JEE Main 2023 Solutions
Jan 24 - Shift 1

Physics

Question 1. From the photoelectric effect experiment, following observation are made. Identify which of these are correct.

A. The stopping potential depends only on the work function of the metal.
B. The saturation current increases as the intensity of incident light increases.
C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light.
D. Photoelectric effect can be explained using wave theory of light.

Choose the correct answer from the options given below:

(1) A,B,D only
(2) B,C only
(3) B only
(4) A,C,D only

Answer. B only

Solution. (A) From Einstein's equation
$K_{max} = eV_s = h\nu - \varphi$
Form the stopping potential (V_s) depends on φ & ν.
(B) Saturation current is proportional to intensity, i.e., number of incident photons.

(C) Kmax only depends on nature of photon and φ.
(D) Einstein used particle behaviour of photon to explain photon electric effect.
Only B is correct.

Question 7. If two charges q1 and q2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force?

(1) 1.5d√k
(2) 2d√k
(3) d√k
(4) k√d

Answer. d√k

Solution. The electrostatic force between two charges in a medium with a dielectric constant (K) is given by Coulomb's law with the medium taken into account:

$F = (1 / (4\pi\varepsilon_0)) * (q_1 * q_2) / (K * d^2)$

Where:

- F is the electrostatic force in the medium.
- ε_0 is the vacuum permittivity (a constant).
- q_1 and q_2 are the charges.
- K is the dielectric constant.
- d is the distance between the charges in the medium.

If we want to find the equivalent distance (d_air) between the charges in air (without the dielectric medium) to have the same electrostatic force, we can set up an equation using Coulomb's law in air (with K = 1) and equate it to the force in the medium:

$(1 / (4\pi\varepsilon_0)) * (q_1 * q_2) / (d\_air^2) = (1 / (4\pi\varepsilon_0)) * (q_1 * q_2) / (K * d^2)$

Now, let's simplify the equation:

$(q_1 * q_2) / (d\_air^2) = (q_1 * q_2) / (K * d^2)$

Now, we can cancel out the charges (q_1 and q_2) on both sides:

$1 / (d\_air^2) = 1 / (K * d^2)$

Now, we can solve for d_air:

$d\_air^2 = K * d^2$

Taking the square root of both sides:

$d\_air = \sqrt{(K * d^2)}$

$d\_air = \sqrt{(K)} * d$

So, the equivalent distance between the charges in air for the same electrostatic force is $d\sqrt{K}$.

Therefore, the correct answer is "$d\sqrt{K}$."

Question 8. The maximum vertical height to which a man can throw a ball is 136m. The maximum horizontal distance upto which he can throw the same ball is:

(1) 192 m
(2) 272 m
(3) 136 m
(4) 68 m

Answer. 272 m

Solution. To find the maximum horizontal distance (d) a man can throw a ball when the maximum vertical height (h) is given, we can use the following equation of motion:

$h=g \cdot d^2/2v^2$

Where:

h is the maximum vertical height (136 m in this case).
g is the acceleration due to gravity (approximately 9.81 m/s²).
d is the maximum horizontal distance (which we want to find).
v is the initial velocity with which the ball is thrown.

We can rearrange this equation to solve for dd:

$d = \sqrt{(2 \cdot h \cdot v2/g)}$

Now, we need to find the value of vv, the initial velocity. To do that, we can use the following equation of motion:

$h = v^2 \sin^2(\theta)/2g$

Where:

h is the maximum vertical height (136 m in this case).
g is the acceleration due to gravity (approximately 9.81 m/s²).
v is the initial velocity (which we want to find).
θ is the launch angle.

We are assuming the ball is thrown vertically upward, so θ=90°, and sin(90°) = 1. Therefore, the equation becomes:

$h = v^2/2g$

Now, solve for v:

$v = \sqrt{(2 \cdot g \cdot h)}$

Substitute the known values:

$v = \sqrt{(2 \cdot 9.81\ m/s^2 \cdot 136\ m)}$

Now, calculate v:

$v = \sqrt{(2677.92\ m^2/s^2)}$

$v \approx 51.75\ m/s$

Now that we have found v, we can calculate dd using the first equation:

$d = \sqrt{(2 \cdot 136\ m \cdot (51.75\ m/s)^2/9.81\ m/s^2}$

Now, calculate d:

$d \approx \sqrt{(2 \cdot 136\ m \cdot 2677.92\ m^2/s^2/9.81\ m/s^2}$

$d \approx 722901.12\ m^3/s^2/9.81\ m/s^2$

$d \approx 73735.37\ m$

$d \approx 271.33\ m$

So, the maximum horizontal distance the man can throw the ball is approximately 271.33 meters. Therefore, the answer is 272 m (rounded to the nearest meter).

Question 12. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth R_e = 6400 km)?

(1) 9.8 N
(2) 19.6 N
(3) 4.9 N
(4) 8 N

Answer. 8 N

Solution. Wearth = 18 N
$mg_{earth} = 18$
Also $mg_h = mg_{earth}(R/(R+h)^2$
$= 18\ (6400/6400+3200)^2$
$= 18 * 4/9 = 8$

Question 13. A conducting circular loop of radius $10/\sqrt{\pi}$ cm is placed perpendicular to a uniform magnetic field of 0.5 T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is:

(1) 5 mV
(2) 10 mV
(3) 1 mV
(4) 100 mV

Answer. 10 mV

Solution. $\varepsilon_{ind} = |-d\varphi/dt|$
$= (dB/dt)A$
$= (0.5/0.5)\pi(1/10\sqrt{\pi})^2$
$= 1*1/100$ V
$= 0.01$ V
$= 10$ mV

Question 14. Two long straight wires P and Q carrying equal current 10A each were kept parallel to each other at 5cm distance. Magnitude of magnetic force experienced by 10 cm length of wire P is F1. If distance between wires is halved and currents on them are doubled, force F2 on 10 cm length of wire P will be:

(1) F1 / 10
(2) F1 / 8
(3) 10 F1
(4) 8 F1

Answer. 8 F1

Solution. The magnetic force experienced by a current-carrying wire due to another parallel current-carrying wire can be calculated using the formula:

$F=\mu 0 \cdot I1 \cdot I2 \cdot L / 2\pi \cdot d$

Where:

FF is the magnetic force between the wires.
$\mu 0$ is the permeability of free space ($\mu 0=4\pi \times 10^{-7}\ T \cdot m/A$).
I1 and I2 are the currents in the two wires.
L is the length of the wire under consideration.
d is the separation distance between the wires.

Given that $I1=I2=10$ A, $L=10$ cm$=0.1$ m and the initial separation distance dd is 5 cm = 0.05 m, we can calculate the initial force F1:

$F1=4\pi \times 10^{-7}\ T \cdot m/A \cdot 10\ A \cdot 10\ A \cdot 0.1\ m / 2\pi \cdot 0.05\ m = 0.08\ N$
Now, let's consider the new scenario where the separation distance d is halved (0.025 m) and the currents on both wires are doubled (20 A each). We want to find the new force F2. Using the same formula:

$F2=4\pi \times 10^{-7}\ T \cdot m/A \cdot 20\ A \cdot 20\ A \cdot 0.1\ m / 2\pi \cdot 0.025\ m=0.32\ N$

So, the force F2 on the 10 cm length of wire P, when the distance between the wires is halved and the currents are doubled, is 0.32 N.

Therefore, $F2=8 \cdot F1$.

Question 15. Given below are two statements:
Statement 1: An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement 2: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.
In the light of the above statements, choose the correct answer from the options given below:

(1) Statement 1 is false but Statement 2 is true
(2) Statement 1 is true but Statement 2 is false
(3) Both Statement 1 and Statement 2 are true
(4) Both Statement 1 and Statement 2 are false

Answer. Statement 1 is true but Statement 2 is false

Solution. Statement 1: An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

This statement is true. When the weight of the elevator is balanced with the tension in the cable, it can move with uniform speed either upward or downward because there is no net force acting on it.

Statement 2: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

This statement is false. When an elevator goes down with increasing speed, the apparent weight of a person inside the elevator decreases. This means that the force exerted by the floor of the elevator on the person's foot is less than their actual weight, not more.

So, the correct answer is: Statement 1 is true but Statement 2 is false.

Question 24. A hole is drilled in a metal sheet. At 27°C, the diameter of hole is 5 cm. When the sheet is heated to 177°C, the change in the diameter of hole is d*10^{-3} cm. What will be the value of d if coefficient of linear expansion of the metal is 1.6*10^{-5} /°C.

Answer. 12

Solution. To find the change in diameter (dd) when the metal sheet is heated from 27°C to 177°C, we can use the formula for linear expansion:

$\Delta L = \alpha \cdot L \cdot \Delta T$

Where:

- ΔL is the change in length (in this case, the change in diameter).
- $\alpha\alpha$ is the coefficient of linear expansion.
- L is the original length (in this case, the original diameter).
- ΔT is the change in temperature.

We are given:

- The original diameter (L) is 5 cm.
- The coefficient of linear expansion (α) is $1.6 \times 10-5\,/°C$.
- The change in temperature (ΔT) is $177°C - 27°C = 150°C$.

Now, let's calculate dd:

$d = \alpha \cdot L \cdot \Delta T = (1.6 \times 10-5\,/°C) \cdot (5\,cm) \cdot (150°C) = 0.00012\,cm = 1.2 \times 10-4\,cm$
So, the value of d is $1.2 \times 10-4$ cm 0r 12× 10^{-3} cm.

Question 25. Assume that protons and neutrons have equal masses. Mass of a nucleon is 1.6*10^{-27} kg and radius of nucleus is 1.5*10^{-15} A$^{1/3}$ m. The approximate ratio of the nuclear density and water density is n*10^{13}. The value of n is?

Answer. 11

Solution. density of nuclei = mass of nuclei / volume of nuclei
$p = 1.6*10^{-27} / 4/3\ \pi\ (1.5*10^{-15})^3$ A
$= 1.6*10^{-27} / (14.14*10^{-45}) = 0.013 * 10^{18}$
$p_w = 10^3$
Hence $p/p_w = 11.31 * 10^{13}$
The value of n is 11

Question 26. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5th second. The force acted on the body is?

Answer. 40 N

Solution. To find the force acting on the spherical body, you can use Newton's second law of motion, which relates force (F), mass (m), and acceleration (a):
F=ma
You already know the mass of the body is 2 kg. To find the acceleration, you can use the formula for kinetic energy (KE):
$KE=1/2mv^2$
Where:

KE is the kinetic energy (given as 10000 J).
m is the mass (2 kg).
v is the final velocity of the body.

You want to find the acceleration, so rearrange this equation to solve for v:

$v^2 = 2KE/m$

Now, calculate the velocity:

$v^2=2\times10000\ J/2\ kg=10000\ m^2/s^2$

$v = \sqrt{10000\ m^2/s^2} = 100\ m/s = 100 m/s$

Now that you have the final velocity, you can calculate the acceleration using the following kinematic equation:

$v=u+at$

Where:

vv is the final velocity (100 m/s).
uu is the initial velocity (which is 0 m/s since the body starts from rest).
aa is the acceleration (what we want to find).
tt is the time (5 seconds).

Rearrange the equation to solve for aa:

$a=v-u/t=100\ m/s-0/5\ s=20\ m/s^2$

Now that you have the acceleration, you can use Newton's second law to calculate the force:

$F=ma=2\ kg\times 20\ m/s^2=40\ N$

So, the force acted on the body is 40 Newtons.

Chemistry

Question 31. The magnetic moment of a transition metal compound has been calculated to be 3.87 B.M. The metal ion is?

(1) V^{2+}
(2) Ti^{2+}
(3) Cr^{2+}
(4) Mn^{2+}

Answer. V^{2+}

Solution. To determine the metal ion in the transition metal compound with a calculated magnetic moment of 3.87 Bohr magnetons (B.M.), you can use the following formula:

Magnetic Moment (μ) = √[n(n+2)]

Where: μ = Magnetic Moment in Bohr magnetons (B.M.) n = Number of unpaired electrons

The number of unpaired electrons (n) in the transition metal ion can be calculated based on its electron configuration. Let's calculate n for each of the given transition metal ions:

1. V^{2+} (Vanadium ion): Electron configuration: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^3$ Number of unpaired electrons (n) = 3
2. Ti^{2+} (Titanium ion): Electron configuration: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^2$ Number of unpaired electrons (n) = 2
3. Cr^{2+} (Chromium ion): Electron configuration: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^4$ Number of unpaired electrons (n) = 4
4. Mn^{2+} (Manganese ion): Electron configuration: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^5$ Number of unpaired electrons (n) = 5

Now, let's calculate the magnetic moments for each of these ions:

1. For V^{2+}: μ = √[3(3+2)] = √[3(5)] = √15 ≈ 3.87 B.M.

2. For Ti^{2+}: $\mu = \sqrt{[2(2+2)]} = \sqrt{[2(4)]} = \sqrt{8} \approx 2.83$ B.M.
3. For Cr^{2+}: $\mu = \sqrt{[4(4+2)]} = \sqrt{[4(6)]} = \sqrt{24} \approx 4.90$ B.M.
4. For Mn^{2+}: $\mu = \sqrt{[5(5+2)]} = \sqrt{[5(7)]} = \sqrt{35} \approx 5.92$ B.M.

The calculated magnetic moment of 3.87 B.M. matches the value for V^{2+}. Therefore, the metal ion in the transition metal compound is V^{2+} (Vanadium ion).

Question 32. Assertion A: Hydrolysis of an alkyl chloride is a slow reaction but in the presence of NaI, the rate of the hydrolysis increases.
Reason R: I^- is a good nucleophile as well as a good leaving group.

In the light of the above statements, choose the correct answer from the options given below

(1) A is true but R is false
(2) Both A and R are true but R is not the correct explanation of A
(3) A is false but R is true
(4) Both A and R are true and R is the correct explanation of A

Answer. Both A and R are true and R is the correct explanation of A

Solution. The given assertion and reason are related to the hydrolysis of alkyl chlorides in the presence of NaI.

Assertion A: Hydrolysis of an alkyl chloride is a slow reaction but in the presence of NaI, the rate of hydrolysis increases.

Reason R: I- is a good nucleophile as well as a good leaving group.

Both Assertion A and Reason R are true, and Reason R is the correct explanation of Assertion A.

In the presence of NaI, the iodide ion (I-) acts as a nucleophile and attacks the alkyl chloride, leading to the formation of an alkyl iodide. Additionally,

iodide ions can also serve as leaving groups. This dual role of iodide ions facilitates the hydrolysis reaction and increases its rate.

So, the correct answer is "Both A and R are true, and R is the correct explanation of A."

Question 34. Statement 1: For colloidal particles, the value of colligative properties are of small order as compared to values shown by true solutions at same concentration.
Statement 2: For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement 1 and Statement 2 are false
(2) Statement 1 is true but Statement 2 is false
(3) Statement 1 is false but Statement 2 is true
(4) Both Statement 1 and Statement 2 are true

Answer. Statement 1 is true but Statement 2 is false

Solution. For colloidal particles value of colligative properties is less as compared to true solutions at same concentration as number of particles are less. But fixed layer and diffused layer have opposite
Charges. So, Statement 1 is true but Statement 2 is false

Question 36. Given below are two statements:
Statement 1: Noradrenaline is a neurotransmitter
Statement 2: Low level of noradrenaline is not the cause of depression in human.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement 1 is correct but Statement 2 is incorrect
(2) Both Statement 1 and Statement 2 are incorrect
(3) Both Statement 1 and Statement 2 are correct
(4) Statement 1 is incorrect but Statement 2 is correct

Answer. Statement 1 is correct but Statement 2 is incorrect

Solution. Statement 1 is correct but Statement 2 is incorrect.

Explanation:

Statement 1: Noradrenaline is a neurotransmitter.

This statement is correct. Noradrenaline (also known as norepinephrine) is indeed a neurotransmitter in the human body. It is a chemical messenger that plays a crucial role in the functioning of the nervous system and is involved in various physiological processes, including the "fight or flight" response, mood regulation, and stress responses.

Statement 2: Low level of noradrenaline is not the cause of depression in humans.

This statement is incorrect. Low levels of noradrenaline have been implicated in some cases of depression. In fact, one of the theories related to depression involves a chemical imbalance in neurotransmitters, including noradrenaline. Depression is a complex condition with multiple contributing factors, and it is not solely caused by low levels of noradrenaline. However, alterations in neurotransmitter levels, including noradrenaline, can contribute to the development and severity of depressive symptoms in some individuals. The relationship between neurotransmitters and depression is an active area of research in psychiatry and neuroscience.

Question 37. Reaction of BeO with ammonia and hydrogen fluoride gives A which on thermal decomposition gives BeF_2 and NH_4F. What is 'A'?

(1) $(NH_4)BeF_3$
(2) $(NH_4)Be_2F_5$
(3) H_3NBeF_3
(4) $(NH_4)_2BeF_4$

Answer. $(NH_4)_2BeF_4$

Solution. The reaction given is:

BeO + NH3 + HF → A → BeF2 + NH4F

To find out what 'A' is, let's analyze the reaction step by step.

1. BeO + NH3 + HF → A (unknown compound)
2. A (unknown compound) → BeF2 + NH4F

In the second step, 'A' decomposes to form BeF2 and NH4F. The compound 'A' must contain both Be and F, as it eventually forms BeF2. Also, it contains NH4, which eventually forms NH4F.

The only option that satisfies these criteria is:

(NH4)2BeF4

So, 'A' is likely (NH4)2BeF4.

Question 38. Order of Covalent bond;
A. KF > KI ; LiF > KF
B. KF < KI ; LiF > KF
C. $SnCl_4 > SnCl_2$; CuCl > NaCl
D. LiF >KF ; CuCl < NaCl
E. KF < KI ; CuCl > NaCl
Choose the correct answer
(1) B,C,E only
(2) C,E only
(3) B,C only

(4) A,B only

Answer. B,C,E only

Solution. Let's analyze the given options:

1. KF > KI; LiF > KF
2. KF < KI; LiF > KF
3. SnCl4 > SnCl2; CuCl > NaCl
4. LiF > KF; CuCl < NaCl
5. KF < KI; CuCl > NaCl

To determine the correct order of covalent bond character, we need to consider the electronegativities of the atoms involved.

Option 1: KF > KI; LiF > KF This option suggests that LiF has the highest covalent character, which is true because Li is the most electronegative among these elements.

Option 2: KF < KI; LiF > KF This option suggests that LiF has the highest covalent character, which is correct.

Option 3: SnCl4 > SnCl2; CuCl > NaCl This option does not provide information about LiF, KF, KI, so it's not relevant to the question.

Option 4: LiF > KF; CuCl < NaCl This option suggests that LiF has the highest covalent character, which is correct.

Option 5: KF < KI; CuCl > NaCl This option suggests that LiF has the highest covalent character, which is not accurate.

So, the correct options are 1, 2, and 4:

1. KF > KI; LiF > KF
2. KF < KI; LiF > KF
3. LiF > KF; CuCl < NaCl

The correct answer is B, C, and E.

Question 39. Which of the following is true about freons?

(1) All radicals are called freons.
(2) These are chlorofluorocarbon compounds.
(3) These are radicals of chlorine and chlorine monoxide.
(4) These are chemicals causing skin cancer.

Answer. These are chlorofluorocarbon compounds

Solution. The correct statement about freons is:

- These are chlorofluorocarbon compounds.

Freons are a group of chlorofluorocarbon (CFC) compounds that were commonly used in refrigeration and air conditioning systems. However, they have been largely phased out due to their detrimental effects on the ozone layer, which led to the development of more environmentally friendly alternatives. The other statements are not accurate descriptions of freons.

Question 40. An ammoniacal metal salt solution gives a brilliant red precipitate on addition of dimethylglyoxime. The metal ion is:

(1) Cu^{2+}
(2) Fe^{2+}
(3) Ni^{2+}
(4) Co^{2+}

Answer. Ni^{2+}

Solution. The metal ion that gives a brilliant red precipitate on addition of dimethylglyoxime is Ni2+ (nickel ions). This reaction is a characteristic test for the presence of nickel ions in a solution.

Question 41. Which of the Phosphorus oxoacid can create silver mirror from $AgNO_3$ solution?

(1) $H_4P_2O_5$
(2) $H_4P_2O_7$

(3) $(HPO_3)_n$
(4) $H_4P_2O_6$

Answer. $H_4P_2O_5$

Solution. Correct option is (2) H4P2O5 as Oxyacid having P–H bond can reduce AgNO3 to Ag.

Question 47. The primary and secondary valencies of cobalt respectively in $[Co(NH_3)_5Cl]Cl_2$ are:
(1) 3 and 5
(2) 2 and 8
(3) 3 and 6
(4) 2 and 6

Answer. 3 and 6

Solution. In [Co(NH3)5Cl]Cl2, the primary valency of cobalt is 3, and the secondary valency is 6.

Mathematics

Question 66. The relation R = {(a,b): gcd (a,b) = 1, 2a ≠ b, a, b, e, ℤ} is:
(1) Reflexive but not symmetric

(2) Neither symmetric nor transitive
(3) Symmetric but not transitive
(4) Transitive but not reflexive

Answer. Neither symmetric nor transitive

Solution. The relation R = {(a,b): gcd (a,b) = 1, 2a ≠ b, a, b ∈ $\mathbb{Z}$} is:

Neither symmetric nor transitive.

Explanation:

1. It is not symmetric because, for example, if (a,b) belongs to R, then gcd(a,b) = 1, which means both a and b are coprime. However, this does not necessarily mean that (b,a) would also belong to R. So, symmetry does not hold.
2. It is not transitive because, for example, if (a,b) and (b,c) belong to R, it means that gcd(a,b) = gcd(b,c) = 1, but it doesn't imply that gcd(a,c) must also be 1. Therefore, transitivity does not hold.
3. It is reflexive because for any integer a, gcd(a,a) = a, and since the condition is gcd(a,a) = 1, it is true for all integers.

So, the relation is reflexive but not symmetric or transitive.

Question 68. The distance of the point (7,-3,-4) from the plane passing through the points (2,-3,1) (-1,1,-2) and (3,-4,2) is:
(1) 4√2
(2) 5

(3) 5√2
(4) 4

Answer. 5√2

Solution. Correct option is 5√2

$$= \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$$x - z\ -1 = 0$$

Distance of P (7, –3, –4) from Plane is

$$d = \left|\frac{7+4-1}{\sqrt{2}}\right| = 5\sqrt{2}$$

Question 69. The distance of the point (-1,9,-16) from the plane 2x+3y-z = 5 measured parallel to the line (x+4)/3 = (2-y)/4 = (z-3)/12 is?

(1) 20√2
(2) 31
(3) 26
(4) 13√2

Answer. 26

Solution. Correct option is 26
Equation of line:

$$\frac{x+1}{3}=\frac{y-9}{-4}=\frac{z+16}{12}$$

G.P on line $(3\lambda-1,\ -4\lambda+9,\ 12\lambda-16)$

point of intersection of line & plane

$6\lambda-2-12\lambda+27-12\lambda+16=5$

$\lambda=2$

Point (5, 1, 8)

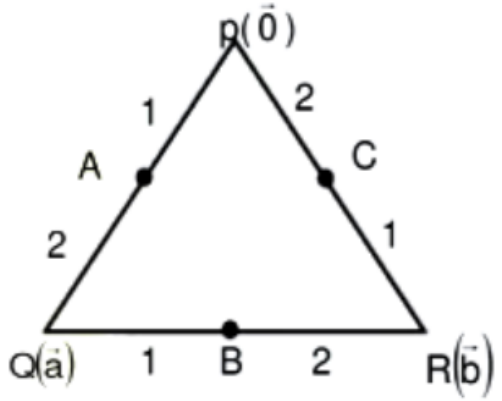
Distance $=\sqrt{36+64+576}=26$

Question 71. Let PQR be a triangle.The points A, B and C are on the sides QR,RP and PQ respectively such that QA/AR = RB/BP = PC/CQ = 1/2 . Then Area(△PQR) / Area(△ABC) is equal to?

(1) 4
(2) 2
(3) 5/2
(4) 3

Answer. 3

Solution. Correct answer is 3



Let position vector of P, Q R be $\bar{o}, \bar{a}$ & $\bar{b}$ respectively

$$\Rightarrow \text{P.V of A} = \frac{\bar{a}}{3},\ \text{P.V of B} = \frac{2\bar{a}+\bar{b}}{3} \text{ and } \text{P.V of C} = \frac{2\bar{b}}{3}$$

$$\therefore \overline{AB} = \frac{\vec{a}+\vec{b}}{3} \ \& \ \overline{BC} = \frac{\bar{b}-2\bar{a}}{3}$$

$$\Delta PQR = \frac{1}{2}\left|\overline{PQ}\times\overline{PR}\right| = \frac{1}{2}\left|\bar{a}\times\bar{b}\right|$$

$$\Delta ABC = \frac{1}{2}\left|\overline{AB}\times BC\right| = \frac{1}{2}\left|\left(\frac{(\bar{a}+\bar{b})}{3}\times\frac{(\bar{b}-2\bar{a})}{3}\right)\right| = \frac{1}{2}\left|\frac{\bar{a}\times\bar{b}}{3}\right|$$

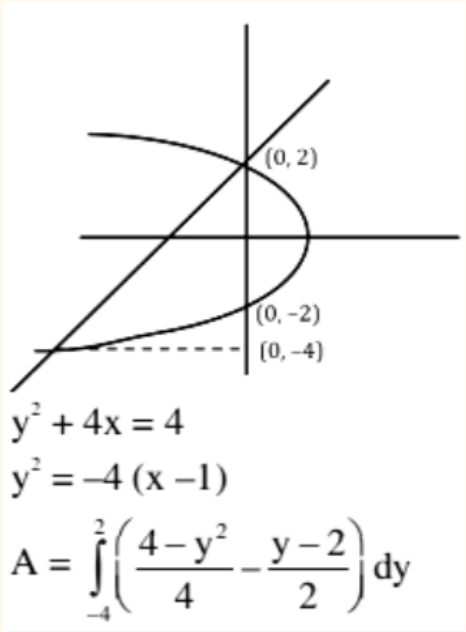
$$= \frac{\Delta PQR}{\Delta ABC} = 3$$

Question 76. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is:

(1) 25/3
(2) 22/3
(3) 23/3
(4) 9

Answer. 9

Solution. Correct option is 9



$y^2 + 4x = 4$

$y^2 = -4(x - 1)$

$$A = \int_{-4}^{2} \left(\frac{4 - y^2}{4} - \frac{y - 2}{2} \right) dy$$

$=[2y-y^3/12-y^2/4]^2_{-4}$
$=(4-2/3-1)-(-8+16/3-4)$
$=3+12-18/3$
$=15-6$
$=9$

Question 77. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the?
(1) Directrix 4x = -3
(2) Length of latus rectum 3/2
(3) Directrix 4x = 3
(4) Length of latus rectum 2

Answer. Directrix 4x = 3

Solution. Correct option is directrix 4x = 3

$y^2 = 24x$

$a = 6$

$xy = 2$

$AB \equiv ty = x + 6t^2$ .....(1)

$AB \equiv T = S_1$

$kx + hy = 2hk$ ..... (2)

From (1) and (2)

$\frac{k}{1} = \frac{h}{-t} = \frac{2hk}{-6t^2}$

$\Rightarrow$ then locus is $y^2 = -3x$

Therefore directrix is $4x = 3$

Question 82. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is?

Answer. 7

Solution. Given curve

$$9x^2 + 16y^2 = 144$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$$

$\therefore a = 4$ and $b = 3$

So, general point on the ellipse is = $(4\cos\theta, 3\sin\theta)$

We know, Equation of tangent to a given ellipse at its point $(a\cos\theta, b\sin\theta)$ is $x\cos\theta/a + y\sin\theta/b = 1$

∴ Here equation of tangent at point (4cos θ, 3sinθ) is xcosθ/4+ysinθ/3=1
When this tangent cut's x axis then y = 0.

$$\therefore \frac{x\cos\theta}{4} + 0 = 1$$

$$\Rightarrow x = 4\sec\theta$$

∴ Point of intersection at x axis is A(4sec θ,0). When this tangent cut's y axis then x = 0.

$$\therefore 0 + \frac{y\sin\theta}{3} = 1$$

$$\Rightarrow y = 3\cos ec\theta$$

∴ Point of intersection at y axis is B(0,3cosec θ).
∴ Length of AB

$$= \sqrt{(4\sec\theta - 0)^2 + (0 - 3\cos ec\theta)^2}$$

$$= \sqrt{16\sec^2\theta + 9\cos ec^2\theta}$$

$$= \sqrt{16(1 + \tan^2\theta) + 9(1 + \cot^2\theta)}$$

$$= \sqrt{25 + 16\tan^2\theta + 9\cot^2\theta}$$

We know, $AM \geq GM$

$$\therefore \frac{16\tan^2\theta + 9\cot^2\theta}{2} \geq \sqrt{(16\tan^2\theta)(9\cot^2\theta)}$$

$$\Rightarrow 16\tan^2\theta + 9\cot^2\theta \geq 2(4\tan\theta)(3\cot\theta)$$

$$\Rightarrow 16\tan^2\theta + 9\cot^2\theta \geq 2 \times 4 \times 3$$

$$\Rightarrow 16\tan^2\theta + 9\cot^2\theta \geq 24$$

$$\therefore AB = \sqrt{25 + 16\tan^2\theta + 9\cot^2\theta}$$

≥ √(25 +24)
≥ √49
≥ 7
∴ Minimum length of AB = 7.

Question 84. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is?

Answer. 546

Solution. For at most two language courses
$= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$

Question 85. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places is?

Answer. 60

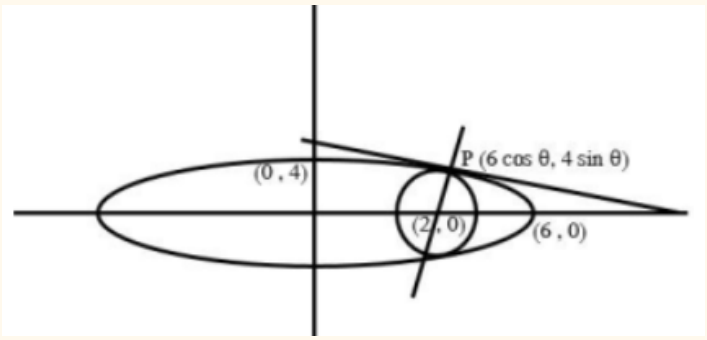
Solution. Even digits occupy at even places

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = \frac{24 \times 120}{4 \times 12} = 60$$

Question 89. Let C be the largest circle centered at (2,0) and inscribed in the ellipse $x^2/36 + y^2/16 = 1$. If $(1,\alpha)$ lies on C, then $10\alpha^2$ is equal to?

Answer. 118

Solution.



Equation of normal of ellipse $x^2/36 + y^2/16=1$ at any point $P(6\cos\theta,4\sin\theta)$ is

$3\sec\theta x-2\operatorname{cosec}\theta y=10$ this normal is also the normal of the circle passing through the point (2,0) So,

$6\sec\theta=10$ or $\sin\theta=0$ (Not possible) $\cos\theta = 3/5$ and $\sin\theta = 4/5$ so point $P=(18/5,16/5)$

So the largest radius of circle $r=\sqrt{320}/5$

So the equation of circle $(x-2)^2+y^2=64/5$

Passing it through $(1,\alpha)$

Then $\alpha^2=59/5$

$10\alpha^2=118$