JEE Main 2023 Solutions
Jan 29 - Shift 2

Physics

Question 1. At 300 K, the rms speed of oxygen molecule is $\sqrt{(\alpha + 5)} / \alpha$ times to that of its average speed in the gas. Then, the value of α will be (used π = 22/7)

(1) 24
(2) 27
(3) 32
(4) 28

Answer. 28

Solution. Correct answer is 28

$$\sqrt{\frac{3RT}{M}} = \sqrt{\frac{\alpha+5}{\alpha}}\sqrt{\frac{8}{\pi}\frac{RT}{M}}$$

$$3 = \frac{\alpha+5}{\alpha}\frac{8}{\pi}$$

$$\alpha = 28$$

Question 3. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is 1/√m , the value of m is?

(1) 8
(2) 4
(3) 2
(4) 16

Answer. 8

Solution. The de Broglie wavelength (λ) of a particle is given by the formula:

λ = h / p

Where h is the Planck constant and p is the momentum of the particle.

For an α particle (helium nucleus), the charge is 2e (where e is the elementary charge) and the mass (m_α) is 4 times the mass of a proton (m_p). When both the α particle and the proton are accelerated by the same potential, their kinetic energies (K) are the same.

The kinetic energy (K) can be calculated using the electric potential (V) and charge (q) of the particle:

K = qV

Now, we can use the kinetic energy to find the momentum of the particles. For the proton:

K_proton = q_proton * V K_proton = e * V

And for the α particle:

K_α = q_α * V K_α = 2e * V

Since the kinetic energy is the same for both particles, we can set these two equations equal to each other:

e * V = 2e * V

The charges cancel out, and we have:

V = 2V

Now, let's consider the de Broglie wavelength for both particles:

For the proton:

λ_proton = h / p_proton

For the α particle:

λ_α = h / p_α

Since both particles have the same kinetic energy and are accelerated by the same potential (V), their momenta are proportional to the square root of their masses (m):

p_proton = √(2 * m_proton * K) p_α = √(2 * m_α * K)

Now, let's calculate the ratio of their wavelengths:

(λ_proton / λ_α) = (p_α / p_proton)

(λ_proton / λ_α) = (√(2 * m_α * K) / √(2 * m_proton * K))

The kinetic energies cancel out:

(λ_proton / λ_α) = (√(2 * m_α) / √(2 * m_proton))

Now, we are given that this ratio is 1/√m:

(1/√m) = (√(2 * m_α) / √(2 * m_proton))

Squaring both sides:

1/m = (2 * m_α) / (2 * m_proton)

Now, we know that m_α = 4 * m_proton:

1/m = (2 * 4 * m_proton) / (2 * m_proton)

1/m = 8

So, the value of m is 1/8 or 8.

Question 4. A point charge 2×10^{-2} C is moved from P to S in a uniform electric field of 30 NC^{-1} directed along positive x-axis. If coordinates of P and S are (1, 2, 0) m and (0, 0, 0) m respectively, the work done by electric field will be?

(1) –600 mJ
(2) –1200 mJ
(3) 1200 mJ
(4) 600 mJ

Answer. –600 mJ

Solution. The work done (W) by an electric field when moving a point charge through a distance can be calculated using the formula:

W = q * E * d

Where:

- W is the work done
- q is the charge
- E is the electric field strength
- d is the distance over which the charge is moved

In this case, we have:

- Charge q = 2 × 10^(-2) C
- Electric field strength E = 30 N/C
- Initial position P(1, 2, 0) m
- Final position S(0, 0, 0) m

The distance (d) between P and S is given by the displacement in the x-direction, which is the difference in x-coordinates between S and P:

d = x_S - x_P = 0 - 1 = -1 m

Now, we can calculate the work done:

W = (2 × 10^(-2) C) * (30 N/C) * (-1 m)

W = -0.6 J

Since 1 J = 1000 mJ (millijoules), we can express the result in millijoules:

W = -0.6 J * 1000 mJ/J = -600 mJ

So, the work done by the electric field is -600 mJ.

Question 5. A square loop of area 25 cm^2 has a resistance of 10. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be?

(1) 1.0×10^{-3} J
(2) 5×10^{-3} J
(3) 2.5×10^{-3} J
(4) 1.0×10^{-4} J

Answer. 1.0×10^{-3} J

Solution. Correct answer is 1.0×10^{-3} J

$\ell = 50$ cm

$t = 1$ sec

$\therefore\ V = \frac{0.05}{1} = 0.05$ m/s.

$i = \frac{40 \times 0.05 \times 0.05}{10} = 0.01A$

$F = Bi\ell = 40 \times 0.01 \times 0.05$

$F = 0.02$ N

$\therefore\quad W = 0.02 \times \ell = 0.02 \times .05$

$\therefore\quad W = 1 \times 10^{-3}$J

Question 6. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is 500 m2. It is in level flight with a speed of 1080 km/h. If the density of air is 1.2 kg m^{-3}, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be. (g = 10 m/s^2)

(1) 16
(2) 6
(3) 8
(4) 10

Answer. 10

Solution. Correct answer is 10

$$P_2A - P_1A = 5.4 \times 10^5 \times g$$

$$P_2 - P_1 = \frac{5.4 \times 10^6}{500} = 5.4 \times 2 \times 10^2 \times 10$$

$$= 10.8 \times 10^3$$

$$P_2 + 0 + \frac{1}{2}\rho V_2^2 = P_1 + 0 + \frac{1}{2}\rho V_1^2$$

$$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

$$= \frac{1}{2}\rho(V_1 - V_2)(V_1 + V_2)$$

$$10.8 \times 10^3 = \frac{1}{2} \times 1.2(V_1 - V_2) \times 2 \times 3 \times 10^2$$

$$10.8 \times 10 = 3.6(V_1 - V_2)$$

$$V_1 - V_2 = 30$$

$$\left(\frac{V_1 - V_2}{V}\right) \times 100 = \frac{30}{300} \times 100 = 10\%$$

Question 7. Heat energy of 184 kJ is given to ice of mass 600 g at –12°C. Specific heat of ice is 2222.3 J $kg^{-1}°C^{-1}$ and latent heat of ice in 336 kJ/kg^{-1}

A. Final temperature of system will be 0°C.

B. Final temperature of the system will be greater than 0°C.

C. The final system will have a mixture of ice and water in the ratio of 5 : 1.

D. The final system will have a mixture of ice and water in the ratio of 1 : 5.

E. The final system will have water only.

Choose the correct answer from the options given below:

(1) A and E only

(2) A and C only

(3) B and D only

(4) A and D only

Answer. A and D only

Solution. To determine the final state of the system, we need to consider the heat transfer and phase changes.

Given:

- Initial temperature of ice (Ti) = -12°C
- Mass of ice (m) = 600 g = 0.6 kg
- Specific heat of ice (c_ice) = 2222.3 J/kg°C
- Latent heat of fusion (L) for ice = 336 kJ/kg = 336000 J/kg
- Heat energy given to the system (Q) = 184 kJ = 184000 J

Let's break down the process:

1. The initial state: The ice is at -12°C.
2. Heat required to raise the temperature of ice from -12°C to 0°C: Q1 = m * c_ice * ΔT1 Q1 = 0.6 kg * 2222.3 J/kg°C * (0°C - (-12°C)) = 15934.8 J
3. Heat required to melt the ice at 0°C: Q2 = m * L Q2 = 0.6 kg * 336000 J/kg = 201600 J
4. Total heat absorbed by the system to reach equilibrium (Q_total): Q_total = Q1 + Q2 Q_total = 15934.8 J + 201600 J = 217534.8 J

Now, we need to compare this to the heat energy given to the system (Q = 184000 J).

Since the system absorbs more heat energy than was supplied, the temperature will be raised to 0°C, but it will still be in the form of a mixture of ice and water.

So, the correct statements are:
A. The final temperature of the system will be 0°C.
D. The final system will have a mixture of ice and water in the ratio of 1 : 5.

Question 8. Substance A has atomic mass number 16 and half life of 1 day. Another substance B has atomic mass number 32 and half life of 1/2 day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days.
(1) 1.69×10^{24}
(2) 6.76×10^{24}
(3) 3.38×10^{24}
(4) 6.76×10^{23}

Answer. 3.38×10^{24}

Solution. To find the total number of atoms of substances A and B left after 2 days, we can calculate the number of half-lives that have passed for each substance.

Substance A:

- Half-life of A = 1 day
- Time (t) = 2 days
- Number of half-lives (n) = t / half-life = 2 / 1 = 2 half-lives

Substance B:

- Half-life of B = 1/2 day
- Time (t) = 2 days
- Number of half-lives (n) = t / half-life = 2 / (1/2) = 4 half-lives

Now, let's calculate the remaining fraction of each substance after the given number of half-lives:

For Substance A: After 2 half-lives, 1/2 * 1/2 = 1/4 of the original substance remains.

For Substance B: After 4 half-lives, 1/2 * 1/2 * 1/2 * 1/2 = 1/16 of the original substance remains.

Now, let's calculate the total number of atoms of A and B combined:

Initial mass of A = 320 g Initial number of moles of A = (320 g) / (atomic mass of A) = (320 g) / (16 g/mol) = 20 moles

Initial mass of B = 320 g Initial number of moles of B = (320 g) / (atomic mass of B) = (320 g) / (32 g/mol) = 10 moles

Now, we can calculate the number of remaining moles for each substance and then the total number of moles remaining:

Remaining moles of A = 20 moles * (1/4) = 5 moles Remaining moles of B = 10 moles * (1/16) = 10/16 moles

Total moles remaining = Remaining moles of A + Remaining moles of B Total moles remaining = 5 moles + 10/16 moles

Now, let's convert the total moles remaining to the total number of atoms using Avogadro's number (6.022 x 10^23 atoms/mol):

Total number of atoms remaining = Total moles remaining * Avogadro's number Total number of atoms remaining = (5 + 10/16) * 6.022 x 10^23 atoms Total number of atoms remaining ≈ 3.38 x 10^24 atoms

So, the total number of atoms of A and B combined that would be left after 2 days is approximately 3.38 x 10^24 atoms.

The correct answer is (3) 3.38 x 10^24.

Question 10. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be
(1) 4 T
(2) 2 T
(3) 8 T
(4) 16 T

Answer. 2 T

Solution. Correct answer is 2T

Question 12. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is:
(1) 0.6
(2) 0.4
(3) 0.2
(4) 1.4

Answer. 0.4

Solution. The modulation index (m) for an amplitude modulation (AM) wave can be calculated using the formula:

m = (V_max - V_min) / (V_max + V_min)

Where:

- V_max is the maximum peak-to-peak voltage of the modulated wave.
- V_min is the minimum peak-to-peak voltage of the modulated wave.

In this case, V_max is 14 mV, and V_min is 6 mV.

m = (14 mV - 6 mV) / (14 mV + 6 mV) m = 8 mV / 20 mV

Now, we can simplify the fraction:

m = (8/20)

m = 2/5

So, the modulation index is 2/5, which as a decimal is 0.4.

The correct answer is (2) 0.4.

Question 13. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.
(1) 4 hours
(2) 6 hours
(3) 12 hours
(4) 3 hours

Answer. 3 hours

Solution. The time period (T) of a satellite in a circular orbit is given by Kepler's third law, which relates the time period to the radius of the orbit. The formula for the time period is:

$T = 2\pi\sqrt{(R^3/GM)}$

Where:

- T is the time period of the satellite.
- R is the radius of the orbit.
- G is the gravitational constant.
- M is the mass of the Earth.

Now, if the separation between the Earth and the satellite is decreased to one fourth of the previous value, the new radius (R') becomes 1/4 of the original radius (R).

So, the new time period (T') is given by:

$T' = 2\pi\sqrt{((R/4)^3/GM)}$

$T' = 2\pi\sqrt{(R^3/(4^3GM))}$

$T' = 2\pi\sqrt{(R^3/GM)} * (1/4)^{(3/2)}$

$T' = 2\pi\sqrt{(R^3/GM)} * 1/8$

$T' = (1/8) * T$

Since the original time period (T) was 24 hours, the new time period (T') is:

$T' = (1/8) * 24$ hours

$T' = 3$ hours

So, the new time period becomes 3 hours.

The correct answer is (4) 3 hours.

Chemistry

Question 31. An indicator ‘X’ is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H 2O 2 at room temp. The indicator ‘X’ forms blue colored complex with compound ‘A’ present in the solution. The indicator ‘X’ and compound ‘A’ respectively are
(1) Starch and iodine
(2) Starch and H_2O_2
(3) Methyl orange and iodine
(4) Methyl orange and H_2O_2

Answer. Starch and iodine

Solution. The indicator "X" that forms a blue colored complex with compound "A" is starch, and compound "A" is iodine.

So the correct answer is (1) Starch and iodine.

Question 35. The major component of which of the following ore is sulphide based mineral?
(1) Malachite
(2) Calamine
(3) Sphalerite
(4) Siderite

Answer. Sphalerite

Solution. The major component of Sphalerite ore is a sulphide-based mineral. So the correct answer is (3) Sphalerite.

Question 36. Given below are two statements:
Statement I : The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.
Statement-II : The d orbitals Ga are in completely filled.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Statement I is incorrect but statement II is correct
(2) Both the statements I and II are incorrect
(3) Both the statements I and II are correct
(4) Statement I is correct but statement II is incorrect

Answer. Both the statements I and II are correct

Solution. The first ionization energies are as follows:
B : 801 kJ/mol
Al : 577 kJ/mol
Ga : 579 kJ/mol
Ga : [Ar]3d104s2 4p1
So, the correct answer is: Both the statements I and II are correct

Question 37. A solution of CrO5 in amyl alcohol has a _____ colour.
(1) Yellow
(2) Green
(3) Blue
(4) Orange-Red

Answer. Blue

Solution. A solution of CrO5 in amyl alcohol has a blue color.

Question 41. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A?
(1) C9H6
(2) C8H6
(3) C6H6
(4) C9H9

Answer. C8H6

Solution. Let's analyze the given information step by step:

1. Hydrocarbon A requires 9.5 equivalents of oxygen for combustion.
2. It produces 3 equivalents of water.

The combustion reaction of hydrocarbons can be represented as follows:
$C_xH_y + (n + m/4)\ O_2 \rightarrow n\ CO_2 + m/2\ H_2O$

Here, n is the number of moles of carbon atoms, and m is the number of moles of hydrogen atoms.

Given: 9.5 equivalents of oxygen are required for combustion. 3 equivalents of water are produced.

This implies: $n + m/4 = 9.5$ $n = 9.5 - m/4$

The number of moles of water produced is equal to m/2. Therefore, the number of equivalents of water produced is m/2.

$m/2 = 3$

Now, we can solve for m: $m = 3 * 2 = 6$

Now that we have found the value of m, we can calculate n: $n = 9.5 - m/4$ $n = 9.5 - 6/4$ $n = 9.5 - 1.5$ $n = 8$

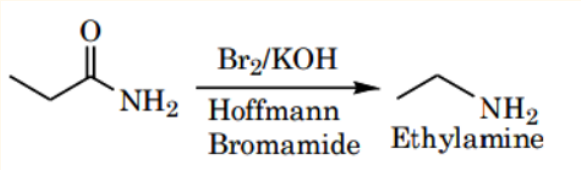
So, the molecular formula of hydrocarbon A is C_8H_6, which corresponds to option (2) - C8H6

Question 43. Reaction of propanamide with Br 2/KOH(aq) produces :

(1) Ethylnitrile
(2) Propylamine
(3) Propanenitrile
(4) Ethylamine

Answer. Ethylamine

Solution. Correct answer is Ethylamine



Question 45. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?
(1) Stomach ulcers
(2) Hyperacidity
(3) Anxiety and stress
(4) Depression and hypertension

Answer. Depression and hypertension

Solution. Equanil is a Tranquilizers which is used in controlling depression and hypertension.

Question 46. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is(excluding rearrangement)
(1) 2-Bromopropane
(2) 1-Bromo-2-methylbutane
(3) 2-Bromopentane
(4) 2-Bromo-3, 3-dimethylpentane

Answer. 2-Bromopentane

Solution. The number of isomeric alkenes formed upon dehydrohalogenation (elimination reaction) of alkyl halides depends on the possibility of forming different constitutional (structural) isomers.

Let's analyze the given options:

1. 2-Bromopropane: This compound can form only one alkene isomer, propene.
2. 1-Bromo-2-methylbutane: This compound can form two isomeric alkenes. Eliminating the bromine atom can yield either 2-methyl-1-butene or 2-methyl-2-butene.
3. 2-Bromopentane: This compound can form two isomeric alkenes. Eliminating the bromine atom can yield either 1-pentene or 2-pentene.
4. 2-Bromo-3,3-dimethylpentane: This compound can form two isomeric alkenes. Eliminating the bromine atom can yield either 2,3-dimethyl-2-pentene or 2,3-dimethyl-1-pentene.

So, the alkyl halide that can give the maximum number of isomeric alkenes on dehydrohalogenation is 2-Bromopentane (option 3), which can form two isomeric alkenes.

Question 48. Given below are two statements:
Statement I : Nickel is being used as the catalyst for producing syn gas and edible fats.
Statement II : Silicon forms both electron rich and electron deficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below :
(1) Statement I is correct but statement II is incorrect
(2) Statement I is incorrect but statement II is correct
(3) Both the statements I and II are correct
(4) Both the statements I and II are incorrect

Answer. Statement I is correct but statement II is incorrect

Solution. The correct answer is:

(1) Statement I is correct but statement II is incorrect.

Statement I is correct because nickel is indeed used as a catalyst in various industrial processes, including the production of synthesis gas (syngas) and hydrogenation of edible fats.

Statement II is incorrect because silicon primarily forms electron-deficient hydrides, not electron-rich hydrides. Examples of silicon hydrides include silane (SiH4) and its derivatives, which are electron-deficient compounds due to the presence of Si-H bonds.

Question 50. According to MO theory the bond orders for O_2^{2-}, CO and NO^+ respectively, are:
(1) 1, 3 and 2
(2) 2, 3 and 3
(3) 1, 3 and 3
(4) 1, 2 and 3

Answer. 1, 3 and 3

Solution. Correct answer is 1, 3 and 3

Mathematics

Question 63. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is
(1) 472
(2) 432
(3) 507
(4) 400

Answer. 432

Solution. Total 3 digit number = 900
Divisible by 3 = 300 (Using 900/3 = 300)
Divisible by 3 & 4 = 75 (Using 900/4 = 225)
Divisible by 3 & 4 = 108, …. (Using = 900/12 = 75)
Number divisible by either 3 or 4 = 300 + 250 – 75 = 450
We have to remove divisible by 48, 144, 192, ….., 18 terms
Required number of numbers = 450 - 18 = 432

Question 66. The shortest distance between the lines (x-1)/2 = (y+8)/-7 = (z-4)/5 and (x-1)/2 = (y-2)/1 = (z-6)/-3 is:
(1) 3√3
(2) 2√3
(3) 5√3
(4) 4√3

Answer. 4√3

Solution. To find the shortest distance between two skew lines, you can use the vector method. The direction vectors of the lines are the coefficients of x, y, and z in the direction ratios.

For the first line: Direction vector = (2, -7, 5)

For the second line: Direction vector = (2, 1, -3)

Now, find the vector between a point on the first line (let's call it P1) and a point on the second line (let's call it P2). The vector between P1 and P2 is the line connecting the two lines.

Let's assume P1 is a point on the first line where x = 1. Then P1 is (1, -8, 4) because we set x = 1 and calculated the corresponding y and z values from the equation.

Let's assume P2 is a point on the second line where x = 1. Then P2 is (1, -1, 3) because we set x = 1 and calculated the corresponding y and z values from the equation.

The vector between P1 and P2 is given by P2 - P1:

P2 - P1 = (1, -1, 3) - (1, -8, 4) = (0, 7, -1)

Now, you want to find the shortest distance between the two lines. To do this, you need to find the projection of the vector P2 - P1 onto the direction vector of one of the lines. You can use the dot product for this:

Shortest distance = |(P2 - P1) · (Direction vector of Line 1)| / |Direction vector of Line 1|

Let's use Line 1 as the reference line:

Shortest distance = |(0, 7, -1) · (2, -7, 5)| / |(2, -7, 5)|

Now, calculate the dot product in the numerator:

(0, 7, -1) · (2, -7, 5) = 02 + 7(-7) + (-1)*5 = -49 - 5 = -54

Calculate the magnitude of the direction vector of Line 1:

|(2, -7, 5)| = √(2^2 + (-7)^2 + 5^2) = √(4 + 49 + 25) = √78

Now, plug these values into the formula:

Shortest distance = |-54| / √78 = 54 / √78

To simplify further, you can rationalize the denominator:

Shortest distance = (54 / √78) * (√78 / √78)

Shortest distance = (54√78) / 78

Now, simplify by canceling common factors:

Shortest distance = (3√78) / 13

So, the shortest distance between the two lines is (3√78) / 13 = 4√3

Question 69. Let R be a relation defined on ℕ as a R b if 2a + 3b is a multiple of 5, a, b Ɛ ℕ. Then R is:
(1) transitive but not symmetric
(2) an equivalence relation
(3) not reflexive
(4) symmetric but not transitive

Answer. an equivalence relation

Solution. To determine the properties of relation R, we need to consider each of the three properties: reflexive, symmetric, and transitive.

1. Reflexive: A relation R is reflexive if, for every element a in the set, aRa holds true. In this case, we need to check if 2a + 3a is a multiple of 5 for all natural numbers a.

2a + 3a = 5a

This is always a multiple of 5 since a is a natural number. Therefore, R is reflexive.

2. Symmetric: A relation R is symmetric if for all a and b in the set, if aRb, then bRa must also hold true. In other words, the order of the elements does not matter. To check symmetry, we need to determine

if for all a and b, if 2a + 3b is a multiple of 5, then 2b + 3a is also a multiple of 5.

2a + 3b being a multiple of 5 implies 2b + 3a is also a multiple of 5, as both involve the same terms.

So, R is symmetric.

3. Transitive: A relation R is transitive if for all a, b, and c in the set, if aRb and bRc, then aRc must hold true. In this case, we need to check if, when 2a + 3b is a multiple of 5 and 2b + 3c is a multiple of 5, then 2a + 3c is also a multiple of 5.

Since both 2a + 3b and 2b + 3c are multiples of 5, their sum 2a + 3b + 2b + 3c is also a multiple of 5.

Therefore, R is transitive.

Based on the analysis, the relation R is reflexive, symmetric, and transitive, which makes it an equivalence relation. So, the correct answer is (2) an equivalence relation.

Question 72. If the lines (x-1)/1 = (y-2)/2 = (z+3)/1 intersect at the point P, then the distance of the point P from the plane z = a is
(1) 10
(2) 22
(3) 28
(4) 16

Answer. 28

Solution.

Point on $L_1 \equiv (\lambda + 1, 2\lambda + 2, \lambda - 3)$

Point on $L_2 \equiv (2\mu + a, 3\mu - 2, \mu + 3)$

$\lambda - 3 = \mu + 3 \quad \Rightarrow \lambda = \mu + 6 \quad \ldots (1)$

$2\lambda + 2 = 3\mu - 2 \Rightarrow 2\lambda = 3\mu - 4 \quad \ldots (2)$

Solving, (1) and (2)

$\Rightarrow \quad \lambda = 22 \ \& \ \mu = 16$

$\Rightarrow \quad P \equiv (23, 46, 19)$

$\Rightarrow \quad a = -9$

Distance of P from z = –9 is 28

Question 74. The plane 2x – y + z = 4 intersects the line segment joining the points A(a, –2, 4) and B(2, b, –3) at the point C in the ratio 2 : 1 and the distance of the point C from the origin is $\sqrt{5}$. If ab < 0 and P is the point (a – b, b, 2b – a) then CP^2 is equal to:

(1) 16/3

(2) 17/3

(3) 73/3

(4) 97/3

Answer. 17/3

Solution. Correct answer is 17/3

$A(a, -2, 4), B(2, b, -3)$

$AC : CB = 2 : 1$

$\Rightarrow C \equiv \left(\frac{a+4}{3}, \frac{2b-2}{3}, \frac{-2}{3}\right)$

C lies on $2x - y + z = 4$

$\Rightarrow \frac{2a+8}{3} - \frac{2b-2}{3} - \frac{2}{3} = 4$

$\Rightarrow a - b = 2 \ldots (1)$

Also $OC = \sqrt{5}$

$\Rightarrow \left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \frac{4}{9} = 5 \quad \ldots (2)$

Solving, (1) and (2)

$(b + 6)^2 + (2b - 2)^2 = 41$

$\Rightarrow 5b^2 + 4b - 1 = 0$

$\Rightarrow b = -1 \text{ or } \frac{1}{5}$

$\Rightarrow a = 1 \text{ or } \frac{11}{5}$

But $ab < 0 \Rightarrow (a, b) = (1, -1)$

$C \equiv \left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3}\right), P \equiv (2, -1, -3)$

$CP^2 = \frac{1}{9} + \frac{1}{9} + \frac{49}{9} = \frac{51}{9} = \frac{17}{3}$

Question 76. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is:

(1) 79

(2) 86
(3) 84
(4) 89

Answer. 89

Solution. Let arrange the letters of OUGHT in alphabetical order. G, H, O, T, U Words starting

$G \text{ - - - - } \rightarrow 4!$

$H \text{ - - - - } \rightarrow 4!$

$O \text{ - - - - } \rightarrow 4!$

$T\,G \text{ - - - } \rightarrow 3!$

$T\,H \text{ - - - } \rightarrow 3!$

$T\,O\,G \text{ - - } \rightarrow 2!$

$T\,O\,H \text{ - - } \rightarrow 2!$

$T\,O\,U\,G\,H \rightarrow 1!$

Total = 89

Question 81. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is _____.

Answer. 3000

Solution. N should be divisible by 2 but not by 3
N = (Numbers divisible by 2) – (Numbers divisible by 6)
N = 9000/2 - 9000/6 = 4500 - 1500 = 3000

Question 82. If the equation of the normal to the curve y = (x-a) / {(x+b)(x-2)} at the point (1,-3) is x-4y = 13, then he value of a + b is equal to _____?

Answer. 4

Solution. Correct answer is 4

$$y = \frac{x-a}{(x+b)(x-2)}$$

At point $(1, -3)$,

$$-3 = \frac{1-a}{(1+b)(1-2)}$$

$$\Rightarrow 1 - a = 3(1 + b) \quad \ldots (1)$$

Now, $y = \frac{x-a}{(x+b)(x-2)}$

$$\Rightarrow \frac{dy}{dx} = \frac{(x+b)(x-2)\times(1)-(x-a)(2x+b-2)}{(x+b)^2(x-2)^2}$$

At $(1, -3)$ slope of normal is $\frac{1}{4}$ hence $\frac{dy}{dx} = -4$,

So, $-4 = \frac{(1+b)(-1)-(1-a)b}{(1+b)^2(-1)^2}$

Using equation (1)

$$\Rightarrow -4 = \frac{(1+b)(-1)-3(b+1)b}{(1+b)^2}$$

$$\Rightarrow -4 = \frac{(-1)-3b}{(1+b)}(b \neq -1)$$

$$\Rightarrow b = -3$$

So, $a = 7$

Hence, $a + b = 7 - 3 = 4$

Question 84. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to _____?

Answer. 10

Solution. A triangle is formed by the tangents at the point (2, 2) on the curves y2 = 2x and x2 + y2 = 4x, and the line x + y + 2 = 0. If r is the radius of its circumcircle, then r2 is equal to 10.

$S_1 : y^2 = 2x \qquad S_2 : x^2 + y^2 = 4x$

P(2,2) is common point on S_1 & S_2

T_1 is tangent to S_1 at P $\Rightarrow T_1 : y.2 = x + 2$

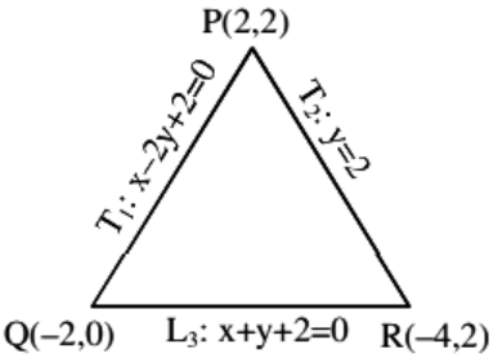
$\Rightarrow T_1 : x - 2y + 2 = 0$

T_2 is tangent to S_2 at P

$\Rightarrow T_2 : x.2 + y.2 = 2(x+2)$

$\Rightarrow T_2 : y = 2$

& $L_3 : x + y + 2 = 0$ is third line



$PQ = a = \sqrt{20}$

$QR = b = \sqrt{8}$

$RP = c = 6$

$\text{Area } (\Delta PQR) = \Delta = \frac{1}{2} \times 6 \times 2 = 6$

$\therefore\ r = \frac{abc}{4\Delta} = \frac{\sqrt{160}}{4} = \sqrt{10} \Rightarrow r^2 = 10$

Question 88. A circle with centre (2, 3) and radius 4 intersects the line x + y = 3 at the points P and Q. If the tangents at P and Q intersect at the point S(α, β), then 4α – 7β is equal to _____?

Answer. 11

Solution. The given circle is x2 + y2 – 4x – 6y – 3 = 0 Chord of contact
The given circle is x2 + y2 – 4x – 6y – 3 = 0 Chord of contact

$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$

$\Rightarrow (\alpha - 2)x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0$ ..... (i)

$\because$ But the equation of chord of contact is given

as : x + y – 3 = 0 ..... (ii)

comparing the coefficients

$$\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = -\left(\frac{2\alpha + 3\beta + 3}{-3}\right)$$

On solving $\alpha = -6$

$\beta = -5$

Now $4\alpha - 7\beta = 11$