# JEE Main 2023 Solutions (Feb 1 - Shift 2)

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## Physics

**Question 1. The escape velocities of two planets A and B are in the ratio 1 : 2. If the ratio of their radii respectively is 1 : 3, then the ratio of acceleration due to gravity of planet A to the acceleration of gravity of planet B will be:**
**(A) 3/2**
**(B) 2/3**
**(C) 3/4**
**(D) 4/3**

**Answer.** 3/4

**Solution.** The escape velocity (v) of a planet is given by the formula:

$v = \sqrt{(2 * g * R)}$

Where:

- v is the escape velocity
- g is the acceleration due to gravity
- R is the radius of the planet

We are given the following ratios:

1. Escape velocity ratio for planet A to planet B: 1 : 2

2. Radius ratio for planet A to planet B: 1 : 3

Let's assume the escape velocity of planet A is vA, the escape velocity of planet B is vB, the radius of planet A is RA, and the radius of planet B is RB.

According to the given ratios:

1. vA / vB = 1 / 2
2. RA / RB = 1 / 3

We can rewrite the escape velocity formula for both planets:

For planet A: vA = √(2 * gA * RA)

For planet B: vB = √(2 * gB * RB)

Now, we can square both sides of the first equation to eliminate the square root:

(vA^2) = 2 * gA * RA

Next, we square both sides of the second equation:

(vB^2) = 2 * gB * RB

Now, let's divide the two equations:

(vA^2) / (vB^2) = (2 * gA * RA) / (2 * gB * RB)

Simplify the equation:

(vA^2) / (vB^2) = (gA * RA) / (gB * RB)

Since we have the ratios of vA/vB and RA/RB from the given information, we can substitute these values:

(1/2)^2 = (gA * (1/3)) / (gB * 1)

1/4 = (1/3) * (gA / gB)

Now, solve for the ratio of gA to gB:

gA / gB = (1/4) / (1/3)

gA / gB = (1/4) * (3/1)

gA / gB = 3/4

So, the ratio of the acceleration due to gravity of planet A to the acceleration due to gravity of planet B is 3/4.

Therefore, the correct answer is (C) 3/4.

**Question 3. Two objects A and B are placed at 15 cm and 25 cm from the pole in front of a concave mirror having radius of curvature 40 cm. The distance between images formed by the mirror is:**
**(1) 100 cm**
**(2) 160 cm**
**(3) 40 cm**
**(4) 60 cm**

**Answer.** 160 cm

**Solution.** To find the distance between the images formed by the concave mirror, we can use the mirror formula:

$1/f=1/v-1/u$

Where:

- ff is the focal length of the concave mirror,
- vv is the image distance (positive for real images),
- uu is the object distance (positive for real objects).

The focal length of a concave mirror with a radius of curvature (R) is half of the radius:

$f=R/2=40\text{ cm}/2=20\text{ cm}$

For Object A (15 cm in front of the mirror): $u_A = -15$ cm (negative because it's in front of the mirror) $f = 20$ cm

Now, we can use the mirror formula to find the image distance ($v_A$) for Object A:

$1/20 = 1/v_A + 1/-15$

Solve for $v_A$:

$1/v_A = 1/20 - 1/15$

$1/v_A = 3/60 - 4/60$

$1v_A = -1/60$

$v_A = -60$ cm (negative because it's a real image on the same side as the object)

For Object B (25 cm in front of the mirror): $u_B = -25$ cm (negative because it's in front of the mirror) $f = 20$ cm

Using the mirror formula for Object B:

$1/20 = 1/v_B + 1/-25$

Solve for $v_B v_B$:

$1/v_B = 1/20 - 1/25$

$1/v_B = 5/100 - 4/100$

$1/v_B = 1/100$

$v_B = 100$ cm (positive because it's a real image on the opposite side of the object)

Now, to find the distance between the images of Object A and Object B, you simply subtract the positions of the images:

Distance between images $= |v_B - v_A| = |100 \text{ cm} - (-60 \text{ cm})| = 160 \text{ cm}$

So, the distance between the images formed by the mirror is 160 cm, and the correct answer is (2) 160 cm.

**Question 4. In an amplitude modulation, a modulating signal having amplitude of X V is superimposed with a carrier signal of amplitude Y V in first case. Then, in second case, the same modulating signal is superimposed with different carrier signal of amplitude 2Y V. The ratio of modulation index in the two cases respectively will be:**
**(1) 2 : 1**
**(2) 4 : 1**
**(3) 1 : 1**
**(4) 1 : 2**

**Answer.** 2 : 1

**Solution.** In amplitude modulation (AM), the modulation index (also known as the modulation depth) is a measure of how much the carrier signal's amplitude is varied by the modulating signal.

The modulation index (MI) is defined as the ratio of the peak amplitude of the modulating signal (X) to the peak amplitude of the carrier signal (Y). Therefore, the formula for modulation index is:

$MI = X / Y$

In the first case, the modulating signal has an amplitude of X V, and the carrier signal has an amplitude of Y V. So, the modulation index in the first case is:

$MI1 = X / Y$

In the second case, the modulating signal is the same (amplitude of X V), but the carrier signal has an amplitude of 2Y V. So, the modulation index in the second case is:

$MI2 = X / (2Y)$

To find the ratio of modulation index in the two cases (MI1:MI2), you can divide MI1 by MI2:

(MI1 / MI2) = (X / Y) / (X / (2Y))

Now, simplify the expression:

(MI1 / MI2) = (X / Y) / (X / (2Y)) = (X / Y) * (2Y / X)

The X and Y terms cancel out:

(MI1 / MI2) = 2

So, the ratio of modulation index in the two cases is 2:1. Therefore, the correct answer is (1) 2:1.

**Question 6. Choose the correct statement about Zener diode**
**(1) It works as a voltage regulator in both forward and reverse bias.**
**(2) It works as a voltage regulator in forward bias and behaves like simple pn junction diode in reverse bias.**
**(3) It works as a voltage regulator only in forward bias**
**(4) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.**

**Answer.** It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

**Solution.** The correct answer is (4) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

**Question 7. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.**
**Assertion A : Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one.**
**Reason R: Capacitance of metallic spheres depend on the radii of spheres.**
**In the light of the above statements, choose the correct answer from the options given below:**
**(1) A is false but R is true**
**(2) A is true but R is false**
**(3) Both A and R are true and R is the correct explanation of A**
**(4) Both A and R are true but R is not the correct explanation of A**

**Answer.** A is false but R is true

**Solution.** (1) A is false but R is true Potential of a conducting sphere is

$$V = \frac{KQ}{R} \text{ (Solid as well as hollow)}$$

$$V_1 = V_2 \text{ and } R_1 = R_2$$

$$\therefore Q_1 = Q_2$$

**Question 9. The ratio of average electric energy density and total average energy density of electromagnetic wave is:**
**(1) 1/2**
**(2) 3**
**(3) 2**
**(4) 1**

**Answer.** 1/2

**Solution.** The correct answer is:

(1) 1/2

The average electric energy density (uE) and the total average energy density (u) of an electromagnetic wave are related by the following equation:

uE = (1/2) * u

So, the ratio of the average electric energy density to the total average energy density is 1/2.

**Question 12. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.**
**Assertion A: For measuring the potential difference across a resistance of 600 , the voltmeter with resistance 1000 will be preferred over voltmeter with resistance 4000 .**
**Reason R: Voltmeter with higher resistance will draw smaller current than voltmeter with lower resistance.**
**In the light of the above statements, choose the most appropriate answer from the options given below:**
**(1) A is correct but R is not correct**
**(2) Both A and R are correct but R is not the correct explanation of A**
**(3) A is not correct but R is correct**
**(4) Both A and R are correct and R is the correct explanation of A**

**Answer.** A is not correct but R is correct

**Solution.** (1) A is not correct but R is correct In this case, the voltmeter with resistance 4000 ohm would be preferred because it would draw less current than the voltmeter with resistance 1000 ohm, leading to a more accurate measurement of the potential difference across the resistance of 600 ohm.

**Question 17. For a body projected at an angle with the horizontal from the ground, choose the correct statement.**
**(1) The Kinetic Energy (K.E.) is zero at the highest point of projectile motion**
**(2) The vertical component of momentum is maximum at the highest point.**
**(3) The horizontal component of velocity is zero at the highest point.**
**(4) Gravitational potential energy is maximum at the highest point**

**Answer.** Gravitational potential energy is maximum at the highest point

**Solution.** Gravitational potential energy is maximum at the highest point. At highest point
$V_y = 0$
$V_x = u_x = u\cos\theta$
$U_g = mgh$, it is maximum at $H_{max}$.

**Question 19. A Carnot engine operating between two reservoirs has efficiency 1/3. When the temperature of cold reservoir raised by x, its efficiency decreases to 1/6. The value of x, if the temperature of hot reservoir is 99°C, will be**
**(1) 33 K**
**(2) 16.5 K**
**(3) 66 K**
**(4) 62 K**

**Answer.** 62 K

**Solution.** The Carnot engine efficiency (η) is given by the formula:

$\eta = 1 - (T_c/T_h)$

Where:

- η is the efficiency of the engine.
- Tc is the absolute temperature of the cold reservoir.
- Th is the absolute temperature of the hot reservoir.

In the first case, the efficiency is 1/3, and the hot reservoir is at 99°C, which is equivalent to 99 + 273.15 = 372.15 K. So, for the first case:

1/3 = 1 - (Tc/372.15)

Now, in the second case, the efficiency is 1/6, and the temperature of the cold reservoir is raised by x, so it becomes (Tc + x). Therefore, for the second case:

1/6 = 1 - ((Tc + x)/372.15)

Now, we can set up a system of equations with these two equations:

1/3 = 1 - (Tc/372.15) 1/6 = 1 - ((Tc + x)/372.15)

Now, let's solve for x. First, solve the first equation for Tc:

Tc = 372.15 * (1 - 1/3) Tc = 372.15 * (2/3) Tc = 248.1 K

Now, plug this value of Tc into the second equation and solve for x:

1/6 = 1 - ((248.1 + x)/372.15)

Now, solve for x:

1/6 = (372.15 - 248.1 - x)/372.15

1/6 = (124.05 - x)/372.15

Now, cross-multiply:

372.15 = 6 * (124.05 - x)

Now, isolate x:

6 * (124.05 - x) = 372.15

124.05 - x = 372.15/6

124.05 - x = 62.025

Now, subtract 124.05 from both sides to find x:

x = 124.05 - 62.025 x = 62.025 K

So, the value of x is 62.025 K. Therefore, option (4) is the correct answer.

**Question 20. A coil is placed in magnetic field such that plane of coil is perpendicular to the direction of magnetic field. The magnetic flux through a coil can be changed:**
**A. By changing the magnitude of the magnetic field within the coil.**
**B. By changing the area of coil within the magnetic field.**
**C. By changing the angle between the direction of magnetic field and the plane of the coil.**
**D. By reversing the magnetic field direction abruptly without changing its magnitude.**
**Choose the most appropriate answer from the options given below:**
**(1) A, B and C only**
**(2) A and B only**
**(3) A, B and D only**
**(4) A and C only**

**Answer.** A, B and C only

**Solution.** The correct answer is (1) A, B and C only

The magnetic flux through a coil is given by the following equation:

$\Phi = B * A * \cos(\theta)$

where:

- $\Phi$ is the magnetic flux in Webers (Wb)

- B is the magnetic field strength in Teslas (T)
- A is the area of the coil in square meters (m^2)
- θ is the angle between the direction of the magnetic field and the plane of the coil

Therefore, the magnetic flux through a coil can be changed by:

- Changing the magnitude of the magnetic field (B)
- Changing the area of the coil (A)
- Changing the angle between the direction of the magnetic field and the plane of the coil (θ)

Reversing the magnetic field direction abruptly without changing its magnitude does not change the magnetic flux through the coil, because the cosine of 180 degrees is -1, which is the same as the cosine of 0 degrees.

Therefore, the most appropriate answer is (1) A, B and C only.

# Chemistry

**Question 31. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**
**Assertion (A): $Cu^{2+}$ in water is more stable than $Cu^{+}$.**
**Reason (R): Enthalpy of hydration for $Cu^{2+}$ is much less than that of $Cu^{+}$.**
**In the light of the above statements, choose the correct answer from the options given below:**
**(1) (A) is not correct but (R) is correct**
**(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)**
**(3) (A) is correct but (R) is not correct**
**(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A)**

**Answer.** Both (A) and (R) are correct and (R) is the correct explanation of (A)

**Solution.** The correct answer is (2): Both (A) and (R) are correct and (R) is the correct explanation of (A).

$Cu^{2+}$ is more stable than $Cu^{+}$ in water because it has a greater hydration energy. Hydration energy is the energy released when an ion attracts water molecules around it. $Cu^{2+}$ has a greater charge density than $Cu^{+}$, which means that it can attract water molecules more strongly. This releases more hydration energy, which makes $Cu^{2+}$ more stable.

Therefore, both Assertion (A) and Reason (R) are correct, and (R) is the correct explanation of (A).

**Question 35. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).**
**Assertion (A): Gypsum is used for making fireproof wall boards.**
**Reason (R): Gypsum is unstable at high temperatures.**
**In the light of the above statements, choose the correct answer from the options given below**
**(1) (A) is not correct but (R) is correct**
**(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)**
**(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)**
**(4) (A) is correct but (R) is not correct**

**Answer.** Both (A) and (R) are correct but (R) is not the correct explanation of (A)

**Solution.** The correct answer is:

(2) Both (A) and (R) are correct, but (R) is not the correct explanation of (A).

Gypsum is indeed used for making fireproof wall boards, which is stated in assertion (A). However, the reason (R) provided, which states that gypsum is unstable at high temperatures, is not the correct explanation for why gypsum is used for fireproofing. Gypsum actually contains water molecules in its chemical structure, and when heated, it releases this water slowly, which helps to retard the spread of fire and makes it a suitable material for fireproofing applications. Therefore, while both (A) and (R) are correct, (R) is not the correct explanation for (A).

**Question 36. Which element is not present in Nessler's reagent?**
**(1) Mercury**
**(2) Iodine**
**(3) Potassium**
**(4) Oxygen**

**Answer.** Oxygen

**Solution.** The correct answer is (4) Oxygen.

Nessler's reagent is a chemical solution used to detect the presence of ammonia. It is made up of potassium tetraiodomercurate(II) (K2HgI4) dissolved in water. The chemical formula of K2HgI4 does not contain any oxygen atoms.

The other three elements, mercury, iodine, and potassium, are all present in Nessler's reagent.

Therefore, the answer is (4) Oxygen.

**Question 37. The starting material for convenient preparation of deuterated hydrogen peroxide ($D_2O_2$) in laboratory is:**
**(1) K2S2O8**
**(2) BaO**
**(3) BaO2**
**(4) 2-ethylanthraquinol**

**Answer.** K2S2O8

**Solution.** Correct option is (1)
K2S2O8 K2S2O8(s) + 2D2O(l) → 2KDSO4(aq.) + D2O2

**Question 39. Given below are two statements :**
**Statement I : Sulphanilic acid gives esterification test for carboxyl group.**
**Statement II : Sulphanilic acid gives red colour in Lassigne's test for extra element detection.**
**In the light of the above statements, choose the most appropriate answer from the options given below :**
**(1) Statement I is incorrect but Statement II is correct**

**(2) Both Statement I and Statement II are correct**
**(3) Both Statement I and Statement II are incorrect**
**(4) Statement I is correct but Statement II is incorrect**

**Answer.** Statement I is incorrect but Statement II is correct

**Solution.** The correct answer is (1) Statement I is incorrect but Statement II is correct.

Sulphanilic acid is a primary aromatic amine that does not contain a carboxyl group. Therefore, it cannot give an esterification test.

However, sulphanilic acid does give a red color in Lassigne's test for extra element detection. This is because sulphanilic acid reacts with sodium fusion extract to form a red-colored azo dye.

Therefore, Statement I is incorrect but Statement II is correct.

**Question 40. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).**
**Assertion (A) : An aqueous solution of KOH when used for volumetric analysis, its concentration should be checked before the use.**
**Reason (R) : On aging, KOH solution absorbs atmospheric $CO_2$.**
**In the light of the above statements, choose the correct answer from the options given below :**
**(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)**
**(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)**
**(3) (A) is correct but (R) is not correct**
**(4) (A) is not correct but (R) is correct**

**Answer.** Both (A) and (R) are correct and (R) is the correct explanation of (A)

**Solution.** The correct answer is (2) Both (A) and (R) are correct and (R) is the correct explanation of (A).

Aqueous solutions of KOH absorb atmospheric $CO_2$ over time, forming potassium carbonate ($K_2CO_3$). This reaction is as follows:

$2KOH + CO_2 \rightarrow K_2CO_3 + H_2O$

As a result, the concentration of KOH in the solution decreases over time. This is why it is important to check the concentration of an aqueous solution of KOH before using it for volumetric analysis.

Therefore, both Assertion (A) and Reason (R) are correct, and (R) is the correct explanation of (A).

**Question 46. The industrial activity held least responsible for global warming is:**
**(1) Electricity generation in thermal power plants**
**(2) industrial production of urea**
**(3) manufacturing of cement**
**(4) steel manufacturing**

**Answer.** industrial production of urea

**Solution.** The industrial activity held least responsible for global warming is industrial production of urea.

Urea is a nitrogenous fertilizer that is essential for crop production. It is produced by the reaction of ammonia and carbon dioxide. The ammonia used in this reaction is typically produced from natural gas, which is a fossil fuel. However, the carbon dioxide used in the reaction is often captured from other industrial processes, such as coal-fired power plants. This means that the overall carbon footprint of urea production is relatively low.

The other industrial activities listed in the question are all major contributors to global warming. Electricity generation in thermal power plants is the largest source of greenhouse gas emissions in the world. Manufacturing of cement and steel are also major emitters of greenhouse gases.

Therefore, the answer is (2) industrial production of urea.

**Question 49. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**
**Assertion (A): -halocarboxylic acid on reaction with dil NH3 gives good yield of -amino carboxylic acid whereas the yield of amines is very low when prepared from alkyl halides.**
**Reason (R): Amino acids exist in zwitter ion form in aqueous medium.**
**In the light of the above statements, choose the correct answer from the options given below:**
**(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)**
**(2) (A) is not correct but (R) is correct**
**(3) (A) is correct but (R) is not correct**
**(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)**

**Answer.** Both (A) and (R) are correct and (R) is the correct explanation of (A)

**Solution.** The correct answer is (4) Both (A) and (R) are correct and (R) is the correct explanation of (A).

When an α-halocarboxylic acid is treated with dilute ammonia, it reacts to form an α-amino carboxylic acid. This reaction is known as the Gabriel Synthesis. The yield of the α-amino carboxylic acid is typically good.

On the other hand, when an alkyl halide is treated with ammonia, it reacts to form a primary amine. However, the yield of the primary amine is

typically low. This is because the primary amine can further react with ammonia to form a secondary amine, and then a tertiary amine.

The reason why α-halocarboxylic acids react with ammonia to form α-amino carboxylic acids in good yield is because the amino acids exist in zwitterion form in aqueous medium. Zwitterions are molecules that have both a positive and a negative charge on them. In the case of amino acids, the positive charge is on the amino group and the negative charge is on the carboxylate group.

The zwitterion form of the amino acid is more stable than the amino acid in its free form. This is because the zwitterion form is less likely to undergo further reactions.

Therefore, when an α-halocarboxylic acid is treated with ammonia, the amino acid that is formed immediately exists in zwitterion form. This prevents the amino acid from further reacting with ammonia to form secondary or tertiary amines.

Therefore, both Assertion (A) and Reason (R) are correct, and (R) is the correct explanation of (A).

**Question 52. A → B**
**The above reaction is of zero order. Half life of this reaction is 50 min. The time taken for the concentration of A to reduce to one-fourth of its initial value is _____ min. (Nearest Integer)**

**Answer.** 75

**Solution.** Assume reaction starts with 1 mole A

$$\left(t_{1/2}=\frac{a}{2k}, K=\frac{1}{2\times 50}\right.$$

For 75% completion

$$a-\frac{a}{4}=kt$$

$$t=\frac{3}{4}\frac{a}{k}=\frac{3}{4}\times\frac{100}{a}=75$$

**Question 57. Among the following, the number of tranquilizer/s is /are _____.**

**A. Chloroliazepoxide**
**B. Veronal**
**C. Valium**
**D. Salvarsan**

**Answer.** 3

**Solution.** Correct option is Valium, chlorodiazepoxide and Veronal are tranquilizer where as salvarsan is antibiotic.

# Mathematics

**Question 61. The number of integral values of k, for which one root of the equation $2x^2 – 8x + k = 0$ lies in the interval (1, 2) and its other root lies in the interval (2, 3) is:**
**(1) 0**
**(2) 1**
**(3) 2**
**(4) 3**

**Answer.** 1

**Solution.** To find the integral values of k for which one root of the equation 2x^2 - 8x + k = 0 lies in the interval (1, 2) and its other root lies in the interval (2, 3), you can use the following approach:

1. The discriminant of the quadratic equation should be greater than zero to have real roots.
2. The first root should be between 1 and 2, and the second root should be between 2 and 3.

Let's break this down:

First root between 1 and 2: 1 < x1 < 2

Second root between 2 and 3: 2 < x2 < 3

For a quadratic equation in the form ax^2 + bx + c = 0, the sum of the roots is given by:

x1 + x2 = -b/a

In this case, a = 2 and b = -8, so:

x1 + x2 = 8/2 = 4

Now, we want one root between 1 and 2 and the other between 2 and 3, which means that the sum of the roots should be between 3 and 5.

$1 + 2 < x1 + x2 < 2 + 3$ $3 < x1 + x2 < 5$

We already know that $x1 + x2 = 4$. To have it fall within the range (3, 5), k must be an integral value such that $3 < x1 + x2 < 5$. Therefore, there is only one integral value of k that satisfies this condition:

$k = 1$

So, the answer is (2) 1.

**Question 70. Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that one the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then**
**(1) B and C are independent**
**(2) The number of favourable cases of the events A, B and C are 15, 6 and 6 respectively**
**(3) A and B are mutually exclusive is 6**
**(4) The number of favourable cases of the event (A U B) ∩ C is 6.**

**Answer.** The number of favourable cases of the event (A U B) ∩ C is 6.

**Solution.** Let's analyze each part of the question:

A is the event that the number appeared on the 1st die is less than the number appeared on the 2nd die.

B is the event that the number appeared on the 1st die is even and that on the second die is odd.

C is the event that the number appeared on the 1st die is odd and that on the 2nd die is even.

(1) B and C are independent: Since event B is defined by the first die being even and the second die being odd, and event C is defined by the first die being odd and the second die being even, they are complementary events. In other words, if B occurs, C cannot, and vice versa. Therefore, B and C are mutually exclusive, not independent.

(2) The number of favorable cases of the events A, B, and C are 15, 6, and 6, respectively: Let's calculate the number of favorable cases for each event:

For event A, where the first die is less than the second die: There are 15 possible outcomes (1-6 on the first die and 2-6 on the second die). So, this statement is correct.

For event B, where the first die is even and the second die is odd: There are 3 even numbers (2, 4, 6) on the first die and 3 odd numbers (1, 3, 5) on the second die. So, there are 3 x 3 = 9 favorable cases, not 6 as stated in the statement.

For event C, where the first die is odd and the second die is even: Similarly, there are 3 odd numbers (1, 3, 5) on the first die and 3 even numbers (2, 4, 6) on the second die. So, there are 3 x 3 = 9 favorable cases, not 6 as stated in the statement.

So, this statement is not correct.

(3) A and B are mutually exclusive: This statement is not correct. Events A and B are not mutually exclusive because an outcome can satisfy both A and B (e.g., the first die shows a 2, which is even and less than any outcome on the second die).

(4) The number of favorable cases of the event (A U B) ∩ C is 6: Let's find the number of favorable cases for the event (A U B) ∩ C. This event

represents cases where the first die is less than or equal to the second die, and the first die is odd while the second die is even.

The possible outcomes that satisfy this condition are: (1, 2), (1, 4), (1, 6), (3, 4), (3, 6), and (5, 6). There are 6 favorable cases, which matches the statement.

So, the correct answer is (4) The number of favorable cases of the event (A U B) ∩ C is 6.

**Question 73. Let the plane P pass through the intersection of the planes 2x + 3y – z = 2 and x + 2y + 3z = 6, and be perpendicular to the plane 2x + y – z + 1 = 0. If d is the distance of P from the point (–7, 1,1), then $d^2$ is equal to :**

**(1) 25/83**
**(2) 250/83**
**(3) 250/82**
**(4) 15/53**

**Answer.** 250/83

**Solution.** Correct answer is 250/83

$P \equiv P_1 + \lambda P_2 = 0$

$(2+\lambda)x + (3+2\lambda)y + (3\lambda - 1)z - 2 - 6\lambda = 0$

Plane P is perpendicular to $P_3 \therefore \vec{n}.\vec{n}_3 = 0$

$2(\lambda + 2) + (2\lambda + 3) - (3\lambda - 1) = 0$

$\lambda = -8$

$P \equiv -6x - 13y - 25z + 46 = 0$

$6x + 13y + 25z - 46 = 0$

Dist from (–7, 1, 1)

$$d = \left|\frac{-42 + 13 + 25 - 46}{\sqrt{36 + 169 + 625}}\right| = \frac{50}{\sqrt{830}}$$

$$d^2 = \frac{50 \times 50}{830} = \frac{250}{83}$$

**Question 80. The sum of absolute maximum and minimum values of the function $f(x) = |x^2 – 5x + 6| – 3x + 2$ in the interval $[–1, 3]$ is equal to:**

**(1) 12**

**(2) 24**

**(3) 13**

**(4) 10**

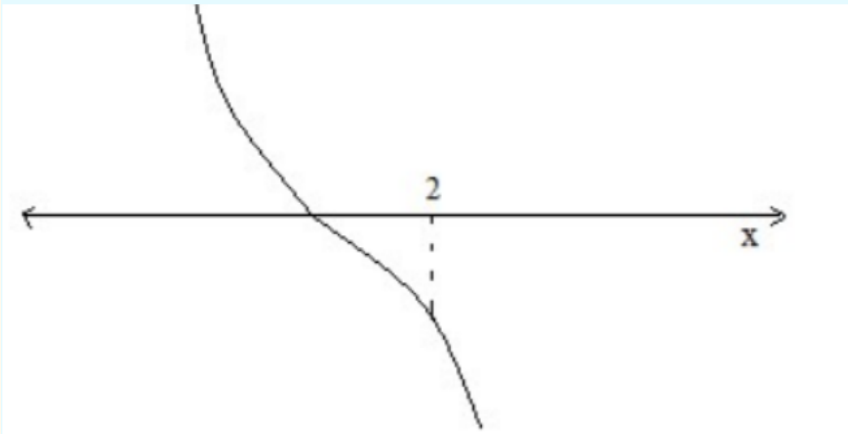
**Answer.** 10

**Solution.** Given $f(x)=|x2-5x+6|-3x+2$

$\Rightarrow f(x) = x^2 -8x+8;\ x\in[-1,2]$

$f(x) = -x^2 +2x-4;\ x\in[2,3]$

f (x) is decreasing function in [-1, 3]




$\therefore$ fmax = f(−1) = 17

fmin = f(3) = -7

$\therefore$ fmax + fmin = 10

**Question 82. If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of the chord is equal to _____.**

**Answer.** 16

**Solution.** Given $y2=8x+4y+4$

⇒y2−4y=8x+4
⇒y2−4y+4=8x+8
⇒(y−2)2=8(x+1)
Parabola with vertex V(-1, 2) and 4a=8
⇒ a = 2
So focus S(a+α,β)=S(2−1,2)=(1,2)
Let the equation of focal chord of the parabola x/a+y/b=1 and given x-intercept of focal chord is '3'.
so x/3+y/b=1 and passing through focus (1, 2)
so 1/3+2/b=1
2/b=1−1/3=2/3⇒b=3
Equation of focal chord x+y=3
So, slope of focal chord
∴ m = −1
tanθ = −1
⇒ θ= 3π/4
We know that length of focal chord in 4acosec2θ = 4(2)(√2)2 = 8(2) = 16

**Question 83. Number of integral solutions to the equation x + y + z = 21, where x ≥ 1, y ≥ 3, z ≥ 4, is equal to _____.**

**Answer.** 105

**Solution.** x+y+z=21,x≥1,y≥3,z≥4
⇒ (x−1)+(y−3)+(z−4)=21−8=13
⇒ α+β+γ=13,α≥0,β≥0,γ≥0
where α=x−1,β=y−3 & γ=z−4
∴ No. of integral solution = n+r−1Cr−1
where n = 13, r = number of variables = 3
∴ No. of integral solution = 13+3−1C3−1=15C2
=15×14/2 =105

**Question 85. The sum of the common terms of the following three arithmetic progressions.**
**3, 7, 11, 15, …, 399,**
**2, 5, 8, 11, …, 359 and**
**2, 7, 12, 17, …, 197,**
**Is equal to _____.**

**Answer.** 321

**Solution.** 3, 7, 11, 15, ……, 399 d1 = 4
2, 5, 8, 11,……., 359 d2 = 3
2, 7, 12, 17,……., 197 d3 = 5
LCM (d1, d2, d3) = 60
Common terms are 47, 107, 167
Sum = 321

**Question 86. The line x = 8 is the directrix of the ellipse E = x2/a2 + y2/b2 = 1 with the corresponding focus (2, 0). If the tangent to E at point P in the first quadrant passes through the point (0, $4\sqrt{3}$ ) and intersects the x-axis at Q, then $(3PQ)^2$ is equal to?**

**Answer.** 39

**Solution.** Correct answer is 39

$\frac{a}{e} = 8$ .....(1) $\quad ae = 2$ .....(2)

$8e = \frac{2}{e}$

$e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

$a = 4$

$b^2 = a^2(1 - e^2)$

$= 16\left(\frac{3}{4}\right) \quad = 12$

$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

$P\left(2\sqrt{3}, \sqrt{3}\right)$

Q (8/√3, 0 )

$(3PQ)^2 = 39$

**Question 88. Let $\alpha$x + $\beta$y + yz = 1 be the equation of a plane passing through the point (3, –2, 5) and perpendicular to the line joining the points (1, 2, 3) and (–2, 3, 5). Then the value of $\alpha\beta$y is equal to _____?**

**Answer.** 6

**Solution.** Normal vector of plane = 3^i−^j−2^k

Plane : 3x – y – 2z + λ = 0

Point (3, –2, 5) satisfies the plan

λ = –1

3x – y – 2z = 1

αβy = 6

**Question 90. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is _____?**

**Answer.** 81

**Solution.** Correct answer is 81

Taking single digit $\rightarrow$ 444444 $\quad \frac{6!}{6!} = 1$

Taking two digit $\rightarrow$

(4, 5) 444555 (4, 9) 444999

$\frac{5!}{3!2!} = 10$ $\quad$ $\frac{5!}{3!2!} = 10$

Taking three digit

$4, 5, 9, 4, 4, 4 \Rightarrow \frac{5!}{3!} = 20$

$4, 5, 9, 5, 5, 5 \Rightarrow \frac{5!}{4!} = 5$

$4, 5, 9, 9, 9, 9 \Rightarrow \frac{5!}{4!} = 5$

$4, 5, 9, 4, 5, 9 \Rightarrow \frac{5!}{2!2!} = 30$

Total = 81