JEE Main 2023 Solutions
Jan 25 - Shift 2

Physics

Question 1. According to law of equipartition of energy the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is:

(1) 3/2 R
(2) 9/2 R
(3) 5/2 R
(4) 7/2 R

Answer. 7/2 R

Solution. The molar specific heat at constant volume (Cv) for a diatomic gas is given by the equipartition of energy theorem. For a diatomic gas, there are three translational degrees of freedom, two rotational degrees of freedom, and one vibrational degree of freedom for each molecule. When the molecule has one additional vibrational mode, there will be two vibrational degrees of freedom.

The equipartition of energy theorem states that each degree of freedom contributes R/2 to the molar specific heat at constant volume, where RR is the gas constant.

For a diatomic gas with three translational, two rotational, and two vibrational degrees of freedom, the molar specific heat at constant volume (Cv) is:

Cv=3/2R+2/2R+2/2R=3/2R+R+R=7/2R

So, when the molecule has one additional vibrational mode, the molar specific heat at constant volume is Cv=7/2R. Therefore, the correct answer is:

Cv=7/2R

Question 2. The light rays from an object have been reflected towards an observer from a standard flat mirror, the image observed by the observer are: -
A. Real
B. Erect
C. Smaller in size than object
D. Laterally inverted
Choose the most appropriate answer from the options given below:
(1) A, C and D Only
(2) A and D Only
(3) B and D Only
(4) B and C Only

Answer. B and D Only

Solution. The correct answer is:

(3) B and D Only

In the case of a flat mirror, the image formed is virtual, erect, the same size as the object, and laterally inverted. Therefore, the options (1), (2), and (4) are not accurate descriptions of the image formed by a standard flat mirror.

Question 5. The resistance of a wire is 5 Ω . It's new resistance in ohm if stretched to 5 times of its's original length will be

(1) 5
(2) 625
(3) 125
(4) 25

Answer. 125

Solution. The resistance of a wire is directly proportional to its length and inversely proportional to its cross-sectional area. When the wire is stretched to 5 times its original length, its resistance will increase.

The resistance (R) is given by the formula:

$R = \rho * (L / A)$

Where:

ρ is the resistivity of the material (a constant)
L is the length of the wire
A is the cross-sectional area of the wire

Since the length is increased to 5 times its original length, the new length (L') will be 5 times the original length (L). The cross-sectional area is reduced proportionally to maintain the same volume of the wire.

Therefore, if the original resistance was 5 ohms, and the length is increased by a factor of 5, the new resistance will be:

$R' = \rho * (5L / (1/5)A) = 25 * \rho * (L / A)$

So the new resistance will be 25 times the original resistance:

R' = 25 * 5 ohms = 125 ohms

The correct answer is:

(3) 125

Question 9. Given below are two statements:
Statement I: Stopping potential in photoelectric effect does not depend on the power of the light source.
Statement II: For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light.
In the light of above statements, choose the most appropriate answer from the options given below.
(1) Both Statement I and Statement II are incorrect
(2) Statement I is incorrect but statement II is correct
(3) Both Statement I and statement II are correct
(4) Statement I is correct but statement II is incorrect

Answer. Both Statement I and statement II are correct

Solution.

$$\text{Stopping potential } V_S = \frac{KE_{max}}{e}$$

$$V_S = \frac{\frac{hC}{\lambda} - \phi}{e}$$

Stopping potential does not depend on intensity or power of light used, it only depends on frequency or wavelength of incident light. So both statements I and II are correct.

Question 10. A wire of length 1m moving with velocity 8 m/s at right angles to a magnetic field of 2T. The magnitude of induced emf, between the ends of wire will be_____?

(1) 16 V
(2) 8 V
(3) 12 V
(4) 20 V

Answer. 16 V

Solution. The magnitude of the induced electromotive force (emf) in a wire moving at right angles to a magnetic field is given by Faraday's law of electromagnetic induction:

emf = B * L * v

Where:

emf is the induced electromotive force.
B is the magnetic field strength (in Tesla, T).
L is the length of the wire (in meters, m).
v is the velocity of the wire (in meters per second, m/s).

In this case, B = 2 T (2 Tesla), L = 1 m (1 meter), and v = 8 m/s (8 meters per second).

emf = 2 T * 1 m * 8 m/s = 16 V (Volts)

So, the magnitude of the induced emf between the ends of the wire is 16 V.

Question 12. Every planet revolves around the sun in an elliptical orbit:
A. The force acting on a planet is inversely proportional to square of distance from sun.
B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun.
C. The Centripetal force acting on the planet is directed away from the sun.
D. The square of time period of revolution of planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.
Choose the correct answer from the options given below:

(1) B and C only
(2) A and C only
(3) A and D only
(4) C and D only

Answer. A and D only

Solution. The correct answer is:

A and D only

Explanation:
A. Kepler's first law of planetary motion states that every planet revolves around the sun in an elliptical orbit. This means that the square of the time period of revolution of a planet around the sun is directly proportional to the cube of the semi-major axis of its elliptical orbit.

D. This statement corresponds to Kepler's third law of planetary motion, also known as the law of harmonies, which describes the relationship between a planet's orbital period (time period) and the size of its orbit. It states that the square of the time period (T) is directly proportional to the cube of the semi-major axis (a) of the elliptical orbit: $T^2 \propto a^3$.

Question 16. Statement I: When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N-type semiconductor such that P-type has excess holes and N-type has excess electrons.
Statement II: When such P-type and N-type semi-conductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ammeter.
In the light of above statements, choose the most appropriate answer from the options given below.

(1) Both statement I and statement II are incorrect
(2) Both statement I and statement II are correct
(3) Statement I is incorrect but statement II is correct
(4) Statement I is correct but statement II is incorrect

Answer. Statement I is correct but statement II is incorrect

Solution. When P-N junction is formed an electric field is generated form N-side to P-side due to which barrier potential arises & majority charge carrier can not flow through the junction due to barrier potential so current is zero unless we apply forward bias voltage.

Question 23. A body of mass 1 kg collides head on elastically with a stationary body of mass 3 kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2 m/s. The initial speed of the smaller body before collision is _____ms^{-1}.

Answer. 4

Solution. Let the initial speed of the smaller body (1 kg) be v m/s. The relative velocity of approach before the collision is the sum of their individual velocities, as they are moving in opposite directions.

Relative velocity of approach = v+0=v m/s

After the collision, the smaller body reverses its direction and moves with a speed of 2 m/s. So, its final velocity is –2 m/s.

Using the law of conservation of linear momentum, the total momentum before the collision is equal to the total momentum after the collision.

Before collision: $1\ kg \cdot v + 3\ kg \cdot 0 = 1kg \cdot v\ kg\ m/s$

After collision: $1\ kg \cdot (-2) + 3\ kg \cdot 2 = -2 + 6 = 4\ kg\ m/s$

According to the law of conservation of linear momentum,

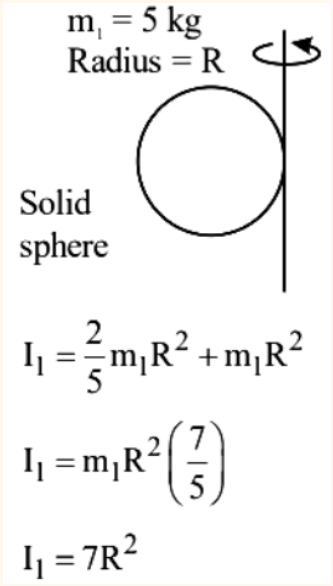
$1\ kg \cdot v = 4\ kg \cdot\ m/s$
Solving for v:

$v = 4\ kg \cdot\ m/s / 1\ kg = 4\ m/s$
The initial speed of the smaller body before the collision is 4 m/s.

Question 25. If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be x/7 . Then the value of x is _____?

Answer. 5

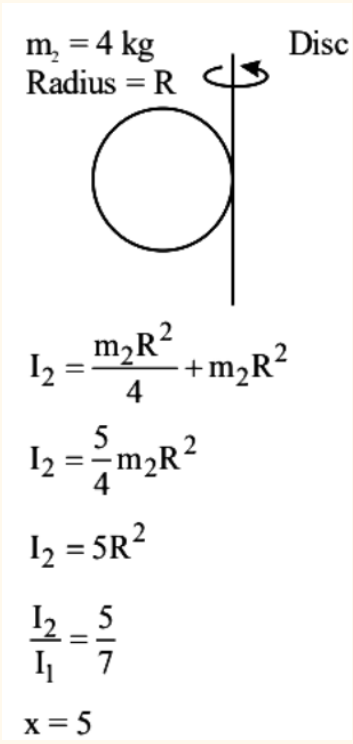
Solution. Correct answer is 5



$$I_1 = \frac{2}{5}m_1R^2 + m_1R^2$$

$$I_1 = m_1R^2\left(\frac{7}{5}\right)$$

$$I_1 = 7R^2$$



$$I_2 = \frac{m_2R^2}{4} + m_2R^2$$

$$I_2 = \frac{5}{4}m_2R^2$$

$$I_2 = 5R^2$$

$$\frac{I_2}{I_1} = \frac{5}{7}$$

$$x = 5$$

Question 27. A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330 ms–1) Hz.

Answer. 400

Solution. The observed frequency (f') of a sound source moving relative to an observer can be calculated using the Doppler effect formula:

$f' = f(v+vo)/v+vs$

Where:

f′ is the observed frequency
f is the source frequency
v is the speed of sound in air
v_o is the observer's velocity relative to the medium (positive if moving towards the source)
v_s is the source's velocity relative to the medium (positive if moving towards the observer)

In this case, the train is approaching the observer, so vsvs is negative, and the observer is stationary with respect to the medium, so vovo is zero.

Given:

Source frequency (f) = 320 Hz
Speed of sound (v) = 330 m/s
Observer's velocity relative to the medium (v_o) = 0 m/s (stationary observer)
Source's velocity relative to the medium (v_s) = 66 m/s (approaching the observer)

Now, plug these values into the Doppler effect formula:

$f' = \{320 \times (330+0)\}/330 - 66 = (320 \times 330)/264 = (320 \times 5)/4 = 400$ Hz

So, the frequency observed by the observer is 400 Hz.

Chemistry

Question 31. Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from

(1) +6 to +3
(2) +6 to +2
(3) +3 to +1
(4) +2 to +1

Answer. +6 to +3

Solution. Potassium dichromate (K2Cr2O7) acts as a strong oxidizing agent in acidic solutions. In this process, the chromium (Cr) changes its oxidation state from +6 to +3.

The half-reaction involved is:
$Cr_2O_7^{2-} \rightarrow 2Cr^{3+}$
So, the oxidation state of chromium goes from +6 to +3 during this redox reaction.

Question 34. Statement I : Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative centre and head pointing towards the positive centre.
Statement II : The crossed arrow of the dipole moment symbolized the direction of the shift of charges in the molecules.
In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct
(2) Statement I is correct but Statement II is incorrect
(3) Both Statement I and Statement II are incorrect
(4) Statement I is incorrect but Statement II is correct

Answer. Statement I is correct but Statement II is incorrect

Solution. Statement II: The crossed arrow symbolizes the direction of the shift of electron density in the molecule.

Question 35. Which one among the following metals is the weakest reducing agent?

(1) Rb
(2) Li
(3) K
(4) Na

Answer. Na

Solution. Sodium (Na) have lowest oxidation potential in alkali metals. Hence it is weakest reducing agent among alkali metals.

Question 37. Which of the following represents the correct order of metallic character of the given elements?

(1) Be < Si < K < Mg
(2) K < Mg < Be < Si
(3) Be < Si < Mg < K
(4) Si < Be < Mg < K

Answer. Si < Be < Mg < K

Solution. Metallic character increases down the group and decreases along the period. So, the correct answer is Si < Be < Mg < K

Question 38. Given below are two statements:
Statement I : In froth floatation method a rotating paddle agitates the mixture to drive air out of it.
Statement II : Iron pyrites are generally avoided for extraction of iron due to environmental reasons.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both Statement I and Statement II are false
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are true
(4) Statement I is false but Statement II is true

Answer. Statement I is false but Statement II is true

Solution. Statement I: In froth floatation method a rotating paddle agitates the mixture to drive air out of it.
Statement II: Iron pyrites are generally avoided for extraction of iron due to environmental reasons.

The correct answer is (4) Statement I is false but Statement II is true.

In froth floatation, a rotating paddle agitates the mixture to create froth, not to drive air out of it.

Iron pyrites are avoided for the extraction of iron due to the presence of sulfur impurities, which can lead to environmental pollution when the iron is extracted and processed**.**

Question 39. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R
Assertion A: The alkali metals and their salts impart characteristic colour to reducing flame.
Reason R: Alkali metals can be detected using flame tests.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) A is not correct but R is correct
(2) Both A and R are correct and R is the correct explanation of A
(3) Both A and R are correct but R is NOT the correct explanation of A
(4) A is correct but R is not correct

Answer. A is not correct but R is correct

Solution. The alkali metals and their salts impart characteristic colour to oxidizing flame.

Question 41. What is the mass ratio of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g of 0.25 molal aqueous solution and 250 mL of 0.25 molal aqueous solution?
(1) 3 : 1
(2) 1 : 2
(3) 2 : 1
(4) 1 : 1

Answer. 2 : 1

Solution. Assume : Mass of solvent ≈ Mass of solution

Case I :-

$$0.25 = \frac{W_1}{62} \times \frac{1000}{500}$$

Case II :-

$$0.25 = \frac{W_2}{62} \times \frac{1000}{250}$$

$$\frac{W_1}{W_2} = \frac{2}{1}$$

Question 45. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : Carbon forms two important oxides – CO and CO_2.CO is neutral whereas CO_2 is acidic in nature.

Reason R : CO_2 can combine with water in a limited way to form carbonic acid, while CO is sparingly soluble in water.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Both A and R are correct and R is the correct explanation of A
(2) Both A and R are correct but R is NOT the correct explanation of A
(3) A is not correct but R is correct
(4) A is correct but R is not correct

Answer. Both A and R are correct and R is the correct explanation of A

Solution. The correct answer is:

(1) Both A and R are correct, and R is the correct explanation of A.

Explanation:

- Assertion A states that carbon forms two important oxides: CO and CO2. It also mentions the nature of these oxides, with CO being neutral, and CO2 being acidic. This is a correct statement.
- Reason R provides the explanation that CO2 can combine with water to form carbonic acid, and CO is sparingly soluble in water. This is also a correct explanation.

So, both Assertion A and Reason R are correct, and Reason R provides a valid explanation for Assertion A.

Question 47. When the hydrogen ion concentration $[H^+]$ changes by a factor of 1000, the value of pH of the solution _____
(1) decreases by 2 units
(2) increases by 2 units
(3) decreases by 3 units
(4) increases by 1000 units

Answer. decreases by 3 units

Solution. When the hydrogen ion concentration [H+] changes by a factor of 1000, the value of pH of the solution decreases by 3 units.

The pH is defined as the negative logarithm (base 10) of the hydrogen ion concentration, so:

pH = -log[H+]

If the [H+] changes by a factor of 1000, it means that the new concentration is 1000 times smaller or larger than the original concentration. Taking the negative logarithm of 1000 is equal to 3 (because 10^3 = 1000). So the pH will change by 3 units.

Question 49.
A. Ammonium salts produce haze in atmosphere.
B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals.
C. Polychlorinated biphenyls act as cleansing solvents.
D. 'Blue baby" syndrome occurs due to the presence of excess of sulphate ions in water.
Choose the correct answer from the options given below.
(1) A and D only
(2) B and C only
(3) A, B and C only
(4) A and C only

Answer. A and C only

Solution. The correct answer is (4) A and C only.

Explanation:
A. Ammonium salts can react with pollutants like sulfur dioxide and nitrogen oxides in the atmosphere to produce fine particles and haze.

B. Ozone is not produced when atmospheric oxygen reacts with chlorine radicals; it is typically produced in reactions involving nitrogen oxides and volatile organic compounds in the presence of sunlight.

C. Polychlorinated biphenyls (PCBs) are not cleansing solvents; they are synthetic organic chemicals used in various industrial applications. They are considered persistent organic pollutants and are harmful to the environment.

D. "Blue baby" syndrome is related to the presence of excess nitrate ions in water, not sulfate ions. Nitrate can interfere with the oxygen-carrying capacity of hemoglobin, causing a bluish coloration in infants, hence the name "blue baby" syndrome.

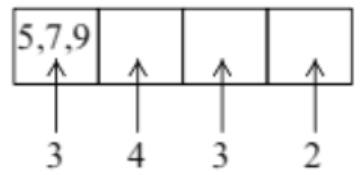
So, options A and C are the correct statements.

Mathematics

Question 61. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9 without repetition, is
(1) 12
(2) 120
(3) 6
(4) 72

Answer. 72

Solution. Numbers between 5000 & 10000 Using digits 1, 3, 5, 7, 9



Total Numbers = 3 × 4 × 3 × 2 = 72

Question 66. The shortest distance between the lines x + 1 = 2y = – 12z and x = y + 2 = 6z – 6 is?
(1) 2
(2) 3/2
(3) 5/2
(4) 3

Answer. 2

Solution.

$x+1=2y=-12z$ and

$x = y + 2 = 6z - 6$

$\Rightarrow \quad \frac{x-(-1)}{1} = \frac{y-0}{\frac{1}{2}} = \frac{z-0}{-\frac{1}{12}}$

and $\frac{x-0}{1} = \frac{y-(-2)}{1} = \frac{z-1}{\frac{1}{6}}$

These lines may be written in vector form as

$\vec{r} = (-\hat{i}+0\hat{j}+0\hat{k}) + \lambda(\hat{i}+\frac{1}{2}\hat{j}-\frac{1}{12}\hat{k}) \quad ...(i)$

and $\vec{r} = (0\hat{i}-2\hat{j}+\hat{k}) + \lambda(\hat{i}+\hat{j}+\frac{1}{6}\hat{k}) \ ...(ii)$

We know that the shortest distance between

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and

$\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$SD = \left| \frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

Here, $\vec{a_1} = -\hat{i}+0\hat{j}+0\hat{k}$,

$\vec{b_1} = \hat{i}+\frac{1}{2}\hat{j}-\frac{1}{12}\hat{k}$

$\vec{a_2} = 0\hat{i}-2\hat{j}+\hat{k}$,

$\vec{b_2} = \hat{i}+\hat{j}+\frac{1}{6}\hat{k}$

Now, $\vec{a_2} - \vec{a_1} = (0\hat{i}-2\hat{j}+\hat{k})$

$-(-\hat{i}+0\hat{j}+0\hat{k}) = \hat{i}-2\hat{j}+\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1/2 & -1/12 \\ 1 & 1 & 1/6 \end{vmatrix}$$

$= (\frac{1}{12}+\frac{1}{12})\hat{i} - (\frac{1}{6}+\frac{1}{12})\hat{j} + (1-\frac{1}{2})\hat{k}$

$$= \left(\frac{1}{12} + \frac{1}{12}\right)\hat{i} - \left(\frac{1}{6} + \frac{1}{12}\right)\hat{j} + \left(1 - \frac{1}{2}\right)\hat{k}$$

$$= \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}$$

$$\therefore \quad |\vec{b_1} \times \vec{b_2}| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(-\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{1}{36} + \frac{1}{16} + \frac{1}{4}} = \sqrt{\frac{4+9+36}{144}}$$

$$= \sqrt{\frac{49}{144}} = \frac{7}{12}$$

$$\therefore \text{ Required } S.D. = \left|\frac{(\hat{i} - 2\hat{j} + \hat{k}).\left(\frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}\right)}{\frac{7}{12}}\right|$$

$$= \left|\frac{\frac{1}{6} + \frac{1}{2} + \frac{1}{2}}{\frac{7}{12}}\right| = \frac{\frac{7}{6}}{\frac{7}{12}} = \frac{7}{6} \times \frac{12}{7} = 2 \text{ units}.$$

Question 77. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that N - 2, $\sqrt{3N}$, N + 2 are in geometric progression be k/48. Then the value of k is?

(1) 16

(2) 2

(3) 4

(4) 8

Answer. 4

Solution. Correct answer is 4

$n(s) = 36$

Given : $N - 2, \sqrt{3N}, N + 2$ are in G.P.

$3N = (N - 2)(N + 2)$

$3N = N^2 - 4$

$\Rightarrow N^2 - 3N - 4 = 0$

$(N - 4)(N + 1) = 0 \Rightarrow \boxed{N = 4}$ or $N = -1$ rejected

$(\text{Sum} = 4) \equiv \{(1, 3), (3,1), (2,2)\}$

$n(A) = 3$

$$P(A) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48} \Rightarrow \boxed{k = 4}$$

Question 83. If the shortest distance between the line joining the points (1,2,3) and (2,3,4), and the line (x-1)/2 = (y+1)/-1 = (z-2)/0 is α, then $28\alpha^2$ is equal to?

Answer. 18

Solution.

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}) \qquad \vec{r} = \vec{a} + \lambda\vec{p}$$

$$\vec{r} = (+\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} - \hat{j}) \qquad \vec{r} = \vec{b} + \mu\vec{q}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$d = \left| \frac{(\vec{b} - \vec{a}).(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$$

$$d = \left| \frac{(-3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{14}} \right|$$

$$= \left| \frac{-6 + 3}{\sqrt{14}} \right| = \frac{3}{\sqrt{14}}$$

$$\alpha = \frac{3}{\sqrt{14}}$$

Now, $28\alpha^2 = 28 * 9/14 = 18$
$28\alpha^2 = 18$

Question 84. For the two positive number a, b, if a, b and 1/18 are in a geometric progression, while 1/a, 10 and 1/b are in arithmetic progression, then 16a + 12b is equal to _____.

Answer. 3

Solution. a,b,1/18 → GP
$a/18 = b^2$ …. (i)

1/a, 10,1/b → AP
1/a + 1/b = 20

$$\Rightarrow a+b=20ab \text{ , from eq. (i) ; we get}$$

$$\Rightarrow 18b^2+b=360b^3$$

$$\Rightarrow 360b^2-18b-1=0 \quad \{\because b\neq 0\}$$

$$\Rightarrow b=\frac{18\pm\sqrt{324+1440}}{720}$$

$$\Rightarrow b=\frac{18+\sqrt{1764}}{720} \qquad \{\because b>0\}$$

$$\Rightarrow b=\frac{1}{12}$$

$$\Rightarrow a=18\times\frac{1}{144}=\frac{1}{8}$$

$$\text{Now, } 16a+12b=16\times\frac{1}{8}+12\times\frac{1}{12}=3$$

16a + 12b is equal to 3.

Question 85. The remainder when $(2023)^{2023}$ is divided by 35 is _____?

Answer. 7

Solution.

$$(2023)^{2023}$$
$$=(2030-7)^{2023}$$
$$=(35\,K-7)^{2023}$$
$$={}^{2023}C_0(35\,K)^{2023}(-7)^0+{}^{2023}C_1(35\,K)^{2022}(-7)+\ldots..+\ldots\ldots+{}^{2023}C_{2023}(-7)^{2023}$$
$$=35\,N-7^{2023}.$$

$$\text{Now, } -7^{2023}=-7\times 7^{2022}=-7\,(7^2)^{1011}$$

$$=-7\,(50-1)^{1011}$$

$$=-7({}^{1011}C_0\,50^{1011}-{}^{1011}C_1(50)^{1010}+\ldots\ldots-{}^{1011}C_{1011})$$

$$=-7\,(5\lambda-1)$$

$$=-35\lambda+7$$

∴ when (2023)2023 is divided by 35 remainder is 7.

Question 86. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is ____?

Answer. 6860 or 3

Solution. 7 Red apple(RA), 5 white apple(WA), 8 oranges (O)
5 fruits to be selected (Note:- fruits taken different)
Possible selections :- (2O, 1RA, 2WA) or (2O, 2RA, 1WA) or (3O, 1RA, 1WA)
⇒ 8C2 7C1 5C2 + 8C2 7C2 5C1 + 8C3 7C1 5C1 = 1960 + 2940 + 1960 = 6860

Question 87. Points P(–3, 2), Q(9, 10) and R(α, 4) lie on a circle C with PR as its diameter. The tangents to C at the points Q and R intersect at the point S. If S lies on the line 2x – ky = 1, then K is equal to?

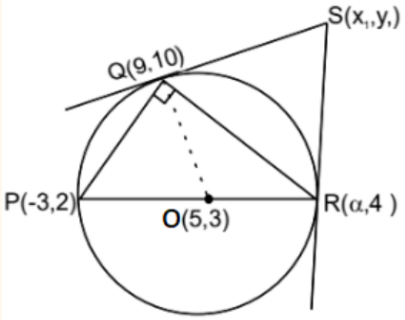
Answer. 3

Solution. Correct answer is 3

$m_{PQ} \cdot m_{QR} = -1$

$\Rightarrow \frac{10-2}{9+3} \times \frac{10-4}{9-\alpha} = -1 \Rightarrow \boxed{\alpha = 13}$

$m_{OQ} \cdot m_{QS} = -1 \Rightarrow m_{QS} = -\frac{4}{7}$



Equation of QS

$y - 10 = -\frac{4}{7}(x-9)$

$\Rightarrow 4x + 7y = 106 \ldots.(1)$

$m_{OR} \cdot m_{RS} = -1 \Rightarrow m_{RS} = -8$

Equation of RS

$y - 4 = -8(x-13)$

$\Rightarrow 8x + y = 108 \ldots.(2)$

Solving eq. (1) & (2)

$x_1 = \frac{25}{2}$ $y_1 = 8$

$S(x_1, y_1)$ lies on $2x - ky = 1$

$25 - 8k = 1$

$\Rightarrow 8k = 24$

$\Rightarrow \boxed{k = 3}$

Question 89. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non-smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is k/10. Then the value of k is _____?

Answer. 9

Solution. E1: Smokers P(E1) = 1/4
E2: non-smokers P(E2) = 3/4
E: diagnosed with lung cancer

$$P(E/E_1) = \frac{27}{28}$$

$$P(E/E_2) = \frac{1}{28}$$

$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E)}$$

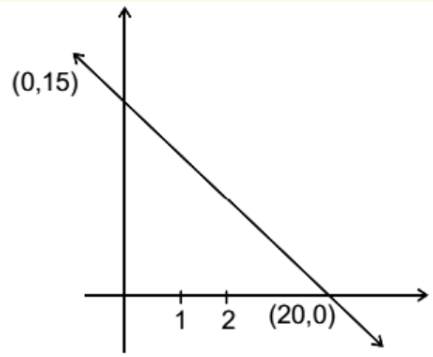
$$= \frac{\frac{1}{4}\times\frac{27}{28}}{\frac{1}{4}\times\frac{27}{28}+\frac{3}{4}\times\frac{1}{28}} = \frac{\cancel{27}^{9}}{\cancel{30}_{10}} = \frac{9}{10}$$

$$k = 9$$

Question 90. A triangle is formed by X-axis, Y-axis and the line 3x + 4y = 60. Then the number of points P(a, b), which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is _____?

Answer. 31

Solution. If x = 1, y = 57/4 = 14.25



(1, 1) (1, 2) .... (1, 14) $\Rightarrow$ 14 pts.

If $x = 2,\ y = \frac{27}{2} = 13.5$

(2, 2) (2, 4) .... (2, 12) $\Rightarrow$ 6 pts.

If $x = 3,\ y = \frac{51}{4} = 12.75$

(3, 3) (3, 6) (3, 9) (3, 12) $\Rightarrow$ 4 pts.

If $x = 4,\ y = 12$

(4, 4) (4, 8) $\Rightarrow$ 2 pts.

If $x = 5.\ y = \frac{45}{4} = 11.25$

(5, 5), (5, 10) $\Rightarrow$ 2 pts.

If $x = 6,\ y = \frac{21}{2} = 10.5$

(6, 6) $\Rightarrow$ 1 pt.

If $x = 7,\ y = \frac{39}{4} = 9.75$

(7, 7) $\Rightarrow$ 1 pt.

If $x = 8,\ y = 9$

(8, 8) $\Rightarrow$ 1 pt.

If $x = 9\ \ y = \frac{33}{4} = 8.25$ $\Rightarrow$ no pt.

Total = 31 pts.