JEE Main 2023 Solutions
Jan 29 - Shift 1

Physics

Question 1. A stone is projected at angle 30º to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be -
(1) 1 : 4
(2) 4 : 1
(3) 1 : 2
(4) 4 : 3

Answer. 4 : 3

Solution. The kinetic energy (KE) of an object is given by the formula:

KE = (1/2) * m * v^2

Where m is the mass of the object, and v is its velocity.

In projectile motion, at the highest point of the trajectory, the vertical component of the velocity becomes zero, while the horizontal component remains the same. Therefore, the kinetic energy at the highest point is solely due to the horizontal motion.

At the point of projection, the stone has both horizontal and vertical velocity components. The total initial kinetic energy includes both components.

Let's denote the horizontal component of velocity as v_x and the vertical component as v_y. The initial kinetic energy (at the point of projection) is:

KE_initial = (1/2) * m * (v_x^2 + v_y^2)

At the highest point, v_y becomes zero, so the kinetic energy at that point is:

KE_highest_point = (1/2) * m * v_x^2

To find the ratio of KE_initial to KE_highest_point:

Ratio = KE_initial / KE_highest_point
= [(1/2) * m * (v_x^2 + v_y^2)] / [(1/2) * m * v_x^2]

The mass 'm' cancels out, and the ratio simplifies to:

Ratio = (v_x^2 + v_y^2) / v_x^2

Now, we can use trigonometric relationships to relate v_x and v_y to the initial velocity 'v' and the launch angle 'θ':

v_x = v * cos(θ)
v_y = v * sin(θ)

Substitute these expressions into the ratio formula:

Ratio = [(v * cos(θ))^2 + (v * sin(θ))^2] / (v * cos(θ))^2

Simplify further:

Ratio = [v^2 * (cos^2(θ) + sin^2(θ))] / [v^2 * cos^2(θ)]

Since cos^2(θ) + sin^2(θ) = 1, the ratio becomes:

Ratio = (v^2 * 1) / (v^2 * cos^2(θ))

Now, notice that the v^2 terms in the numerator and denominator cancel out:

Ratio = 1 / cos^2(θ)

The cosine of 30 degrees (cos(30°)) can be found using trigonometric values, which is (√3/2). So:

Ratio = 1 / (cos(30°))^2
Ratio = 1 / [(√3/2)^2]
Ratio = 1 / (3/4)
Ratio = 4/3

So, the ratio of the kinetic energy of the stone at the point of projection to its kinetic energy at the highest point of flight is 4 : 3 (option 4).

Question 4. If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be:
Given: Earth's radius = 6.4×10^6 m
(1) 36 km
(2) 28 km
(3) 64 km
(4) 32 km

Answer. 64 km

Solution. Maximum line of sight distance between two antennas

$$\text{antennas, } d_M = \sqrt{2Rh_T} + \sqrt{2R.h_R}$$

$$d_M = 2 \times \sqrt{2 \times 6.4 \times 10^6 \times 80} = 64 \text{ km}$$

Question 6. If a radioactive element having half-life of 30 min. is undergoing beta decay, the fraction of radioactive element remains undecayed after 90 min will be?

(1) 1/4
(2) 1/16
(3) 1/2
(4) 1/8

Answer. ⅛

Solution. The fraction of a radioactive element that remains undecayed after a certain time can be calculated using the formula:

Remaining Fraction = (1/2)^(t / half-life)

Where:

t is the time in question.
half-life is the half-life of the radioactive element.

In this case, the half-life of the element is 30 minutes, and we want to find the fraction remaining after 90 minutes (t = 90 minutes). Plug these values into the formula:

Remaining Fraction = (1/2)^(90 / 30)

Remaining Fraction = (1/2)^(3)

Remaining Fraction = (1/2)^3

Remaining Fraction = 1/8

So, the fraction of the radioactive element that remains undecayed after 90 minutes is 1/8.

Question 8. A bicycle tyre is filled with air having pressure of 270 kPa at 27°C. The approximate pressure of the air in the tyre when the temperature increases to 36°C is?
(1) 278 kPa
(2) 360 kPa
(3) 262 kPa
(4) 270 kPa

Answer. 278 kPa

Solution. You can use the ideal gas law to solve this problem, which relates pressure (P), volume (V), temperature (T), and the gas constant (R). The ideal gas law is given by:

PV = nRT

Where:
P = pressure
V = volume
n = number of moles
R = gas constant
T = temperature (in Kelvin)

In this case, we need to calculate the new pressure when the temperature changes from 27°C to 36°C. To do this, we first convert the temperatures to Kelvin:

Initial temperature (T1) = 27°C + 273.15 = 300.15 K
Final temperature (T2) = 36°C + 273.15 = 309.15 K

Since the number of moles and volume remain constant for the bicycle tire, we can write:

P1/T1 = P2/T2

Where:
P1 = initial pressure (270 kPa)
T1 = initial temperature in Kelvin (300.15 K)
P2 = final pressure (which we want to find)
T2 = final temperature in Kelvin (309.15 K)

Now, solve for P2:

P2 = (P1 * T2) / T1

P2 = (270 kPa * 309.15 K) / 300.15 K

P2 ≈ 278 kPa

So, the approximate pressure of the air in the tire when the temperature increases to 36°C is approximately 278 kPa. Therefore, option (1) is the closest to this value.

Question 10. Ratio of thermal energy released in two resistors R and 3R connected in parallel in an electric circuit is:
(1) 3 : 1
(2) 1 : 1
(3) 1 : 27
(4) 1 : 3

Answer. 3 : 1

Solution. Correct answer is 3 : 1

$$H = \frac{V^2}{R} \times t$$

$$\frac{H_1}{H_2} = \frac{\frac{V^2 t}{R}}{\frac{V^2 t}{3R}} = 3:1$$

Question 11. The threshold wavelength for photoelectric emission from a material is 5500 Å. Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a
A. 75 W infra-red lamp
B. 10 W infra-red lamp
C. 75 W ultra-violet lamp
D. 10 W ultra-violet lamp
Choose the correct answer from the options given below:
(1) B and C only
(2) C only
(3) A and D only
(4) C and D only

Answer. C and D only

Solution. The threshold wavelength for photoelectric emission from a material is 5500 Å. The energy of the incident radiation must be greater than the work function (hv) of the material to cause photoemission.

The relationship between energy (E), wavelength (λ), and frequency (v) of electromagnetic radiation is given by:

$E = hv = hc/\lambda$

Where:
E = energy of the radiation
h = Planck's constant
v = frequency of the radiation
c = speed of light
λ = wavelength of the radiation

We are given the threshold wavelength ($\lambda_{threshold}$) as 5500 Å. Let's convert this to meters:

λ_threshold = 5500 Å = 5500 × 10^(-10) meters

Now, we can calculate the energy of the incident radiation with this threshold wavelength:

E_threshold = hc/λ_threshold

Next, we need to determine the minimum energy (work function) required for photoemission. This is given as the threshold energy (E_threshold).

Now, let's consider the options:
A. 75 W infra-red lamp
B. 10 W infra-red lamp
C. 75 W ultra-violet lamp
D. 10 W ultra-violet lamp

Infra-red lamps emit radiation with low energy, and their wavelengths are longer. Ultra-violet lamps emit higher-energy radiation with shorter wavelengths. For photoemission to occur, the incident radiation must have energy greater than or equal to the threshold energy (E_threshold).

Given the threshold wavelength, we can compare the options:

A. Infra-red lamp with 75 W (low energy)
B. Infra-red lamp with 10 W (low energy)
C. Ultra-violet lamp with 75 W (high energy)
D. Ultra-violet lamp with 10 W (high energy)

Based on the comparison, options C and D are the ones that can provide sufficient energy for photoemission since they are ultra-violet lamps (higher energy) with either 75 W or 10 W.

So, the correct answer is:
(4) C and D only

Question 13. Which one of the following statement is not correct in the case of light emitting diodes?
A. It is a heavily doped p-n junction.
B. It emits light only when it is forward biased.
C. It emits light only when it is reverse biased.
D. The energy of the light emitted is equal to or slightly less then the energy gap of the semiconductor used.
Choose the correct answer from the options given below:
(1) A
(2) B
(3) C and D
(4) C

Answer. C

Solution. The correct statement regarding light emitting diodes (LEDs) is:

C. It emits light only when it is reverse biased.

LEDs emit light when they are forward-biased (not reverse-biased), and this is the basic principle of their operation. When a forward voltage is applied to the LED, electrons recombine with holes in the semiconductor material, releasing energy in the form of photons (light).

Therefore, the statement "It emits light only when it is reverse-biased" is not correct.

So, the correct answer is (4) C.

Question 16. A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of 30 m/s. If both trains emit sounds with frequency 300 Hz, (Speed of sound: 330 m/s) approximate difference of frequencies heard by the person will be:
(1) 55 Hz

(2) 80 Hz
(3) 33 Hz
(4) 10 Hz

Answer. 55 Hz

Solution. To calculate the Doppler effect on the frequency observed by the person, we can use the formula:

f' = f ((v + vo) / (v + vs))

Where:

f' is the observed frequency
f is the source frequency (300 Hz in this case)
v is the speed of sound (330 m/s)
vo is the observer's speed (in this case, 0 m/s since the observer is stationary)
vs is the source's speed (in this case, 30 m/s)

For train 'A,' which is reaching the station, the relative speed is (0 + 30) m/s = 30 m/s.
For train 'B,' which is leaving the station, the relative speed is (0 - 30) m/s = -30 m/s.

Now, we can calculate the frequencies observed:

For train 'A' (reaching the station):
f_A' = 300 ((330 + 0) / (330 + 30)) = 300 (330 / 360) = 275 Hz

For train 'B' (leaving the station):
f_B' = 300 ((330 + 0) / (330 - 30)) = 300 (330 / 300) = 330 Hz

The difference in frequencies heard by the person is:
Δf = |f_B' - f_A'| = |330 Hz - 275 Hz| = 55 Hz

So, the approximate difference in frequencies heard by the person is 55 Hz, and the correct answer is (1) 55 Hz.

Question 19. Which of the following are true?
A. Speed of light in vacuum is dependent on the direction of propagation.
B. Speed of light in a medium is independent of the wavelength of light.
C. The speed of light is independent of the motion of the source.
D. The speed of light in a medium is independent of intensity.
Choose the correct answer from the options given below:
(1) A and C only
(2) B and C only
(3) B and D only
(4) C and D only

Answer. C and D only

Solution. Speed of light does not depend on the motion of source as well as intensity. So , the correct answer is C and D

Question 22. A solid sphere of mass 2 kg is making pure rolling on a horizontal surface with kinetic energy 2240 J. The velocity of centre of mass of the sphere will be ____ ms^{-1}.

Answer. 40

Solution. To find the velocity of the center of mass (v) of the solid sphere making pure rolling, you can use the following formula:

Kinetic Energy (KE) = 1/2 * I * ω^2 + 1/2 * M * v^2

Where:

KE = Kinetic Energy
I = Moment of Inertia of the sphere
ω = Angular velocity
M = Mass of the sphere
v = Velocity of the center of mass

For a solid sphere, the moment of inertia (I) is (2/5) * M * R^2, where R is the radius of the sphere.

Given:
Mass of the sphere (M) = 2 kg
Kinetic Energy (KE) = 2240 J

Now, you need to calculate the angular velocity (ω). Since it's making pure rolling, the linear velocity at the edge is equal to the angular velocity times the radius (v = ω * R). Therefore, we have:

v = R * ω

Also, you know that the total energy in rolling motion is equally divided between translational (1/2 * M * v^2) and rotational (1/2 * I * ω^2) kinetic energy.

KE = (1/2) * M * v^2 + (1/2) * [(2/5) * M * R^2] * ω^2

Substituting the values:

2240 = (1/2) * 2 * v^2 + (1/2) * [(2/5) * 2 * R^2] * ω^2

Simplify:

2240 = v^2 + (2/5) * R^2 * ω^2

Now, substitute v = R * ω:

$2240 = (R * \omega)^2 + (2/5) * R^2 * \omega^2$

$2240 = \omega^2 * (R^2 + (2/5) * R^2)$

$2240 = \omega^2 * (7/5 * R^2)$

Now, solve for ω^2:

$\omega^2 = 2240 / (7/5 * R^2)$

$\omega^2 = 1600 / R^2$

Now, you can find the angular velocity ω:

$\omega = \sqrt{(1600 / R^2)}$

Now, calculate the velocity of the center of mass (v) using $v = R * \omega$:

$v = R * \sqrt{(1600 / R^2)}$

$v = R * (40 / R)$

$v = 40$ m/s

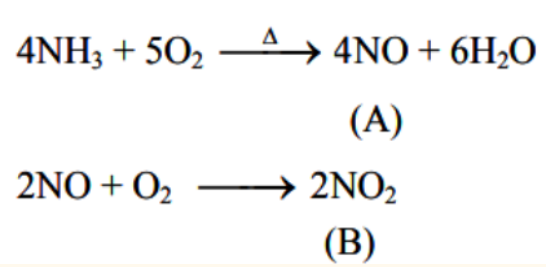
So, the velocity of the center of mass of the sphere is 40 m/s.

Chemistry

Question 31. "A" obtained by Ostwald's method involving air oxidation of NH3, upon further air oxidation produces "B". "B" on hydration forms an oxoacid of Nitrogen along with evolution of "A". The oxoacid also produces "A" and gives positive brown ring test. Identify A and B, respectively.
(1) NO2, N2O5
(2) NO, NO2
(3) NO2, N2O4
(4) N2O3, NO2

Answer. NO, NO2

Solution. Correct option is (3) NO, NO2



Question 33. During the borax bead test with CuSO4, a blue green colour of the bead was observed in oxidising flame due to the formation of?
(1) CuO
(2) Cu
(3) $Cu(BO_2)_2$
(4) Cu_3B_2

Answer. $Cu(BO_2)_2$

Solution. The blue-green color observed during the borax bead test with CuSO4 in the oxidizing flame is due to the formation of Cu(BO2)2, which is a copper borate complex.

So, the correct answer is option (3): Cu(BO2)2.

Question 34. Compound that will give positive Lassaigne's test for both nitrogen and halogen is?
(1) NH4Cl
(2) NH2OH.HCl
(3) CH3NH2.HCl
(4) N2H4.HCl

Answer. CH3NH2.HCl

Solution. Lassaigne test for both Nitrogen & Halogen is given by the compound which have C, N as well as X atom in the compound. So, the correct answer is CH3NH2.HCl

Question 35. Correct statement about smog is?
(1) Classical smog also has high concentration of oxidizing agents
(2) Photochemical smog has high concentration of oxidizing agents
(3) Both NO_2 and SO_2 are present in classical smog
(4) NO_2 is present in classical smog

Answer. Photochemical smog has high concentration of oxidizing agents

Solution. The correct statement about smog is:

(2) Photochemical smog has high concentration of oxidizing agents

Photochemical smog, unlike classical smog, contains high concentrations of oxidizing agents, primarily ozone (O3), which is formed through photochemical reactions involving pollutants like nitrogen oxides (NOx) and volatile organic compounds (VOCs) in the presence of sunlight. This type of

smog is often associated with urban areas and is characterized by the presence of ozone and other oxidizing agents.

Question 36. The correct order of hydration enthalpies is
(A) K^+
(B) Rb^+
(C) Mg^{2+}
(D) Cs^+
(E) Ca^{2+}
Choose the correct answer from the options given below:
(1) E > C > A > B > D
(2) C > E > A > D > B
(3) C > A > E > B > D
(4) C > E > A > B > D

Answer. C > E > A > B > D

Solution. The correct order of hydration enthalpies is as follows:

(B) Rb+ < (D) Cs+ < (A) K+ < (C) Mg2+ < (E) Ca2+

So, the correct answer is:

(4) C > E > A > B > D

Question 37. The standard electrode potential (M^{3+}/M^{2+}) for V, Cr, Mn & Co are –0.26 V, –0.41 V, +1.57 V and +1.97 V, respectively. The metal ions which can liberate H_2 from a dilute acid are
(1) V2+ and Cr2+
(2) Cr2+ and Co2+
(3) V2+ and Mn2+
(4) Mn2+ and Co2+

Answer. V2+ and Cr2+

Solution. The metal ions that can liberate H2 from a dilute acid are the ones with a more negative standard electrode potential (E°) than the standard hydrogen electrode potential (0.00 V). In this case, the metal ions that can do that are V2+ and Cr2+.

So, the correct option is (1) V2+ and Cr2+.

Question 38. Number of cyclic tripeptides formed with 2 amino acids A and B is:
(1) 2
(2) 4
(3) 5
(4) 3

Answer. 4

Solution. Cyclic tripeptides are composed of three amino acids. In this case, we have two choices for each position in the peptide, either A or B.

Therefore, the number of cyclic tripeptides that can be formed with amino acids A and B is 2^3, which is equal to 8.

However, one of the combinations is the same as its reverse (e.g., ABA and ABA), so we need to subtract these duplicates, leaving us with 8 - 4 = 4 unique cyclic tripeptides.

So, the correct answer is (2) 4.

Question 44. The bond dissociation energy is highest for
(1) Br2
(2) F2
(3) I2
(4) Cl2

Answer. Cl2

Solution. As bond length increases, bond dissociation enthalpy decreases. Hence, bond dissociation enthalpy decreases from F2 to I2. But the bond dissociation enthalpy of F2 is lower than Cl2 and Br2. It is due to the presence of lone pair of e on fluorine atom which creates greater repulsion due to small size of fluorine. Hence the bond dissociation enthalpy of F, decreases. I2 < F2 < Br2 < Cl2

Question 48. Which of the given compounds can enhance the efficiency of hydrogen storage tank?
(1) Li/P4
(2) Di-isobutylaluminium hydride
(3) SiH4
(4) NaNi5

Answer. NaNi5

Solution. To enhance the efficiency of a hydrogen storage tank, it is important to consider materials or compounds that can absorb and release hydrogen gas effectively. Among the options provided:

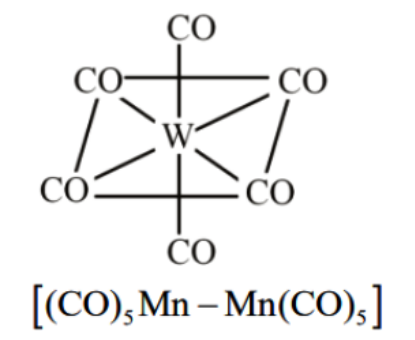
1. Li/P4: This is lithium phosphide, which can be used as a hydrogen storage material.
2. Di-isobutylaluminium hydride: This is a chemical compound, typically used in organic synthesis and not as a hydrogen storage material.
3. SiH4: This is silane, which is used as a precursor in the production of silicon materials. It is not typically used for hydrogen storage.
4. NaNi5: This is sodium nickel hydride, which can be used as a hydrogen storage material.

So, the compounds that can enhance the efficiency of a hydrogen storage tank are (1) Li/P4 and (4) NaNi5.

Question 54. The sum of bridging carbonyls in W(CO)6 and Mn2(CO)10 is_____?

Answer. 0

Solution. The sum of bridging carbonyls in W(CO)6 and Mn2 (CO)10 is 0.



$[(CO)_5Mn - Mn(CO)_5]$

Mathematics

Question 67. A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line x + y = 1, if this ray intersects x-axis at Q, then the abscissa of Q is:

(1) 2 / (3-√3)
(2) 2 / (3+√3)
(3) √3 / 2(√3+1)
(4) 2 / (√3-1)

Answer. 2 / (3+√3)

Solution. Slope of reflected ray = tan 60° = √3

Line $y = \frac{x}{\sqrt{3}}$ intersect

$y + x = 1$ at $\left(\frac{\sqrt{3}}{\sqrt{3}+1}, \frac{1}{\sqrt{3}+1}\right)$

Equation of reflected ray is

$$y - \frac{1}{\sqrt{3}+1} = \sqrt{3}\left(x - \frac{\sqrt{3}}{\sqrt{3}+1}\right)$$

Put $y = 0 \Rightarrow x = \frac{2}{3+\sqrt{3}}$

Question 72. Let the tangents at the points A(4, –11) and B(8, –5) on the circle $x^2 + y^2 – 3x + 10y – 15 = 0$, intersect at the point C. Then the radius of the circle, whose centre is C and the line joining A and B is its tangent, is equal to?

(1) 3√3 / 4
(2) √13
(3) 2√13 / 3
(4) 2√13

Answer. 2√13 / 3

Solution. Equation of tangent at A (4, –11) on circle is

$$\Rightarrow 4x - 11y - 3\left(\frac{x+4}{2}\right) + 10\left(\frac{y-11}{2}\right) - 15 = 0$$

$$\Rightarrow 5x - 12y - 152 = 0 \ \ldots\ldots (1)$$

Equation of tangent at B (8, –5) on circle is

$$\Rightarrow 8x - 5y - 3\left(\frac{x+8}{2}\right) + 10\left(\frac{y-5}{2}\right) - 15 = 0$$

$$\Rightarrow 13x - 104 = 0 \Rightarrow x = 8$$

$$\text{put in (1)} \Rightarrow y = \frac{28}{3}$$

$$r = \left|\frac{3.8 + \frac{2.28}{3} - 34}{\sqrt{13}}\right| = \frac{2\sqrt{13}}{3}$$

Question 75. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is?

(1) 2/15
(2) 1/6
(3) 5/36
(4) 5/24

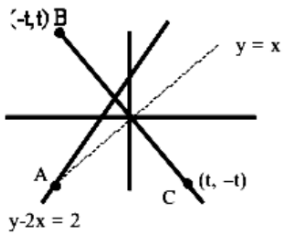
Answer. 1/6

Question 76. Let B and C be the two points on the line y + x = 0 such that B and C are symmetric with respect to the origin. Suppose A is a point on y – 2x = 2 such that ABC is an equilateral triangle. Then, the area of the △ABC is?

(1) 8/√3
(2) 2√3
(3) 10 /√3
(4) 3√3

Answer. 8/√3

Solution. Correct answer is 8/√3



At A $x = y$

$Y - 2x = 2$

$(-2, -2)$

Height from line $x + y = 0$

$$h = \frac{4}{\sqrt{2}}$$

Area of $\Delta = \frac{\sqrt{3}}{4} \frac{h^2}{\sin^2 60} = \frac{8}{\sqrt{3}}$

Question 79. For two non-zero complex numbers z1 and z2, if Re(z1z2) = 0 and Re(z1 + z2), then which of the following are possible?

A. Im(z1) > 0 and Im(z2) > 0

B. Im(z1) < 0 and Im(z2) > 0

C. Im(z1) > 0 and Im(z2) < 0

D. Im(z1) < 0 and Im(z2) < 0

Choose the correct answer from the options given below

(1) A and C

(2) B and D

(3) B and C

(4) A and B

Answer. B and C

Solution. Correct answer is B and C

$$z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

$$\text{Re}(z_1 z_2) = x_1 x_2 - y_1 y_2 = 0$$

$$\text{Re}(z_1 + z_2) = x_1 + x_2 = 0$$

x_1 & x_2 are of opposite sign

y_1 & y_2 are of opposite sign

Question 81. Let the coefficients of three consecutive terms in the binomial expansion of $(1 + 2x)^n$ be the ratio 2 :5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is _____?

Answer. 1120

Solution. Correct answer is 1120

$$t_{r+1} = {}^nC_r (2x)^r$$

$$\Rightarrow \frac{{}^nC_{r-1}(2)^{r-1}}{{}^nC_r (2)^r} = \frac{2}{5}$$

$$\Rightarrow \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!(2)}{r!(n-r)!}} = \frac{2}{5}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{4}{5}$$

$$\Rightarrow 5r = 4n - 4r + 4$$

$$\Rightarrow 9r = 4(n+1) \quad \ldots (1)$$

$$\Rightarrow \frac{{}^nC_r (2)^r}{{}^nC_{r+1}(2)^{r+1}} = \frac{5}{8}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{5}{4}$$

$$\Rightarrow \frac{r+1}{n-r} = \frac{5}{4}$$

$$\Rightarrow 4r+4 = 5n-5r$$

$$\Rightarrow 5n-4 = 9r \quad \ldots (2)$$

From (1) and (2)

$$\Rightarrow 4n+4 = 5n-4 \Rightarrow n = 8$$

$$(1) \Rightarrow r = 4$$

so, coefficient of middle term is

$${}^{8}C_4\, 2^4 = 16 \times \frac{8\times7\times6\times5}{4\times3\times2\times1}$$

$$= 16 \times 70 = 1120$$

Question 82. If all the six digit numbers x1 x2 x3 x4 x5 x6 with 0 < x1 < x2 < x3 < x4 < x5 < x6 are arranged in the increasing order, then the sum of the digits in the 72th number is _____?

Answer. 32

Solution. Correct answer is 32

1	2					$= {}^{7}C_4 = 35$
1	3					$= {}^{6}C_4 = 15$
1	4					$= {}^{5}C_4 = 5$
1	5					$= {}^{4}C_4 = 1$
2	3					$= {}^{6}C_4 = 15$

71 words

2 4 5 6 7 8 $\rightarrow$ 72th word

$2 + 4 + 5 + 6 + 7 + 8 = 32$

Question 85. Let a1, a2, a3, … be a G.P of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then a1a9 + a2a4a9 + a5 + a7 is equal to _____?

Answer. 60

Solution. Correct answer is 60

$$a_4 \cdot a_6 = 9 \Rightarrow (a_5)^2 = 9 \Rightarrow a_5 = 3$$

$$\& \; a_5 + a_7 = 24 \Rightarrow a_5 + a_5 r^2 = 24$$

$$\Rightarrow (1 + r^2) = 8 \Rightarrow r = \sqrt{7}$$

$$\Rightarrow a = \frac{3}{49}$$

$$\Rightarrow a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$$

$$= 9 + 27 + 3 + 21 = 60$$

Question 89. Let the equation of the plane P containing the line x + 10 = (8-y)/2 = z be ax + by + 3z = 2(a + b) and the distance of the plane P from the point (1, 27, 7) be c. Then $a^2 + b^2 + c^2$ is equal to _____?

Answer. 355

Solution. Correct answer is 355

The line $\frac{x+10}{1}=\frac{y-8}{-2}$

$=\frac{z}{1}$ have a point (–10, 8, 0)

with d. r. (1, –2, 1)

$\because$ the plane ax + by + 3z = 2 (a + b)

$\Rightarrow b = 2a$

& dot product of d.r.'s is zero

$\therefore a - 2b + 3 = 0$

$\therefore a = 1$ & $b = 2$

Distance from (1, 27, 7) is

$$c=\frac{1+54+21-6}{\sqrt{14}}=\frac{70}{\sqrt{14}}=5\sqrt{14}$$

$\therefore a^2 + b^2 + c^2 = 1 + 4 + 350$

$= 355$

Question 90. Five digit numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial number. For example, the number 77777 has serial number 1. Then the serial number of 35337 is _____?

Answer. 1436

Solution. No of 5 digit numbers starting with digit 1 = 5 × 5 × 5 × 5 = 625
No of 5 digit numbers starting with digit 2 = 5 × 5 × 5 × 5 = 625
No of 5 digit numbers starting with 31 = 5 × 5 × 5 = 125
No of 5 digit numbers starting with 32 = 5 × 5 × 5 = 125
No of 5 digit numbers starting with 33 = 5 × 5 × 5 = 125
No of 5 digit numbers starting with 351 = 5 × 5 = 25
No of 5 digit numbers starting with 352 = 5 × 5 = 25
No of 5 digit numbers starting with 3531 = 5
No of 5 digit numbers starting with 3532 = 5
Before 35337 will be 4 numbers,
So rank of 35337 will be 1690
So, in descending order serial number will be 3125 – 1690 + 1 = 1436