) 200 N

(2) Zero

(3) 100 N
(4) 300 N

Answer (1)
Sol. g' = g(1- d/R)
W' =w(1- (R/2)/R)
= 200 N

Ques 39. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is ($g = 10\ m/s^2$)
(1) 4 : 9
(2) 2: 3
(3) 3:2
(4) 1 : 1

Answer (1)
Sol. $R = u^2\sin 2\theta/g$
$\Rightarrow$Ratio= $(40^2 \times \sin 60°)/(60^2 \times \sin 120°) = 4/9 \times 1 = 4/9$

Ques 40. A cylindrical wire of mass (0.4 ± 0.01) g has length (8 ± 0.04) cm and radius (6 ± 0.03) mm. The maximum error in its density will be
(1) 3.5%
(2) 5%
(3) 1%
(4) 4%
Answer (4)
Sol. Density $\rho = M/\pi R^2 L$

$$\Rightarrow \frac{d\rho}{\rho} = \frac{dM}{M} + \frac{2dR}{R} + \frac{dL}{L}$$

$$= \left[\frac{0.01}{0.4} + \frac{2\times 0.03}{6} + \frac{0.04}{8}\right] \times 100$$

= 2.5 + 1 + 0.5% = 4%

Ques 41. A TV transmitting antenna is 98 m high and the receiving antenna is at the ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

(1) 1240 km^2

(2) 3942 km^2

(3) 4868 km^2

(4) 1549 km^2

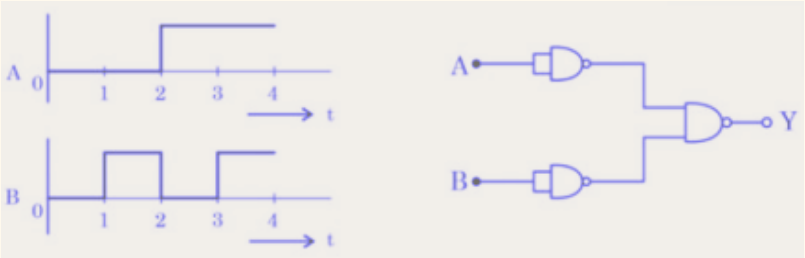
Answer (2)

Sol. $d = \sqrt{2Rh}$

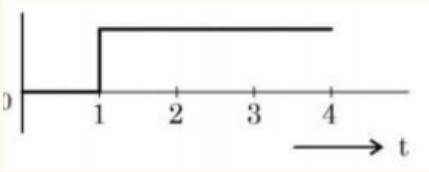
$\Rightarrow$ Area $= \pi d^2 = \pi \times 2 \times R \times h$

$= 3940\ km^2$

Ques 42. For the logic circuit shown, the output waveform at Y is



(1)



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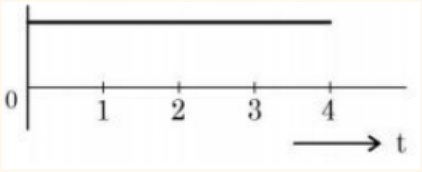

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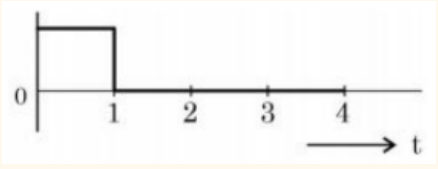

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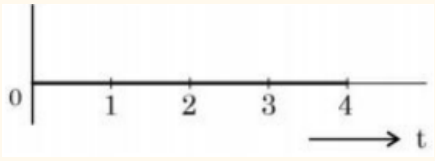


Answer (1)

Sol. $y = (A' \cdot B')$

$= A + B$

$\Rightarrow$ OR gate

$\Rightarrow$ Correct waveform is first.

Ques 43. For a nucleus ${}^{A}_{Z}X$ having mass number A and atomic number Z

A. The surface energy per nucleon (bs) = $-a_1A^{2/3}$.

B. The Coulomb contribution to the binding energy $b_c = -a_2 \frac{Z(Z-1)}{A^{4/3}}$

C. The volume energy $b_v = a_3A$

D. Decrease in the binding energy is proportional to surface area.

E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons. (a_1, a_2 and a_3 are constants)

Choose the most appropriate answer from the options given below:

(1) B, C, E only

(2) C, D only

(3) A, B, C, D only

(4) B, C only

Answer (2)

Sol. $R = R_0A^{1/3}$

$\Rightarrow$ Surface area $\propto R^2 \propto A^{2/3}$

For coulomb contribution, we choose pairs of protons

$\Rightarrow \propto (Z)(Z-1)$

Volume $\propto R^3 \propto A^1$

$\Rightarrow$ Option (2) is correct

Ques 44. Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume, $m_p = 1849\ M_e$):

(1) 1 : 1

(2) 1 : 1849

(3) 1 : 43

(4) 43 : 1

Answer (1)

Sol. $\lambda = h/mv$

⇒ If $\lambda = \lambda'$ then $p = p'$.
⇒ Option 1.

Ques 45. In a reflecting telescope, a secondary mirror is used to:
(1) reduce the problem of mechanical support
(2) make chromatic aberration zero
(3) move the eyepiece outside the telescopic tube
(4) remove spherical aberration

Answer (3)
Sol. A secondary mirror is used to move the eyepiece outside the telescopic tube.

Ques 46. Dimension of $\frac{1}{\mu_0 \varepsilon_0}$ should be equal to
(1) L/T
(2) T^2/L^2
(3) L^2/T^2
(4) T/L

Answer (3)

Sol. We know that $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\Rightarrow \frac{1}{\mu_0 \varepsilon_0} \equiv [LT^{-1}][LT^{-1}]$
$= L^2T^2$

Ques 47. Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is
(1) to oscillate the coil in magnetic field for longer period of time
(2) to bring the coil to rest quickly
(3) to produce large deflecting torque on the coil
(4) to make the magnetic field radial

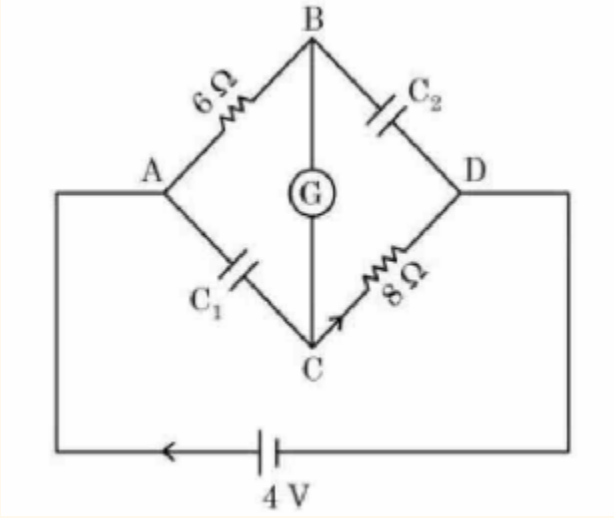
Answer (2)

Sol. Function of magnetic material is to bring the coil to rest quickly through eddy currents.

Ques 48. A charge particle moving in magnetic field B, has the components of velocity along B as well as perpendicular to B. The path of the charge particle will be
(1) helical path with the axis perpendicular to the direction of magnetic field B
(2) helical path with the axis along magnetic field B
(3) circular path
(4) straight along the direction of magnetic field B

Answer (2)
Sol. Perpendicular component results in circular motion. Parallel component results in linear motion.
⇒ Helical path with axis along magnetic field.

Ques 49. In this figure the resistance of the coil of galvanometer G is 2 Ω. The emf of the cell is 4 V. The ratio of potential difference across C1 and C2 is:



(1) 1 ‘
(2) 4/5
(3) 5/4
(4) ¾

Answer (2)

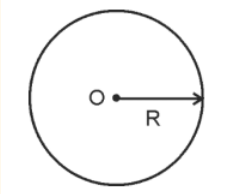
Sol. Capacitors would behave as open circuits

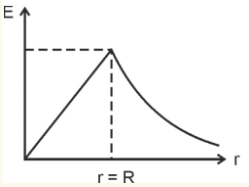
$$\Rightarrow \quad V_{C_1} = i[6\Omega + R_G]$$

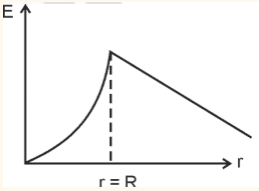
$$\text{And } V_{C_2} = i[R_G + 8\Omega]$$

$$\Rightarrow \quad \text{Ratio} = \frac{8}{10} = \frac{4}{5}$$

Ques 50. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the centre O is represented by:



(1)


(2)


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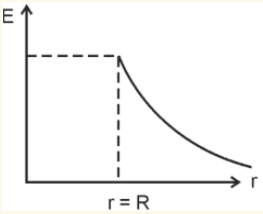

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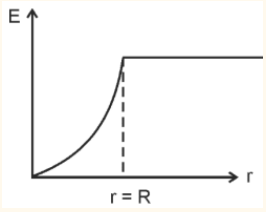


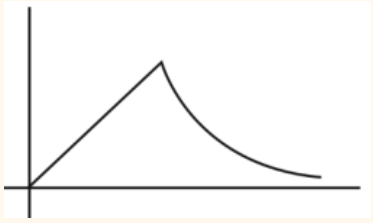
Answer (1)

Sol.

$$E_{inside} = \frac{\rho r}{3\varepsilon_0} \propto r^1$$

$$E_{outside} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \propto \frac{1}{r^2}$$

=>



Physics Section B

Ques 51. An organ pipe 40 cm long is open at both ends. The speed of sound in air is 360 ms^{-1}. The frequency of the second harmonic is _____ Hz.

Answer (900)

Sol. f = 2 · V/ 2L =2 · 360 Hz /(2 x 0.4)= 900 Hz

Ques 52. An air bubble of diameter 6 mm rises steadily through a solution of density 1750 kg/m3 at the rate of 0.35 cm/s. The co-efficient of viscosity of the solution (neglect density of air) is _____ Pas (given, g = 10 ms−2).

Answer (10)

Sol. Buoyancy = 6πηrvT

=>

$$\Rightarrow \frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_T$$

$$\Rightarrow \eta = \frac{\frac{4}{3}r^2\rho g}{6v_T}$$

$$= \frac{2}{9}\times\left(3\times10^{-3}\right)^2 \times \frac{1750\times10}{\frac{0.35}{100}}$$

$$= 2\times10^{-6}\times\frac{175\times10000}{0.35} = 10$$

Ques 53. The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is 1 2 MR x , where R is the radius and M is the mass of the semicircular ring. The value of x will be _____.

Answer (1)

Sol. $I = MR^2$

$\Rightarrow x = 1$

Ques 54. The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is_____%.

Answer (125)

Sol. $K = p^2/2m$

$\Rightarrow K' = (1.5p)^2/2m = 2.25k$

$\Rightarrow (K' - K) / K * 100 = 125$

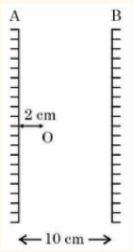
Ques 55. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is _____ MeV.

Answer (121)

Sol. BE = (8.1 – 7.6) × 242 MeV

= 121 MeV

Ques 56. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is _____ cm.



Answer (18)

Sol. Object distance = 8 cm [For image by B]

⇒ Required distance = 8 + 8 + 2 cm

= 18 cm

Ques 57. An oscillating LC circuit consists of a 75 mH inductor and a 1.2 μF capacitor. If the maximum charge to the capacitor is 2.7 μC. The maximum current in the circuit will be _____ mA.

Answer (9)

Sol. $i_{max} = q_{max} \cdot \omega$

$$= 2.7\times10^{-6}\times\frac{1}{\sqrt{LC}}$$

$$= \frac{2.7\times10^{-6}}{\sqrt{75\times10^{-3}\times1.2\times10^{-6}}}\ A$$

$$= \frac{2.7\times10^{-6}}{30\times10^{-5}}A = 9\ mA$$

Ques 58. The magnetic intensity at the center of a long current carrying solenoid is found to be 1.6 × 103 Am^{-1}. If the number of turns is 8 per cm, then the current flowing through the solenoid is _____A.

Answer (2)

Sol. $B/\mu_0 = ni$

⇒ 1.6 × 1000 = 8× i / (1/ 100)

⇒ i = 2 A

Ques 59. A current of 2 A flows through a wire of crosssectional area 25.0 mm2. The number of free electrons in a cubic meter are 2.0 × 1028. The drift velocity of the electrons is _____ × 10–6 ms–1 (given, charge on electron = 1.6 × 10–19C).

Answer (25)

Sol. $I = nAev_d$

$\Rightarrow 2 = 2 \times 10^{28} \times 25 \times 10^{-6} \times 1.6 \times 10^{-19} \times v_d$

$\Rightarrow v_d = 1/(25*1.6*1000)$ m/s $= 25*10^{-6}$ m/s

Ques 60. An electric dipole of dipole moment is 6.0 × 10–6Cm placed in a uniform electric field of 1.5 × 103 NC–1 in such a way that dipole moment is along electric field. The work done in rotating dipole by 180° in this field will be_____mJ.

Answer (18)

Sol. $W = \Delta U = U_f - U_i$

Also, $U = -p \cdot E$

$\Rightarrow W = 2 \times p \times E = 2 \times 6 \times 10^{-6} \times 1.5 \times 10^3$ J

= 18 mJ

Chemistry Section A

Ques 61. $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2$ **What is the value of x?**

(1) 2

(2) 12

(3) 10

(4) 6

Answer (3)

Sol. $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

By balancing the charge, we can easily find out that x is 10.

Ques 62. The reaction ½ H (g) + AgCl(s) ⇋ H^+ (aq) + Cl^- (aq)+Ag(s) occurs in which of the given galvanic cell?
(1) Pt|H2(g)|HCl(sol^n)|AgCl(s)|Ag
(2) Ag|AgCl(s)|KCl(sol^n)|AgNO3|Ag
(3) Pt|H2(g)|HCl(sol^n)|AgNO3(soln)|Ag
(4) Pt|H2(g)|KCl(sol^n)|AgCl(s)|Ag

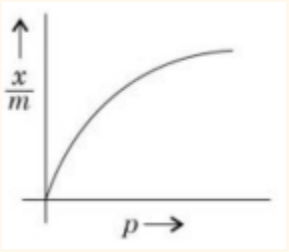
Answer (1)

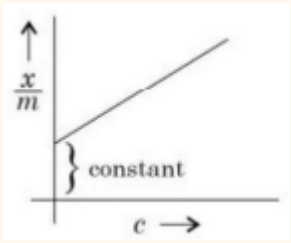
Sol. At anode : $\frac{1}{2}H_2(g) \longrightarrow \underset{(aq)}{H^+} + e^-$

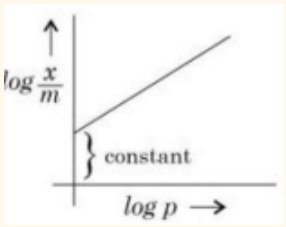
At cathode: $\underset{(S)}{AgCl} + \bar{e} \longrightarrow Ag + \underset{(aq)}{Cl^-}$

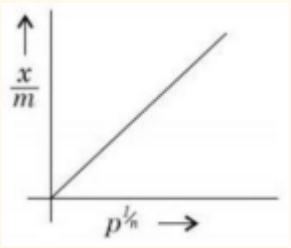
Overall reaction ½ H (g) + AgCl(s)⟶ H^+ + Cl^- + Ag(s)
The cell representation for the above cell will be
Pt|H2(g)|HCl(sol^n)|AgCl(s)|Ag

Ques 63. Which of the following represents the Freundlich adsorption isotherms?

(A) 

(C) 

(B) 

(D) 

Choose the correct answer from the options given below:
(1) A, C, D only
(2) A, B only
(3) B, C, D only
(4) A, B, D only

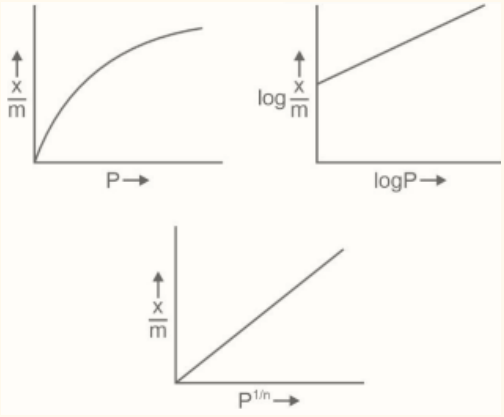
Answer (4)

Sol. Freundlich adsorption isotherm equation is

$$\frac{x}{m} = KP^{1/n} \quad \ldots(i)$$

$$\Rightarrow \log\frac{x}{m} = \log K + \frac{1}{n}\log P \quad \ldots(ii)$$

With the help of the above two equation, following plots are obtained



Ques 64. The correct order of electronegativity for given elements is
(1) P > Br > C > At
(2) Br > P > At > C
(3) Br > C > At > P
(4) C > P > At > Br

Answer (3)

Sol.

Element	Electronegativity
P	2.1
C	2.5
Br	3.0
At	2.2

Ques 65. Which of the following metals can be extracted through alkali leaching technique?
(1) Sn
(2) Pb

(3) Au
(4) Cu

Answer (1)
Sol. $SnO_2 + NaOH \rightarrow Na_2[Sn(OH)_6]$

Ques 66. The water gas on reacting with cobalt as a catalyst forms
(1) Methanal
(2) Methanoic acid
(3) Ethanol
(4) Methanol

Answer (4)
Sol. $\underset{\text{Water gas}}{CO(g) + 2H_2(g)} \xrightarrow[\text{Catalyst}]{\text{Cobalt}} \underset{\text{Methanol}}{CH_3OH(l)}$

Ques 67. What is the purpose of adding gypsum to cement?
(1) To facilitate the hydration of cement
(2) To slow down the process of setting
(3) To give a hard mass
(4) To speed up the process of setting

Answer (2)
Sol. When mixed with water the setting of cement takes place to give a hard mass. The is due to the hydration of molecules of the constituents and their rearrangement. The purpose of adding gypsum is only to slow down the process of setting of the cement so that it gets sufficiently hardened.

Ques 68. Given below are two statements : Statement I : Lithium and Magnesium do not form superoxide Statement II : The ionic radius of Li+ is larger than ionic radius of Mg2+ In the light of the above statements, choose the most appropriate answer from the questions given below :
(1) Statement I is incorrect but Statement II is correct
(2) Statement I is correct but Statement II is incorrect
(3) Both statement I and Statement II are incorrect
(4) Both Statement I and Statement II are correct

Answer (4)

Sol. Ionic radius of Li+ = 76 pm Ionic radius of Mg2+ = 72 pm Both Li+ and Mg2+ do not form superoxide because their ionic radius is very small. Hence, both the given statements are correct

Ques 69. Which halogen is known to cause the reaction given below: $2Cu^{2+} + 4X^{-} \rightarrow Cu_2X_2(s) + X2$

(1) All halogens

(2) Only Bromine

(3) Only Iodine

(4) Only Chlorine

Answer (3)

Sol. $2Cu^{2+} + 4I^{-} \rightarrow Cu_2I_2 + I_2$

Ques 70. Which of the following complex is octahedral, diamagnetic and the most stable?

(1) Na3[CoCl6]

(2) [Ni(NH3)6]Cl2

(3) K3[Co(CN)6]

(4) [Co(H2O)6]Cl2

Answer (3)

Sol. (1) Na3[CoCl6] – Paramagnetic

(2) [Ni(NH3)6]Cl2 – Paramagnetic

(3) K3[Co(CN)6] – Diamagnetic

(4) [Co(H2O)6]Cl2 – Paramagnetic

Ques 71. . Match List I with List II

List I – (Species)		List II – (Maximum allowed concentration in ppm in drinking water)	
A.	F^{-}	I.	< 50 ppm
B.	SO_4^{2-}	II.	< 5 ppm
C.	NO_3^{-}	III.	< 2 ppm
D.	Zn	IV.	< 500 ppm

Choose the correct answer from the options given below.

(1) A-I, B-II, C-III, D-IV

(2) A-II, B-I, C-III, D-IV
(3) A-IV, B-III, C-II, D-I
(4) A-III, B-II, C-I, D-IV

Answer (DROP)
Solu. A → III ; B → IV; C → I; D → II

Ques 72. The correct order of spin only magnetic moments for the following complex ions is
(1) [Fe(CN)6] 3– < [CoF6] 3– < [MnBr4] 2– < [Mn(CN)6] 3–
(2) [CoF6] 3– < [MnBr4] 2– < [Fe(CN)6] 3– < [Mn(CN)6] 3–
(3) [Fe(CN)6] 3– < [Mn(CN)6] 3– < [CoF6] 3– < [MnBr4] 2–
(4) [MnBr4] 2– < [CoF6] 3– < [Fe(CN)6] 3– < [Mn(CN)6] 3–

Answer (3)
Solu.

$[MnBr_4]^{2-} = Mn^{2+}(td) = e^2t_2^3$ = 5 unpaired electrons

$\therefore \mu = \sqrt{5(5+2)} = \sqrt{35}$ B. M.

$[CoF_6]^{3-} = Co^{3+}(Oh)$

(High spin)

$t_{2g}^4 eg^2$ = 4 unpaired electrons

$\therefore \mu = \sqrt{4(4+2)} = \sqrt{24}$ B.M.

$[Mn(CN)_6]^{3-} = Mn^{3+}$ (Oh) = $t_{2g}^4 eg^0$

(Low spin)

= 2 unpaired electrons

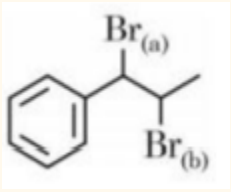
$\therefore \mu = \sqrt{2(2+2)} = \sqrt{8}$ B.M

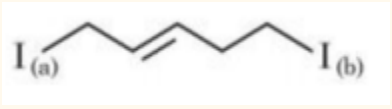
$[Fe(CN)_6]^{3-} = Fe^{3+}(Oh) = t_{2g}^5 eg^0$ = 1 unpaired electron

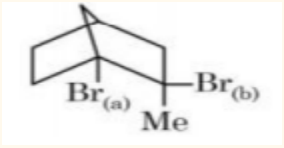
(Low spin)

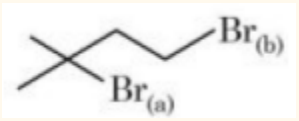
$\therefore \mu = \sqrt{1(1+2)} = 2\sqrt{3}$ B.M

Ques 73. Choose the halogen which is most reactive towards SN1 reaction in the given compounds (A, B, C & D)

A. 

B. 

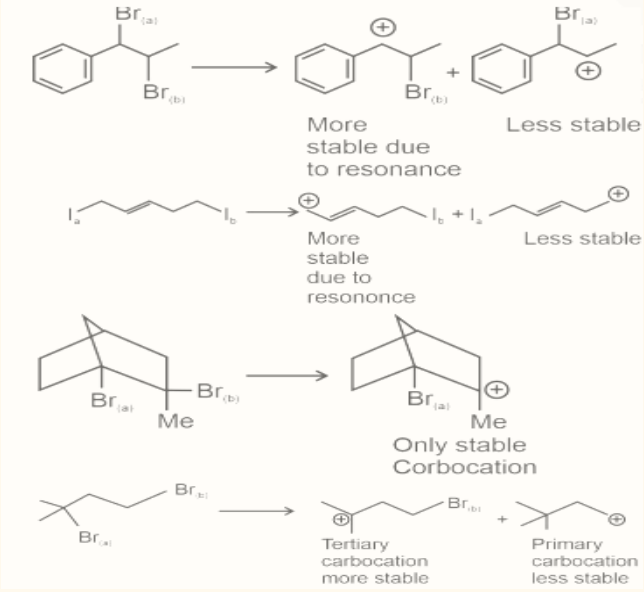
C. 

D. 

(1) A – Br(b) ; B – I(a) ; C – Br(a) ; D – Br(a)
(2) A – Br(b) ; B – I(b) ; C – Br(b) ; D – Br(b)
(3) A – Br(a) ; B – I(a) ; C – Br(b) ; D – Br(a)
(4) A – Br(a) ; B – I(a) ; C – Br(a) ; D – Br(a)

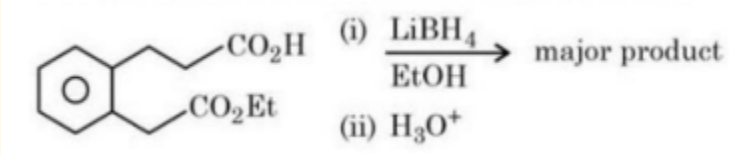
Answer (3)

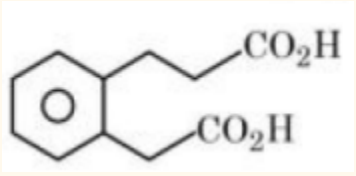
Sol. The leaving group which results in the formation of more stable carbocation will be more reactive towards SN1 reaction

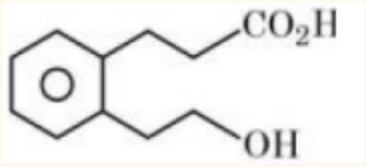


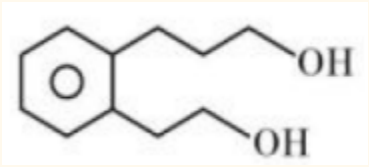
Hence, the correct answer is option (3).

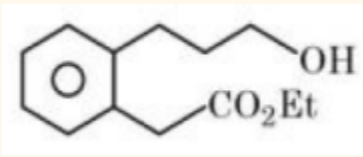
Ques 74. The major product formed in the following reaction is



1. 

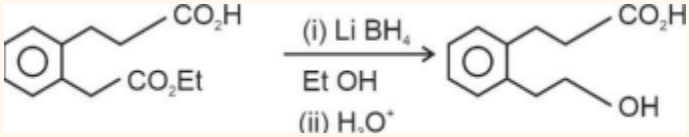
2. 

3. 

4. 

Ans. 2

Sol. Selective reduction of ester group.



Ques 75. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Butan–1–ol has higher boiling point than ethoxyethane.

Reason R : Extensive hydrogen bonding leads to stronger association of molecules.

In the light of the above statements, choose the correct answer from the options given below :

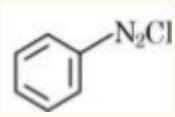
(1) A is true but R is false

(2) Both A and R are true and R is the correct explanation of A

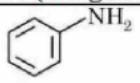
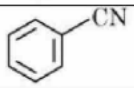
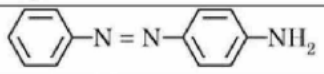
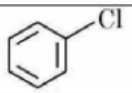
(3) Both A and R are true but R is not the correct explanation of A
(4) A is false but R is true

Answer (2)
Sol. Owing to intermolecular hydrogen bonding in butanol, it has higher boiling point than ethoxyethane.

Ques 76. Match List I with List II :

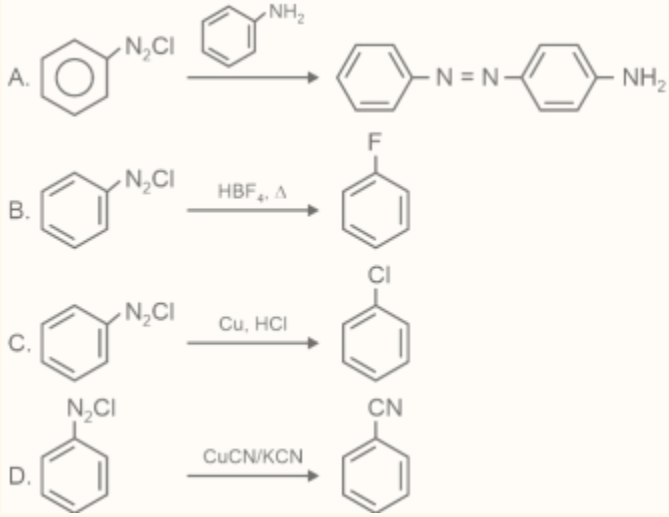


is reacted with reagents in List I to form products in List II.

List I (Reagent)	List II (Product)
A. 	I. 
B. HBF_4, Δ	II. 
C. Cu, HCl	III. 
D. CuCN / KCN	IV. 

Choose the correct answer from the options given below:
(1) A-IV, B-III, C-II, D-I
(2) A-III, B-I, C-II, D-IV
(3) A-I, B-III, C-IV, D-II
(4) A-III, B-I, C-IV, D-II

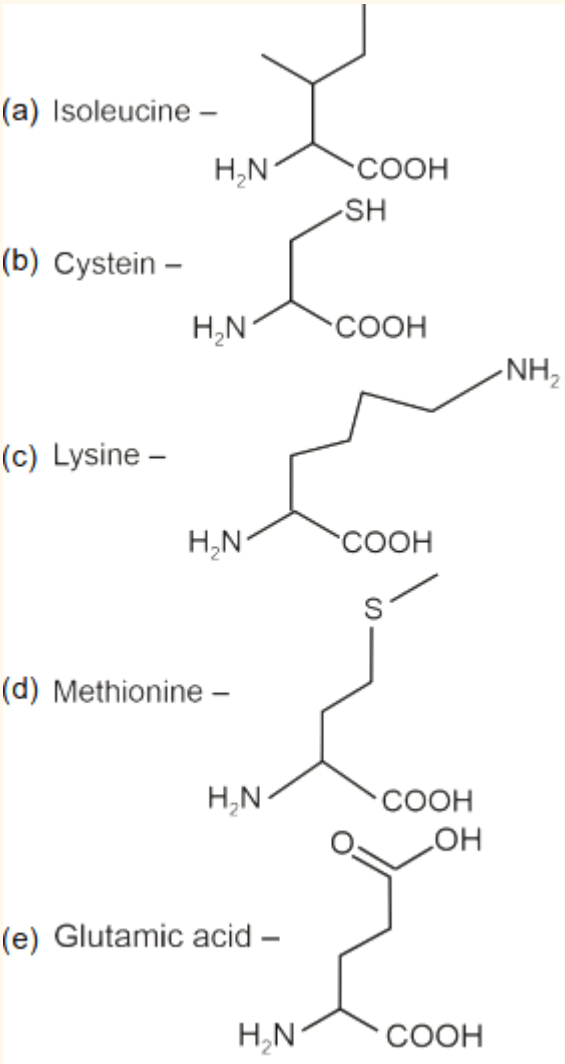
Answer (4)



Ques 77. Sulphur (S) containing amino acids from the following are:
(a) isoleucine
(b) cysteine
(c) lysine
(d) methionine
(e) glutamic acid
(1) b, c, e
(2) a, b, c
(3) b, d
(4) a, d

Answer (3)

Solu.



Ques 78. Match List I with List II :

List I		List II	
A.	Saccharin	I.	High potency sweetener
B.	Aspartame	II.	First artificial sweetening agent
C.	Alitame	III.	Stable at cooking temperature
D.	Sucralose	IV.	Unstable at cooking temperature

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-I, D-II

(2) A-II, B-III, C-IV, D-I

(3) A-II, B-IV, C-III, D-I

(4) A-II, B-IV, C-I, D-III

Answer (4)

Sol. A→II, B→IV, C→I, D→III

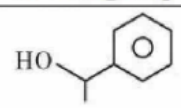
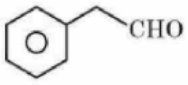
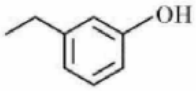
Saccharin is the first popular artificial sweetening agent.

Aspartame use is limited to cold foods and soft drinks because it is unstable at cooking temperature.

Alitame is high potency sweetener.

Sucralose is stable at cooking temperature.

Ques 79. Match List I with List II :

List I (Reagents used)		List II (Compound with Functional group detected)	
A.	Alkaline solution of copper sulphate and sodium citrate	I.	
B.	Neutral $FeCl_3$ solution	II.	
C.	Alkaline chloroform solution	III.	
D.	Potassium iodide and sodium hypochlorite	IV.	

Choose the correct answer from the options given below :

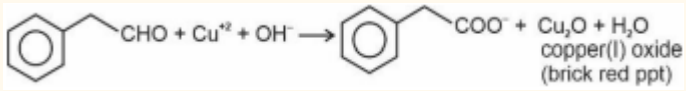
(1) A-III, B-IV, C-I, D-II

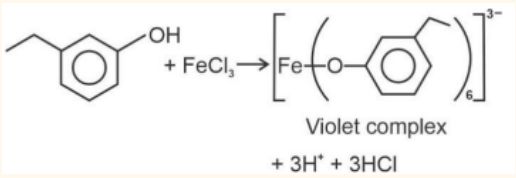
(2) A-III, B-IV, C-II, D-I

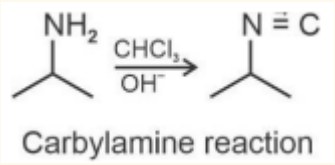
(3) A-IV, B-I, C-II, D-III

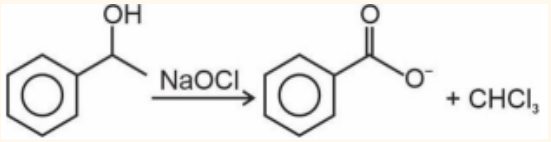
(4) A-II, B-IV, C-III, D–I

Answer (2)

Sol. (A) 

(B) 

(C) 

(D) 

Ques 80. In chromyl chloride, the number of d-electrons present on chromium is same as in (Given at no. of Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)

(1) V (IV)
(2) Mn (VII)
(3) Fe (III)
(4) Ti (III)

Answer (2)

Sol. Chromyl chloride CrO_2Cl_2

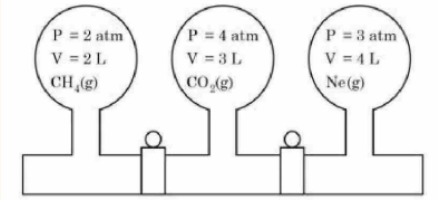
O.S. of Cr = +6

d_0 configuration

Mn (+7) = d^0

Chemistry Section B

Ques 81. Three bulbs are filled with CH4, CO2 and Ne as shown in the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be_____ atm. (Nearest integer).



Answer (3)

Sol. Using the formula

$2*2+4*3+3*4 = P_f*9$

$P_f = 28/9 = 3$

Ques 82. The number of following statement/s which is/are incorrect is _____

(A) Line emission spectra are used to study the electronic structure

(B) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet.

(C) An absorption spectrum is like the photographic negative of an emission spectrum

(D) The element helium was discovered in the sun by spectroscopic method

Answer (1)

Sol. Except (B) all other statements are correct.

Ques 83. The number of following factors which affect the percent covalent character of the ionic bond is_____

(A) Polarising power of cation

(B) Extent of distortion of anion

(C) Polarisability of the anion

(D) Polarising power of anion

Answer (3)

Sol. Percentage covalent character of an ionic bond depends upon

- Polarising power of cation
- Extent of distortion of anion
- Polarisability of the anion

Ques 84. When a 60 W electric heater is immersed in a gas for 100s in a constant volume container with adiabatic walls, the temperature of the gas

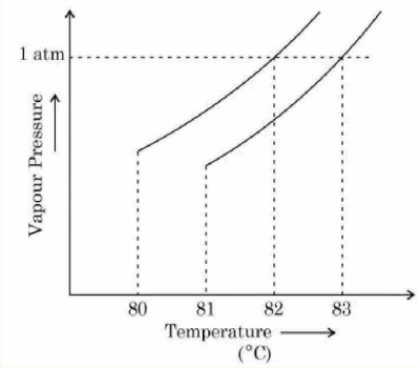
rises by 5°C. The heat capacity of the given gas is _____ J K–1 (Nearest integer)

Answer (1200)

Sol. Heat capacity =Heat absorbed / change in temperature = 60 * 100 J / 5

= 1200 JK^{-1}

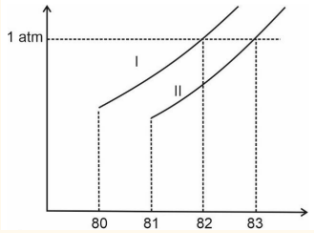
Ques 85. The vapour pressure vs. temperature curve for a solution solvent system is shown below.



The boiling point of the solvent is _____°C.

Answer (82)

Sol. We know that the temperature at which vapour pressure of a liquid becomes 1 atm. is called the boiling point of the liquid.



From the figure, we can clearly see that Graph I is for pure solvent whereas graph II is for solution. Hence the boiling point of solvent and solution are 82°C and 83°C respectively.

Ques 86. . The titration curve of weak acid vs. strong base with phenolphthalein as indicator) is shown below. The Kphenolphthalein = 4 × 10–10 Given: log2 = 0.3 The number of following statement/s which is/are correct about phenolphthalein is _____

(A) It can be used as an indicator for the titration of weak acid with weak base.

(B) It begins to change colour at pH = 8.4
(C) It is a weak organic base
(D) It is colourless in acidic medium

Answer (02)
Sol. Phenolphthalein is an organic acid and can be represented as HPh.

H Ph(colourless) $\leftrightharpoons H^+ + Ph^-$ (Pink)
Using Henderson equation for phenolphthalein
pH = pK +log{[Ph –]/[HPh]}
At equivalence point [Ph –]= [HPh]
$pH_2 = pK_{In}$ +log [4] +10 = -0.6 + 10 =9.4
Hence at (9.4 +- 1) PH, phenolphthalein starts changing colour.
Phenolphthalein is colourless in acidic medium and pink in basic medium.
Phenolphthalein indicator distinguish the pH change between 8 to 10. Therefore, it is used for strong acid and strong base titration or weak acid and strong base titration. Hence, A and C are incorrect statements.

Ques 87. The number of given statement/s which is/are correct is_____
(A) The stronger the temperature dependence of the rate constant, the higher is the activation energy.
(B) If a reaction has zero activation energy, its rate is independent of temperature.
(C) The stronger the temperature dependence of the rate constant, the smaller is the activation energy.
(D) If there is no correlation between the temperature and the rate constant then it means that the reaction has negative activation energy.

Answer (02)
Sol. Rate constant is given by Arrhenius equation k
$= Ae^{-Ea/RT}$
= Using the above equation, we can clearly see that only option (A) and (B) are correct.

Ques 88. XeF_4 reacts with SbF_5 to form $[XeFm]^{n+}[SbFy]^{z-}$.
m + n + y + z = _____??

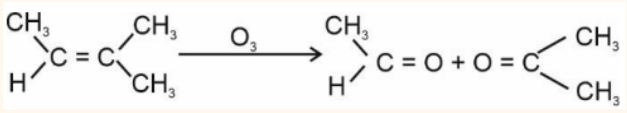
Answer (11)

Sol. $XeF_4 + SbF_5\ SbF \rightarrow [XeF_3]^+[SbF_6]^-$

m = 3, n = 1, y = 6, z = 1

Ques 89. Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of O3 per mole of (X) and gives one mole each of ethanal and propanone is _____ g mol–1 (Molar mass of C : 12 g mol–1 , H : 1 g mol–1)

Answer (70)

Sol. 

M.wt. = 70

Ques 90. 0.5 g of an organic compound (X) with 60% carbon will produce _____ $\times 10^{-1}$ g of CO_2 on complete combustion.

Answer (11)

Sol. $C\% = 12/44 * \frac{\text{Wt. of } CO_2}{\text{Wt. of organic compound}} * 100$

$60 = 12/44 * \frac{\text{Wt. of } CO_2}{0.5} * 100$

Wt. of CO_2 = 1.1