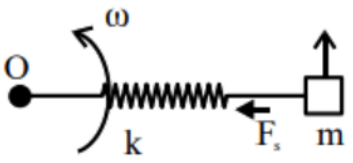
JEE Main 2023 Solutions
Jan 24 - Shift 2

Physics

Question 1. A body of mass 200 g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5 rad/s. Then the ratio of extension in the spring to its natural length will be:
(1) 2 : 3
(2) 1 : 2
(3) 1 : 1
(4) 2 : 5

Answer. 2 : 3

Solution. Correct option is 2:3



Natural length = L_0

Extension = x

$$kx = m(L_0 + x)\omega^2$$

$$\Rightarrow 12.5x = \frac{1}{5}(L_0 + x)25 \quad \Rightarrow 1.5x = L_0$$

$$\Rightarrow \frac{x}{L_0} = \frac{2}{3}$$

Question 3. The frequency (v) of an oscillating liquid drop may depend upon radius (r) of the drop, density (p) of liquid and the surface tension (s) of the liquid as : v = ra pb sc. The values of a, b and c respectively are:

(1) (3/2, 1/2, -1/2)
(2) (-3/2, 1/2, 1/2)
(3) (-3/2, -1/2, 1/2)
(4) (3/2, -1/2, 1/2)

Answer. (-3/2, -1/2, 1/2)

Solution. To determine the values of a, b, and c, we can use the method of dimensional analysis.

We know that in dimensional analysis, each physical quantity can be expressed in terms of fundamental dimensions such as length (L), mass (M), and time (T).

In this case, we have the following variables:

v (frequency, T^{-1})
r (radius, L)
p (density, ML^{-3})
s (surface tension, MT^{-2})

Now, let's express vv in terms of r, p, and s using dimensional analysis:

$v=r^a \cdot p^b \cdot s^c$
Now, let's analyze the dimensions of each term:

r^a has dimensions $[L]^a$
p^b has dimensions $[M]^b \cdot [L]^{-3b}$
s^c has dimensions $[M]^c \cdot [T]^{-2c}$

For the equation to be dimensionally consistent, the dimensions on both sides must match. Therefore, we have:

On the left side, the dimension of v is $[T]^{-1}$.

On the right side, the dimension will be:

$[L]^a \cdot [M]^b \cdot [L]-3^b \cdot [M]^c \cdot [T]^{-2c}$

Now, we equate the dimensions:

$[-1]=[L]^a \cdot [M]^b \cdot [L]-3^b \cdot [M]^c \cdot [T]^{-2c}$

Now, let's equate the dimensions separately for length, mass, and time:

For length (L):

$0=a-3b$

For mass (M):

$0=b+c$

For time (T):

$-1=-2c$

Solving these equations simultaneously:

From the first equation, we have $a=3b$.

From the second equation, we have $c=-b$.

From the third equation, we have $c=1/2$.

Now, substitute the values of a and c into the equation for b:

c=−b becomes 1/2=−b.

So, b=−1/2.

Now, we can find the values of a, b, and c:

a=3b=3(−1/2)=−3/2

b=−1/2

c=1/2
Therefore, the values of aa, bb, and cc are −3/2, −1/2, and 1/2 respectively:
a=−3/2
b=−1/2
c=1/2

Question 5. If the distance of the earth from Sun is 1.5 × 106 km. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is:
(1) 3×10^6 km
(2) 3×10^7 km
(3) 6×10^7 km
(4) 6×10^6 km

Answer. 3×10^6 km

Solution. We can use Kepler's third law of planetary motion to find the distance of the imaginary planet from the Sun. Kepler's third law states that the square of the period of revolution (T) of a planet is directly proportional to the cube of the semi-major axis of its orbit (a).

Mathematically, it can be expressed as:

$T2 \propto a^3$

Let's denote the period of revolution of the imaginary planet as T_p and the distance of the planet from the Sun as a_p. We are given that:

T_p=2.83 years

We also know that the period of Earth's revolution around the Sun (Te) is 1 year, and the distance of the Earth from the Sun (ae) is 1.5×10^6 km.

Now, let's use the ratios of T and aa for the Earth and the imaginary planet:

$T_p/T_e=(a_pa_e)^{3/2}$

Substitute the known values:

2.83 years/1 year=(ap/1.5×10^6 km)$^{3/2}$

Now, let's solve for a_p:

(a_p/1.5×10^6 km)$^{3/2}$ = 2.83
Take the square root of both sides:

a_p/1.5×10^6 km=√2.83

Now, solve for a_p:

ap=1.5×106 km×2.83ap=1.5×106 km×2.83

Calculate the value:

$a_p\approx1.5\times10^6$ km×1.68≈2.52×10^6 km

So, the distance of the imaginary planet from the Sun is approximately 2.52×10^6 km.

None of the given options matches this value. It's possible that there may be a rounding error in the question or options. However, the closest option to the calculated value is (1) 3×10^6 km, so that would be the best choice.

Question 6. Let γ1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio γ1/γ2 is:

(1) 21/25
(2) 35/27
(3) 27/35
(4) 25/21

Answer. 25/21

Solution. To find the ratio γ1/γ2 for monoatomic and diatomic gases, we'll use the concept of degrees of freedom.

For a monoatomic gas, like a noble gas (e.g., helium, argon), there are three translational degrees of freedom and zero rotational degrees of freedom. Therefore, the total degrees of freedom (f_1) for a monoatomic gas is 3.

For a diatomic gas, like nitrogen (N_2) or oxygen (O_2), there are three translational degrees of freedom, two rotational degrees of freedom (since it can rotate about two axes perpendicular to the bond), and two vibrational degrees of freedom (if we consider vibration at higher temperatures). However, for a rigid rotator approximation, we'll consider only the rotational degrees of freedom. Therefore, the total degrees of freedom (f2) for a diatomic gas is 5.

Now, the molar specific heat at constant pressure (C_p) and molar specific heat at constant volume (C_v) for an ideal gas are related to the degrees of freedom (f) by the equation:

$\gamma=C_p/C_v=f+2/f$

For the monoatomic gas (f1=3), we have:

$\gamma1=3+2/3=5/3$

For the diatomic gas (f_2=5), we have:

$\gamma2=5+2/5=7/5$
Now, we can find the ratio $\gamma1/\gamma2$:

$\gamma1/\gamma2 = 5/3/7/5=(5/3)\times(5/7)=25/21$

So, the ratio $\gamma1/\gamma2$ is 25/21.

Question 7. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: A pendulum clock when taken to Mount Everest becomes fast.
Reason R: The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) A is correct but R is not correct
(2) Both A and R are correct but R is NOT the correct explanation of A
(3) A is not correct but R is correct
(4) Both A and R are correct and R is the correct explanation of A

Answer. A is not correct but R is correct

Solution. Correct answer is A is not correct but R is correct.
$T\propto 1/\sqrt{g}$
Time period of pendulum is inversely proportional to acceleration due to gravity.

Question 8. Given below are two statements:
Statement I: Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.
Statement II: Acceleration due to earth's gravity is same at a height 'h' and depth 'd' from earth's surface, if h = d.
In the light of above statements, choose the most appropriate answer from the options given below.
(1) Statement I is incorrect but statement II is correct
(2) Both Statement I and Statement II are incorrect
(3) Both Statement I and Statement II are correct
(4) Statement I is correct but statement II is incorrect

Answer. Statement I is correct but statement II is incorrect

Solution. The correct answer is:

(4) Statement I is correct but statement II is incorrect.

Statement I is correct. The acceleration due to Earth's gravity decreases as you move away from the Earth's surface, whether going up or down. This is because the gravitational force weakens with distance from the Earth's center.

Statement II is incorrect. Acceleration due to Earth's gravity is not the same at a height 'h' and depth 'd' from Earth's surface, even if h = d. Gravity's strength varies with distance from the Earth's center, so the acceleration due to gravity will be different at different heights and depths.

Question 14. When a beam of white light is allowed to pass through convex lens parallel to principal axis, the different colours of light converge at different point on the principle axis after refraction. This is called:
(1) Chromatic aberration
(2) Polarisation

(3) Spherical aberration
(4) Scattering

Answer. Chromatic aberration

Solution. The correct answer is:

(1) Chromatic aberration

Chromatic aberration is the phenomenon in which different colors of light are bent by different amounts when passing through a lens, causing them to converge at different points on the principal axis. This results in a blurring or color fringing effect in images produced by the lens.

Question 17. A photon is emitted in transition from n = 4 to n = 1 level in hydrogen atom. The corresponding wavelength for this transition is (given, h = 4 × 10^{-15} eVs) :
(1) 94.1 nm
(2) 99.3 nm
(3) 974 nm
(4) 941 nm

Answer. 94.1 nm

Solution. The wavelength (λ) of the emitted photon during a transition in a hydrogen atom can be calculated using the formula:

λ=hc/E

Where:
λ = wavelength
h = Planck's constant (4 × 10^-15 eVs)
c = speed of light (approximately 3 × 10^8 m/s)
E = energy difference between the two energy levels (in this case, between n = 4 and n = 1)

The energy difference (ΔE) between two energy levels in a hydrogen atom is given by:

$\Delta E = E1 - E2 = -13.6eV(1/n_1^2 - 1/n^2_2)$
Where:
E1 and E2 are the energies of the initial and final energy levels (in this case, n1 = 4 and n2 = 1).

Let's calculate ΔE:

$\Delta E = -13.6eV(1/4^2 - 1/1^2)$
$\Delta E = -13.6eV(1/16 - 1)$
$\Delta E = -13.6eV(1/16 - 16/16)$
$\Delta E = -13.6eV(-15/16)$
$\Delta E = 13.6eV(15/16)$
Now, we can calculate the wavelength (λ):

$\lambda = \{(4\times10^{-15}eVs)\cdot(3\times10^8m/s)\}/13.6eV\cdot(15/16)$
Now, calculate λ:

$\lambda = \{(1.2\times10^{-6}eV\cdot m)\cdot(3\times10^8m/s)\}/13.6eV\cdot(15/16)$
$\lambda =)3.6\times10^2eV\cdot m\cdot m/s)/13.6eV\cdot(15/16)$

$\lambda = (3.6\times10^2eV\cdot m\cdot m/s)13.6eV\cdot(1516)$
$\lambda \approx 94.1nm$
So, the wavelength of the emitted photon is approximately 94.1 nm, which corresponds to option (1).

Question 18. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R
Assertion A : Steel is used in the construction of buildings and bridges.
Reason R : Steel is more elastic and its elastic limit is high.

In the light of above statements, choose the most appropriate answer from the options given below.
(1) A is correct but R is not correct
(2) A is not correct but R is correct
(3) Both A and R are correct but R is not the correct explanation of A
(4) Both A and R are correct and R is the correct explanation of A

Answer. Both A and R are correct and R is the correct explanation of A

Solution. Option (4) is the correct answer.

Explanation:

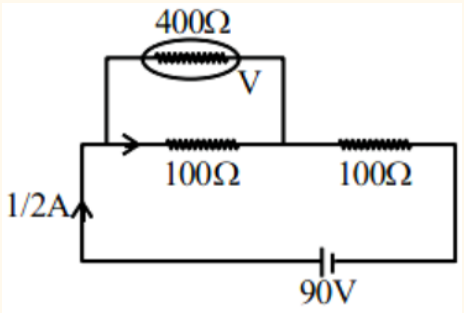
- Assertion A states that steel is used in the construction of buildings and bridges, which is a correct statement. Steel is commonly used in construction due to its high strength and durability.
- Reason R states that steel is more elastic and its elastic limit is high. This is also a correct statement. Steel is known for its elasticity and high elastic limit, which means it can deform without permanently bending or breaking, making it suitable for various structural applications.

Both Assertion A and Reason R are correct statements, and Reason R provides a valid explanation for why steel is used in construction. Therefore, option (4) is the appropriate choice.

Question 20. A cell of emf 90 V is connected across series combination of two resistors each of 100 Ω resistance. A voltmeter of resistance 400 Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be :
(1) 45 V
(2) 80 V
(3) 90 V
(4) 40 V

Answer. 40 V

Solution. Correct option is 40 V



$$R_{eq} = \frac{400 \times 100}{500} + 100$$

$$= 180\Omega$$

$$i = \frac{90}{180} = \frac{1}{2} A$$

$$\text{Reading} = \frac{1}{2} \times \frac{400}{500} \times 100$$

$$= 40 \text{volt}$$

Chemistry

Question 31. Which one amongst the following are good oxidizing agents?
A. Sm2+
B. Ce2+
C. Ce4+
D. Tb4+
Choose the most appropriate answer from the options given below.
(1) C and D only
(2) A and B only
(3) D only
(4) C only

Answer. C and D only

Solution. Good oxidizing agents are substances that readily accept electrons and get reduced in a chemical reaction. The key to identifying good oxidizing agents is to look for elements or ions with high positive oxidation states, as they tend to have a greater affinity for electrons. Let's analyze the options:

A. Sm^{2+} - This ion has a +2 oxidation state, indicating that it is already in a reduced form. It is not a good oxidizing agent.

B. Ce^{2+} - Similar to Sm^{2+}, this ion also has a +2 oxidation state and is not a good oxidizing agent.

C. Ce^{4+} - This ion has a +4 oxidation state, which is a higher oxidation state compared to +2. It is a good oxidizing agent because it can readily accept electrons to become Ce^{2+}.

D. Tb^{4+} - Similar to Ce^{4+}, this ion also has a +4 oxidation state and is a good oxidizing agent for the same reasons.

Based on the analysis:

Good oxidizing agents are C (Ce^{4+}) and D (Tb^{4+}).

So, the correct answer is:

(1) C and D only

Question 32. Which of the following cannot be explained by crystal field theory?
(1) The order of spectrochemical series
(2) Magnetic properties of transition metal complexes
(3) Colour of metal complexes
(4) Stability of metal complexes

Answer. The order of spectrochemical series

Solution. Correct answer is The order of spectrochemical series.
Crystal field theory introduce spectrochemical series based upon the experimental values of $\triangle$ but can't explain it's order. While other three points are explained by CFT. Specially when the CFSE increases thermodynamic stability of the complex increases.

Question 33. $K_2Cr_2O_7$ paper acidified with dilute H_2SO_4 turns green when exposed to
(1) Sulphur dioxide
(2) Carbon dioxide
(3) Sulphur trioxide
(4) Hydrogen sulphide

Answer. Sulphur dioxide

Solution. $K_2Cr_2O_7$ paper, which is acidified with dilute H_2SO_4, turns green when exposed to sulfur dioxide (SO_2).

So, the correct answer is option (1) - Sulphur dioxide.

Question 35. Identify the correct statements about alkali metals.
A. The order of standard reduction potential (M+ | M) for alkali metal ions is Na > Rb > Li.
B. CsI is highly soluble in water.
C. Lithium carbonate is highly stable to heat.
D. Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.
E. All the alkali metal hydrides are ionic solids.
Choose the correct answer from the options given below.
(1) C and E only
(2) A, B, D only
(3) A, B and E only
(4) A and E only

Answer. A and E only

Solution. Let's analyze each statement:

A. The order of standard reduction potential (M+ | M) for alkali metal ions is Na > Rb > Li.

This statement is incorrect. The correct order is Li > Na > K > Rb > Cs. So, this statement is false.

B. CsI is highly soluble in water.

This statement is true. CsI (Cesium iodide) is highly soluble in water.

C. Lithium carbonate is highly stable to heat.

This statement is true. Lithium carbonate (Li_2CO_3) is stable to heat.

D. Potassium dissolved in concentrated liquid ammonia is blue in color and paramagnetic.

This statement is true. Potassium dissolved in concentrated liquid ammonia is indeed blue in color and paramagnetic.

E. All the alkali metal hydrides are ionic solids.

This statement is true. Alkali metal hydrides are ionic solids.

So, the correct answer is (4) A and E only.

Question 37. The metal which is extracted by oxidation and subsequent reduction from its ore is:
(1) Al
(2) Cu
(3) Ag
(4) Fe

Answer. Ag

Solution. Correct option is Ag.

Ag.

$$4Ag + 8CN^- + O_2 + 2H_2O \rightarrow 4[Ag(CN)_2]^- + 4OH^-$$

$$2[Ag(CN)_2]^{-1} + Zn \rightarrow 2Ag\downarrow + [Zn(CN)_4]^{-2}$$

Question 38. Given below are two statements:

Statement-I : Pure Aniline and other arylamines are usually colourless.
Statement-II : Arylamines get coloured on storage due to atmospheric reduction.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both Statement-I and Statement-II are incorrect
(2) Statement-I is incorrect but Statement-II is correct
(3) Statement-I is correct but Statement-II is incorrect
(4) Both Statement-I and Statement-II are correct

Answer. Statement-I is correct but Statement-II is incorrect

Solution. The correct answer is (3) Statement-I is correct but Statement-II is incorrect.

Pure aniline and other arylamines are indeed usually colorless. However, they get colored upon exposure to air, but it's not due to atmospheric reduction. Instead, they tend to undergo oxidation in the presence of oxygen in the air, which leads to the formation of colored compounds.

Question 40. Given below are two statements, one is labelled as Assertion A : and the other is labelled as Reason R.
Assertion A : Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.
Reason R: Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Both A and R are correct and R is the correct explanation of A
(2) Both A and R are correct but R is NOT the correct explanation of A
(3) A is correct but R is not correct
(4) A is not correct but R is correct

Answer. Both A and R are correct and R is the correct explanation of A

Solution. The correct answer is (1) Both A and R are correct and R is the correct explanation of A.

Beryllium does indeed have a less negative reduction potential compared to other alkaline earth metals. This is due to the combination of its small size (leading to a high hydration energy) and a relatively large value of atomization enthalpy. The small size of Be2+ makes it highly polarizing, which contributes to its unique properties in comparison to other alkaline earth metals.

Question 41. Correct statement is:
(1) An average human being consumes more food than air
(2) An average human being consumes equal amount of food and air
(3) An average human being consumes 100 times more air than food
(4) An average human being consumes nearly 15 times more air than food

Answer. An average human being consumes nearly 15 times more air than food

Solution. An average human being consumes nearly 15 times more air than food

Question 45. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : Benzene is more stable than hypothetical cyclohexatriene.
Reason R : The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms.
In the light of the above statements, choose the correct answer from the options given below.
(1) A is false but R is true
(2) Both A and R are correct but R is NOT the correct explanation of A

(3) A is true but R is false
(4) Both A and R are correct and R is the correct explanation of A

Answer. Both A and R are correct and R is the correct explanation of A

Solution. Both A and R are correct and R is the correct explanation of A.
Assertion – A: Benzene is more stable than cyclohexatriene (True)
Reason – R: Delocalised π-e cloud lies B.M.O so more attracted by nuclei of carbon atom. (True & Correct Explanation)

Question 46. The number of s-electrons present in an ion with 55 protons in its unipositive state is:
(1) 10
(2) 9
(3) 12
(4) 8

Answer. 10

Solution. Correct option is 10
$Z = 55$ [Cs] ⇒ [Xe] $6s^1$
[Cs+] ⇒ [Xe] i.e. upto 5s count e– of s-subshell
i.e. 1s, 2s, 3s, 4s, 5s ⇒ 10 electrons

Mathematics

Question 61. The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

(1) 4
(2) 2
(3) $2\sqrt{2}$
(4) $\sqrt{6}$

Answer. $2\sqrt{2}$

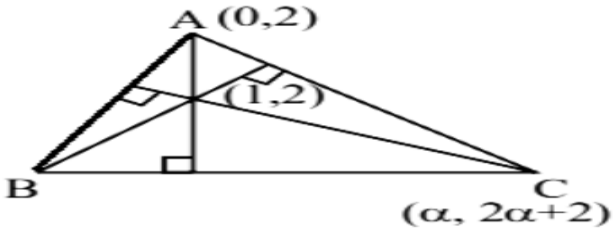
Solution. Correct answer is $2\sqrt{2}$

$AB : (\lambda+1)x + \lambda y = 4$

$AC : \lambda x + (1-\lambda)y + \lambda = 0$

Vertex A is on y-axis

$\Rightarrow x = 0$



So $y = \frac{4}{\lambda}, y = \frac{\lambda}{\lambda - 1}$

$\Rightarrow \frac{4}{\lambda} = \frac{\lambda}{\lambda - 1}$

$\Rightarrow \lambda = 2$

$AB : 3x + 2y = 4$

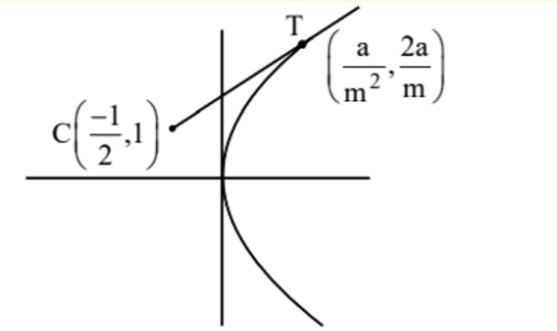
$AC : 2x - y + 2 = 0$

$\Rightarrow A(0,2)$ Let C $(\alpha, 2\alpha + 2)$

Now (Slope of Altitude through C) (-3/2) = -1

$(2\alpha / \alpha - 1)(-3/2) = -1 \Rightarrow \alpha = -½$

So C(-1/2 , 1)



Let Equation of tangent be $y = mx + \frac{3}{2m}$

$m^2 + 2m - 3 = 0$

$\Rightarrow m = 1, -3$

So tangent which touches in first quadrant at T is

$T \equiv \left(\frac{a}{m^2},\frac{2a}{m}\right)$

$\equiv \left(\frac{3}{2},3\right)$

$\Rightarrow CT = \sqrt{4+4} = 2\sqrt{2}$

Question 63. Let y = y(x) be the solution of the differential Equation $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$, $y(1) = 1$. Then $6\,y^2(e)$ is equal to:

(1) $3/2\ e^2$

(2) e^2

(3) $2e^2$

(4) $3e^2$

Answer. $2e^2$

Solution. Correct option is $2e^2$

$$(x^2 - 3y^2)\,dx + 3xy\,dy = 0$$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy} \Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3}\frac{x}{y} \quad (1)$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$(1) \Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$$

$$\Rightarrow vdv = \frac{-dx}{3x}$$

Integrating both side

$$\frac{v^2}{2} = \frac{-1}{3}\ln x + c$$

$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + c$$

$y(1) = 1$

$$\Rightarrow \boxed{\frac{1}{2} = c}$$

$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + \frac{1}{2}$$

$$\Rightarrow y^2 = -\frac{2}{3}x^2 \ln x + x^2$$

$$y^2(e) = -\frac{2}{3}e^2 + e^2 = \frac{e^2}{3}$$

$$\Rightarrow \boxed{6y^2(e) = 2e^2}$$

Question 64. The number of real solutions of the equation $3(x^2 + 1/x^2) - 2(x + 1/x) + 5 = 0$, is

(1) 3

(2) 4

(3) 0

(4) 2

Answer. 0

Solution. Let's solve the given equation:

$3(x^2 + 1/x^2) - 2(x + 1/x) + 5 = 0$

First, let's simplify the equation by multiplying through by x^2 to get rid of the fractions:

$3(x^4 + 1) - 2(x^3 + 1) + 5x^2 = 0$

Now, expand the terms:

$3x^4 + 3 - 2x^3 - 2 + 5x^2 = 0$

Rearrange the terms:

$3x^4 - 2x^3 + 5x^2 + 1 = 0$

Now, let's make a substitution to simplify this quartic equation. Let $y = x^2$:

$3y^2 - 2y + 5y + 1 = 0$

Combine like terms:

$3y^2 + 3y + 1 = 0$

Now, we have a quadratic equation in terms of y. Let's solve it using the quadratic formula:

$y = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$

In this case, $a = 3$, $b = 3$, and $c = 1$. Plugging in these values:

y = (-3 ± √(3^2 - 4(3)(1))) / (2(3))

y = (-3 ± √(9 - 12)) / 6

y = (-3 ± √(-3)) / 6

The discriminant (the value inside the square root) is negative, which means there are no real solutions for y. Therefore, there are no real solutions for x^2.

Since there are no real solutions for x^2, there are also no real solutions for x, and the answer is 0 real solutions.

Question 74. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is
(1) 48
(2) 168
(3) 220
(4) 120

Answer. 168

Solution. Four digit numbers greater than 7000 = 2 × 4 × 3 × 2 = 48
Five digit number = 5! = 120
Total number greater than 7000 = 120 + 48 = 168

Question 79. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is
(1) 225
(2) 125
(3) 150
(4) 120

Answer. 120

Solution. To count the number of square matrices of order 5 with entries from the set {0, 1} such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, we can use combinatorics.

For each row, we have 5 elements, and the sum of elements in each row must be 1. This means that we need to select one of the 5 positions in each row to place the '1', and the remaining 4 positions will have '0'. There are 5 choices for the position of '1' in each row.

So, for each row, there are 5 possible configurations.

Since there are 5 rows in the matrix, the total number of such matrices can be calculated by multiplying the number of possibilities for each row:

Total number of matrices = (Number of possibilities for row 1) x (Number of possibilities for row 2) x ... x (Number of possibilities for row 5) = 5^5 = 3125

However, this counts all matrices, including those with rows that are identical, and we need to exclude those cases.

To exclude cases where rows are identical, we need to count the number of unique row patterns. There are 5 possibilities for the first row, but once that is chosen, the second row must be different, so there are 4 possibilities for the second row. Similarly, there are 3 possibilities for the third row, 2 for the fourth row, and only 1 possibility for the fifth row.

So, the number of matrices with unique row patterns is:

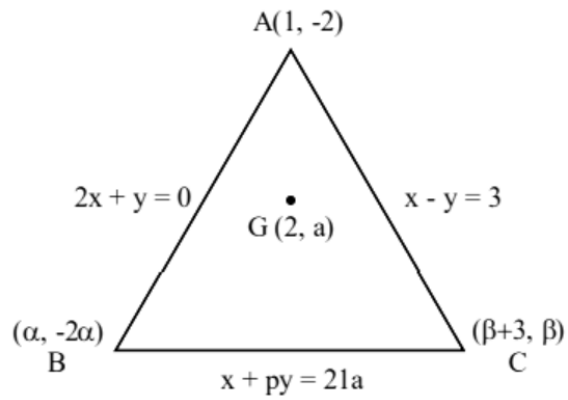
Number of matrices with unique row patterns = 5 x 4 x 3 x 2 x 1 = 120

Therefore, the number of square matrices of order 5 with entries from the set {0, 1} such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1 is (4) 120.

Question 83. The equations of the sides AB, BC and CA of a triangle ABC are : 2x + y = 0, x + py = 21a, (a ≠ 0) and x – y = 3 respectively. Let P(2, a) be the centroid of △ABC. Then $(BC)^2$ is equal to?

Answer. 122

Solution.



Assume $B(\alpha, -2\alpha)$ and $C(\beta + 3, \beta)$

$\frac{\alpha+\beta+3+1}{3}=2$ also $\frac{-2\alpha-2+\beta}{3}=a$

$\Rightarrow \alpha+\beta=2$ $-2\alpha-2+\beta=3a$

$\Rightarrow \beta=2-\alpha$ $-2\alpha-\cancel{2}+\cancel{2}-\alpha=3a \Rightarrow \alpha=-a$

Now both B and C lies as given line

$\alpha - p \cdot 2\alpha = 21a$

$\alpha(1-2p)=21\,a \qquad \ldots.. (1)$

$-a(1-2p)=21\,a \Rightarrow p=11$

$\beta+3+p\beta=21\,a$

$\beta+3+11\beta=21\,a$

$21\alpha+12\beta+3=0$

Also $\beta=2-\alpha$

$21\alpha+12(2-\alpha)+3=0$

$21\alpha+24-12\alpha+3=0$

$9\alpha+27=0$

$\alpha=-3, \beta=5$

So $BC=\sqrt{122}$ and $(BC)^2=122$

Question 86. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is _____.

Answer. 13

Solution. Given R = {(a, b), (b, c), (b, d)}
In order to make it equivalence relation as per given set, R must be {(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)}
There already given so 13 more to be added.

Question 87. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black, and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is?

Answer. 432

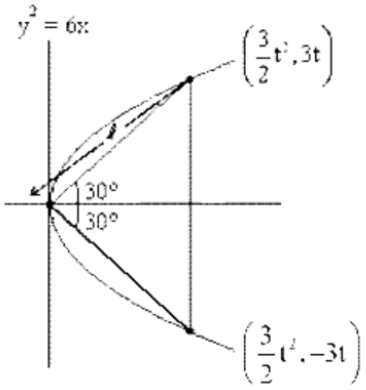
Solution.

Urn A		Urn B		Urn C	
Red	Black	Red	Black	Red	Black
4	6	5	5	λ	4

$$P\left(\frac{C}{R}\right)=\frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right)+P(B)P\left(\frac{R}{B}\right)+P(C)P\left(\frac{R}{C}\right)}$$

$$0.4=\frac{\frac{1}{3}\times\frac{\lambda}{(\lambda+4)}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\frac{\lambda}{(\lambda+4)}}$$

$$\Rightarrow \lambda=6$$



$$\tan 30^\circ = 3t \div \frac{3}{2}t^2$$

$$\frac{1}{\sqrt{3}} = \frac{2}{t}$$

$$t = 2\sqrt{3}$$

$$\left(\frac{3}{2}t^2, 3t\right) = (18, 6\sqrt{3})$$

$$\ell^2 = 18^2 + (6\sqrt{3})^2$$

$$= 324 + 108$$

$$= 432$$

Question 89. Let the sum of the coefficients of the first three terms in the expansion of $(x - 3/x^2)$, $x \neq 0$. $n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is?

Answer. 405

Solution.

Given Binomial $\left(x-\frac{3}{x^2}\right)^n, x \neq 0, n \in N,$

Sum of coefficients of first three terms

$${}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 3^2 = 376$$

$$\Rightarrow 3n^2 - 5n - 250 = 0$$

$$\Rightarrow (n-10)(3n+25) = 0$$

$$\Rightarrow n = 10$$

Now general term ${}^{10}C_r x^{10-r}\left(\frac{-3}{x^2}\right)^r$

$$= {}^{10}C_r x^{10-r}(-3)^r \cdot x^{-2r}$$

$$= {}^{10}C_r (-3)^r \cdot x^{10-3r}$$

Coefficient of $x^4 \Rightarrow 10 - 3r = 4$

$$\Rightarrow r = 2$$

$${}^{10}C_2(-3)^2 = 405$$

Question 90. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8 A is equal to?

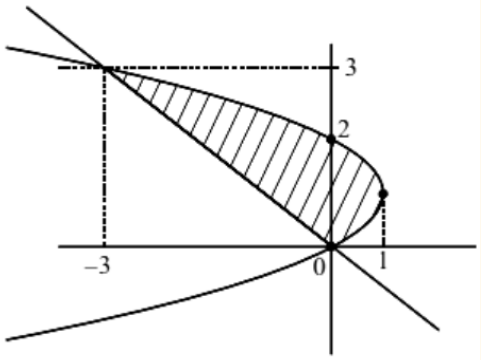
Answer. 36

Solution. $y^2-2y = -x$
$\Rightarrow y^2-2y + 1 = -x + 1$
$(y -1)^2 = -(x-1)$
$y = -x$
Point of intersection

$x^2 + 2x = -x$

$x^2 + 3x = 0$

$x = 0, -3$



$$A = \int_0^3 (-y^2 + 2y + y)dy$$

$$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3 = \frac{9}{2}$$

$8A = 36$