

# JEE Main 2024 Question Paper with Solution

## Jan 27 Shift 1 (B.E./B.Tech)

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### JEE Main Physics Questions

**Ques 1.** A body of mass 1000 kg has a velocity of 6 m/s. If an extra 2000 kg mass is embedded in it, then what will be the velocity of the combined mass?

- A. 5 m/s
- B. 4 m/s
- C. 2 m/s
- D. 3 m/s

**Ans.** C

**Solution.** To determine the velocity of the combined mass after embedding the extra 2000 kg, we can use the principle of conservation of linear momentum. This principle states that in a closed system, the total linear momentum remains constant unless acted upon by an external force. In this case, the system consists of the 1000 kg body and the embedded 2000 kg mass. Before embedding, the total momentum is:

$$p_1 = m_1 v_1 = (1000 \text{ kg}) * (6 \text{ m/s}) = 6000 \text{ kg m/s}$$

After embedding, the total mass becomes 3000 kg (1000 kg + 2000 kg). Let  $v_2$  be the final velocity of the combined mass. Applying the conservation of momentum, we have:

$$p_1 = p_2$$

$$6000 \text{ kg m/s} = (3000 \text{ kg}) * v_2$$

Solving for  $v_2$ , we get:

$$v_2 = (6000 \text{ kg m/s}) / (3000 \text{ kg}) = 2 \text{ m/s}$$

Therefore, the velocity of the combined mass after embedding the extra 2000 kg is 2 m/s. This means that the overall system slows down due to the increase in mass while maintaining the total momentum.

**Ques 2. Two infinite current carrying wires having current  $I$  in opposite directions are shown below. Find the magnetic field in S.I units, at point**

- A.  $7\mu_0 I / \pi$
- B.  $10\mu_0 I / \pi$
- C.  $5\mu_0 I / \pi$
- D.  $\mu_0 I / \pi$

**Ans. B**

**Solution.** Given:

- Distance between the wires  $d = 20 \text{ cm} = 0.20 \text{ m}$

- Current  $I$  in each wire

- Permeability of free space  $\mu = 4\pi \text{ times } 10^{-7} \text{ T}\cdot\text{m/A}$

The distance from each wire to the midpoint is  $d/2 = 0.10 \text{ m}$ .

Magnetic Field due to Each Wire:

For each wire, the distance to the midpoint is  $r = 0.10 \text{ m}$ .

$$B_1 = \mu_0 \cdot I / 2\pi \cdot 0.10$$

$$B_2 = \mu_0 \cdot I / 2\pi \cdot 0.10$$

Total Magnetic Field at Midpoint:

Since the wires are carrying currents in opposite directions, the magnetic fields they generate at the midpoint will add up:

$$B_{\text{total}} = B_1 + B_2$$

$$B_{\text{total}} = \mu_0 \cdot I / 2\pi \cdot 0.10 + \mu_0 \cdot I / 2\pi \cdot 0.10$$

$$B_{\text{total}} = 2\mu_0 \cdot I / 2\pi \cdot 0.10$$

$$B_{\text{total}} = \mu_0 \cdot I / \pi \cdot 0.10$$

$$B_{\text{total}} = 10\mu_0 \cdot I / \pi$$

So, the correct option is:

$$B. 10\mu_0 \cdot I / \pi$$

**Ques 3. If the diameter of earth becomes half keeping mass to be constant, then the acceleration due to gravity at surface of earth becomes**

- A. half**
- B. four times**
- C. twice**
- D. three times**

**Ans. B**

**Solution.** If the diameter of Earth becomes half while its mass remains constant, the acceleration due to gravity at the surface will increase four times, not half, twice, or three times. Here's why:

1. Gravity and distance: The strength of gravity between two objects is inversely proportional to the square of the distance between their centers.

This is based on Newton's Law of Universal Gravitation:

$$F = G * (m_1 * m_2) / r^2$$

F is the gravitational force

G is the gravitational constant

$m_1$  and  $m_2$  are the masses of the objects

r is the distance between their centers

Imagine the Earth as a sphere. Now, imagine shrinking that sphere to half its diameter while keeping the same amount of mass inside. This would be like squeezing the same amount of clay into a smaller ball.

Distance: Because the diameter is halved, the radius (distance from the center to the surface) is also halved. This means the "r" in the equation above is halved.

Force: Since  $r^2$  becomes four times smaller, the gravitational force (F) will quadruple.

2. Constant mass: In this scenario, the Earth's mass ( $m_1$ ) remains constant even though its diameter shrinks. This means the factor of  $m_1$  in the gravity equation stays the same.

3. Resulting increase: Combining the effects of halved distance and constant mass, the gravitational force at the surface of Earth will be four

times what it was before. Since acceleration due to gravity ( $g$ ) is directly proportional to the gravitational force, it will also increase fourfold. Therefore, the new acceleration due to gravity at the surface of Earth would be approximately  $4 * 9.81 \text{ m/s}^2 \approx 39.24 \text{ m/s}^2$  (where  $9.81 \text{ m/s}^2$  is the approximate value of Earth's gravity at sea level).

Remember, this is a hypothetical scenario. In reality, it is not possible for the Earth to suddenly change its diameter while maintaining its mass without violating physical laws.

I hope this explanation with the visual of shrinking Earth clarifies why the acceleration due to gravity would increase fourfold in this hypothetical scenario.

**Ques 4. Two masses  $m_1 = 4 \text{ gm}$  and  $m_2 = 25 \text{ gm}$  are having same kinetic energy; find the ratio of linear momentum**

- A. 1:5
- B. 2:5
- C. 1:1
- D. 1:6

**Ans. B**

**Solution.** Here's how to find the ratio of linear momentum for the two masses with the same kinetic energy:

Kinetic Energy equation: We know both masses have the same kinetic energy (KE). Let's denote it as KE. The Kinetic Energy formula is:

$$KE = \frac{1}{2} * m * v^2$$

where:

KE is the kinetic energy

$m$  is the mass

$v$  is the velocity

Equating KEs: Since both masses have the same KE, we can write:

$$KE_1 = KE_2$$

Substitute the equation for KE for both sides:

$$\frac{1}{2} * m_1 * v_1^2 = \frac{1}{2} * m_2 * v_2^2$$

Solve for velocity ratio: To find the ratio of linear momentum, we need to express the velocities ( $v_1$  and  $v_2$ ) in terms of each other. Rearrange the equation above:

$$v_2^2 = (m_1 / m_2) * v_1^2$$

Take the square root of both sides to find the ratio of velocities:

$$v_2 / v_1 = \sqrt{m_1 / m_2}$$

Find mass ratio: We are given that  $m_1 = 4$  gm and  $m_2 = 25$  gm. Plug these values into the ratio equation:

$$v_2 / v_1 = \sqrt{4 \text{ gm} / 25 \text{ gm}} = \sqrt{4/25} = 2/5$$

Linear momentum ratio: Linear momentum ( $p$ ) is the product of mass and velocity:

$$p = m * v$$

Therefore, the ratio of linear momenta ( $p_2$ ) to ( $p_1$ ) is:

$$p_2 / p_1 = (m_2 * v_2) / (m_1 * v_1) = (m_2 * (2/5) * v_1) / (m_1 * v_1) = 2m_2 / 5m_1$$

Substitute the given mass values:

$$p_2 / p_1 = (2 * 25 \text{ gm}) / (5 * 4 \text{ gm}) = 50 \text{ gm} / 20 \text{ gm} = 5/2$$

Therefore, the ratio of linear momentum for the two masses is 5:2. In simpler terms, the lighter mass (4 gm) has two and a half times the linear momentum of the heavier mass (25 gm) despite having the same kinetic energy.

**Ques 5. A rod of length  $l$  having resistance  $R$ , is cut into two equal parts. These parts are connected in parallel then new resistance shall be?**

- A.  $R$
- B.  $R/2$
- C.  $R/4$
- D.  $2R$

**Ans. B**

**Solution.** when a rod of resistance  $R$  is cut into two equal parts and connected in parallel, the new resistance will be  $R/4$ .

Here's why:

Resistance in parallel: When resistors are connected in parallel, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances. This property can be expressed as:

$$1/R_{\text{total}} = 1/R_1 + 1/R_2 + \dots \text{ (for parallel connection)}$$

Equal parts: Since the rod is cut into two equal parts, each part will have a resistance of  $R/2$ .

Applying formula: Substitute the individual resistances in the parallel connection formula:

$$1/R_{\text{total}} = 1/(R/2) + 1/(R/2) = 2/R$$

Solve for total resistance: Invert both sides of the equation to find the total resistance:

$$R_{\text{total}} = R / 2$$

Therefore, the new resistance of the rod when connected in parallel after being cut into two equal parts is  $R/2$ .

**Ques 6. A charge  $Q=10^{-6}\text{C}$  is placed at origin. Find the potential difference between two points A and B whose position vectors are  $(\sqrt{3}\hat{i} + \sqrt{3}\hat{j})$  m and  $\sqrt{6}\hat{j}$  respectively  $\hat{i}$**

- A. zero**
- B. 1000 Volts**
- C. 2000 Volts**
- D. 500 Volts**

**Ans. A**

**Solution.** Let's calculate each step:

Potential at Point \ ( A \):

$$r_A = \sqrt{6}$$

$$V_A = (8.9875 \cdot 10^9) \cdot (10^{-6}) / \sqrt{6}$$

$$V_A \approx (8.9875 \cdot 10^3) \cdot \sqrt{6}$$

$$V_A \approx (8.9875 \cdot 10^3) \cdot 2.45$$

$$V_A \approx 3667.35 \text{ V}$$

Potential at Point B :

$$r_B = \sqrt{6}$$

$$V_B = (8.9875 \times 10^9) \times (10^{-6}) / \sqrt{6}$$

$$V_B \approx (8.9875 \times 10^3) \times \sqrt{6}$$

$$V_B \approx (8.9875 \times 10^3) \times 2.45$$

$$V_B \approx 3667.35 \text{ V}$$

Potential Difference  $\Delta V$ :

$$\Delta V = V_B - V_A$$

$$\Delta V = 3667.35 \text{ V} - 3667.35 \text{ V}$$

$$\Delta V \approx 0 \text{ V}$$

So, the potential difference between points A and B is approximately 0V.

**Ques 7. Consider the system shown. Find the moment of inertia about the diagonal shown.**

**A.  $1 \text{ kg.m}^2$**

**B.  $2 \text{ kg.m}^2$**

**C.  $4 \text{ kg.m}^2$**

**D.  $6 \text{ kg.m}^2$**

**Ans. C**

**Ques 8. Statement 1: Linear momentum and moment of force have the same dimensions.**

**Statement 2: Planck's constant and angular momentum have same Dimension.**

**A. Statement 1 is correct while statement 2 is false**

**B. Statement 1 is false while Statement 2 is correct**

**C. Both statements are correct**

**D. Both statements are false**

**Ans. B**

**Solution.** Let's analyze each statement:

Statement 1: "Linear momentum and moment of force have the same dimensions."

Linear momentum ( $p$ ) is defined as mass ( $m$ ) multiplied by velocity ( $v$ ), so its dimensions are  $[\text{mass}] * [\text{velocity}] = [M][LT^{-1}]$ .

Moment of force (torque) is defined as force ( $F$ ) multiplied by distance ( $r$ ), so its dimensions are  $[\text{force}] * [\text{distance}] = [MLT^{-2}] * [L] = [ML^2T^{-2}]$ .

Since the dimensions of linear momentum and moment of force are not the same ( $[M][LT^{-1}]$  vs.  $[ML^2T^{-2}]$ ), Statement 1 is false.

Statement 2: "Planck's constant and angular momentum have the same dimensions."

Planck's constant ( $h$ ) is an action quantum and has dimensions of energy multiplied by time, denoted as  $[\text{energy}] * [\text{time}] = [ML^2T^{-2}] * [T] = [ML^2T^{-1}]$ .

Angular momentum ( $L$ ) is defined as the product of moment of inertia ( $I$ ) and angular velocity ( $\omega$ ), so its dimensions are  $[\text{moment of inertia}] * [\text{angular velocity}] = [ML^2] * [T^{-1}] = [ML^2T^{-1}]$ .

Since the dimensions of Planck's constant and angular momentum are the same ( $[ML^2T^{-1}]$ ), Statement 2 is correct.

Therefore, the correct answer is: Statement 1 is false while Statement 2 is correct.

**Ques 9. A prism has a refractive index  $\cot(A/2)$ , where  $A$  is the refractive angle of the prism. The minimum deviation due to this prism is**

- A.  $\pi-3A$
- B.  $\pi-2A$
- C.  $A$
- D.  $A/2$

**Ans. B**

**Solution.** The correct option is **B**  $\pi-2A$

By using  $\mu = \frac{\sin(A+\delta_m)/2}{\sin(A/2)}$



$$\Rightarrow \cot(A/2) = \sin((A+\delta m)/2) / (\sin A/2) \Rightarrow \cos(A/2) / \sin(A/2) = \sin((A+\delta m)/2) / \sin(A/2)$$

$$\Rightarrow \sin(90^\circ - A/2) = \sin((A+\delta m)/2)$$

$$\Rightarrow \delta m = 180 - 2A$$

**Ques 10. A particle performing simple harmonic motion is such that its amplitude is 4 m and speed of particle at mean position is 10 m/s. Find the distance of the particle from mean position where velocity becomes 5 m/s.**

- A.  $\sqrt{3}$  m
- B.  $2\sqrt{3}$  m
- C.  $\sqrt{3/2}$  m
- D.  $1/\sqrt{2}$  m

**Ans. B**

**Solution.** Given,

Amplitude ,  $A=4\text{cm}$

velocity of the particle at mean position ,  $v_0=10\text{cm/s}$

We know the relation between speed of particle , amplitude and distance of particle from mean position is given by,

$$v = \omega \sqrt{A^2 - x^2}$$

Here  $\omega$  is angular frequency,

speed at mean position ,  $v = \omega \sqrt{A^2 - X^2}$

speed at x distance from mean position ,  $v = \omega \sqrt{A^2 - X^2} = 5\text{cm/s}$

$$\text{So, } \omega A \sqrt{A^2 - X^2} = 105$$

$$\Rightarrow A \sqrt{A^2 - X^2} = 2$$

$$\Rightarrow A \sqrt{4^2 - X^2} = 2$$

$$\Rightarrow 4 = 4^2 - X^2$$

$$\Rightarrow X^2 = 12$$

$$\Rightarrow X = \pm 2\sqrt{3}\text{cm}$$

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## JEE Main Chemistry Questions

**Ques 1. Which of the following can not show variable oxidation state?**

- A. Chlorine
- B. Fluorine
- C. Bromine
- D. Iodine

**Ans. B**

**Solution.** The element that cannot show variable oxidation state among the listed options is:

Fluorine (F)

Here's why:

Chlorine (Cl): Can exhibit oxidation states ranging from -1 to +7, commonly observed in compounds like NaCl (-1),  $\text{KClO}_3$  (+5), and  $\text{ClF}_7$  (+7).

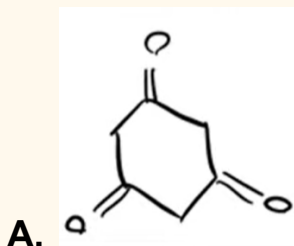
Bromine (Br): Shows variable oxidation states from -1 to +7, similar to chlorine, found in compounds like NaBr (-1),  $\text{KBrO}_3$  (+5), and  $\text{BrF}_7$  (+7).

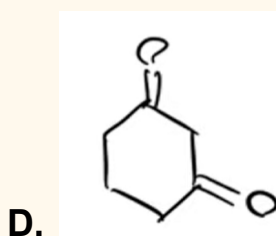
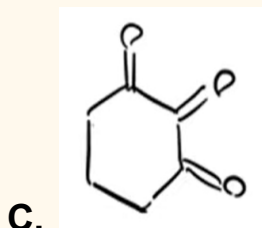
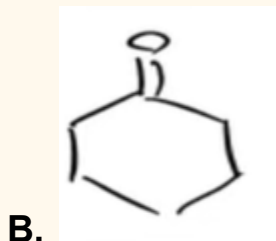
Iodine (I): Exhibits several oxidation states from -1 to +7, like NaI (-1),  $\text{KIO}_3$  (+5), and  $\text{IF}_7$  (+7).

Fluorine (F): Almost exclusively exists in the -1 oxidation state due to its high electronegativity. It readily gains an electron to achieve a stable octet configuration and rarely loses electrons to form higher oxidation states.

Therefore, the element displaying the least propensity for variable oxidation states among the options is fluorine (F).

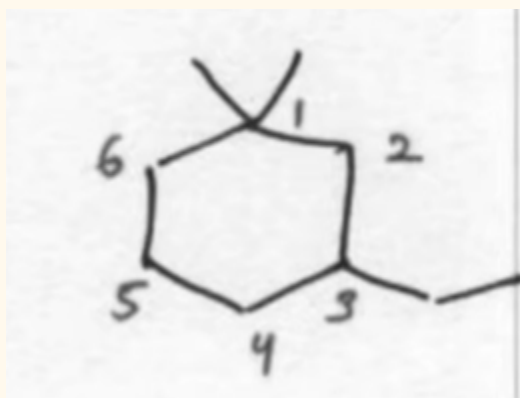
**Ques 2. Which of the following has the highest enol content.**





Ans. A

Ques 3. IUPAC name of this compound is?



A. 1 - ethyl - 3,3-dimethyl cyclohexane

B. 3 - ethyl - 1,1- dimethyl cyclohexane

C. 1 - ethyl - 3,3-dimethyl cyclohexene

D. 3 - ethyl - 1,1- dimethyl cyclohexene

Ans. B

Ques 4. The compound given below is?



- A. Alicyclic
- B. Aromatic
- C. Antiaromatic
- D. Acyclic

Ans. A

Ques 5. Which of the following is a Polar molecule?



A.  $\text{CH}_2=\text{CH}_2$

- B.  $\text{CHCl}_3$
- C.  $\text{CCl}_4$
- D.  $\text{CH}_4$

Ans. B

**Ques 6.** In which of the following compound central atoms has +4 oxidation state?

- A.  $\text{SO}_3$
- B.  $\text{H}_2\text{SO}_3$
- C.  $\text{H}_2\text{S}_2\text{O}_7$
- D.  $\text{BaSO}_4$

Ans. B

**Solution.** To determine the oxidation state of the central atom in each compound, let's analyze each one:

A.  $\text{SO}_3$  (Sulfur trioxide): In  $\text{SO}_3$ , the oxidation state of sulfur (S) can be calculated as follows:

$3(-2) = -6$  (oxygen has an oxidation state of -2, and there are three oxygen atoms)

Let  $x$  be the oxidation state of sulfur.

Therefore,  $x - 6 = 0$  (since the overall charge of  $\text{SO}_3$  is 0)

$$x = +6$$

So, sulfur in  $\text{SO}_3$  has an oxidation state of +6.

B.  $\text{H}_2\text{SO}_3$  (Sulfurous acid): In  $\text{H}_2\text{SO}_3$ , the sulfur is in a lower oxidation state than in  $\text{SO}_3$ . Let's denote the oxidation state of sulfur as  $x$ .

$2(1) + x + 3(-2) = 0$  (considering the charges on hydrogen and oxygen)

$$2 + x - 6 = 0$$

$$x = +4$$

So, sulfur in  $\text{H}_2\text{SO}_3$  has an oxidation state of +4.

C.  $\text{H}_2\text{S}_2\text{O}_7$  (Peroxydisulfuric acid): Here, there are two sulfur atoms. Let's denote the oxidation state of sulfur as  $x$ .

$$2(1) + 2x + 7(-2) = 0$$

$$2 + 2x - 14 = 0$$

$$2x - 12 = 0$$

$$2x = 12$$

$$x = +6$$

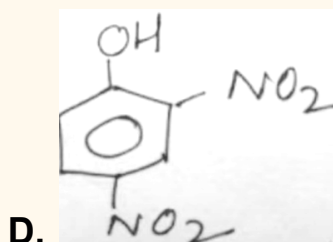
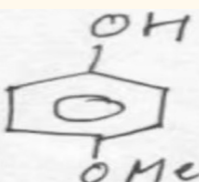
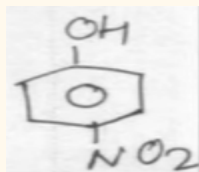
So, sulfur in  $\text{H}_2\text{S}_2\text{O}_7$  has an oxidation state of +6.

D.  $\text{BaSO}_4$  (Barium sulfate): In  $\text{BaSO}_4$ , sulfur is in a higher oxidation state than in  $\text{SO}_3$ . However, in  $\text{BaSO}_4$ , sulfur is not the central atom. The central atom is barium (Ba), and sulfur's oxidation state doesn't influence the overall oxidation state of the compound.

So, among the options given, the compound in which the central atom has a +4 oxidation state is B.  $\text{H}_2\text{SO}_3$

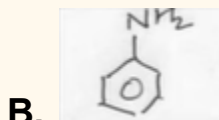
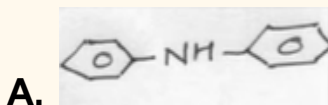
**Ques 7. Which of the following is most acidic ?**

**A. Bu-OH**



**Ans. D**

**Ques 8. Select the strongest base**





Ans. D

**Ques 9. The electronic configuration of Neodymium (60) Nd is**

- A.  $[\text{Xe}] 4f^4 6s^2$
- B.  $[\text{Xe}] 5f^1$
- C.  $[\text{Xe}] 4f^2 6s^2$
- D.  $[\text{Xe}] 5f^4 4d^1$

Ans. A

**Solution.** Here's a breakdown of the reasoning:

- Neon (Ne) with atomic number 20 acts as the "core" electron configuration for elements beyond it.
- From Ne, electrons fill shells in the following order: 4f, 5d, 6s, 4p, 5p, 6d, 7s, etc. (following Aufbau principle).
- Neodymium has an atomic number of 60. Filling the shells according to the order mentioned above:
  - First 54 electrons fill the shells up to  $5p^5$ .
  - The next electron goes to the 4f orbital (as per Aufbau principle).
  - Neodymium has a total of 6 electrons in the 4f subshell.
  - Finally, the remaining two electrons fill the 6s orbital.

Therefore, the correct electronic configuration for Neodymium is:

$[\text{Xe}]$  (core configuration)  $4f^4 6s^2$

Options B, C, and D are incorrect because:

- B: 5f orbitals are filled after 4f orbitals.
- C: Only 2 electrons are present in the 4f subshell, while Nd has 6.
- D: 4d orbitals are filled after 5f orbitals and before 6s.

**Ques 10. Ethanol shows turbidity with lucas reagent (conc.HCl + anhydrous ZnCl<sub>2</sub>) ?**

- A. Immediately**
- B. After 5 to 7 mins**
- C. Upon heating**
- D. After 10 - 12 mins**

**Ans. C**

**Solution.** Lucas reagent (concentrated HCl + anhydrous ZnCl<sub>2</sub>) is used to test the presence of an alcohol functional group in a compound. It reacts with primary and secondary alcohols to form alkyl halides.

Ethanol is a primary alcohol. When ethanol is treated with Lucas reagent, it does not show turbidity immediately because primary alcohols do not react readily with Lucas reagent at room temperature. However, if the reaction mixture is heated, turbidity is observed due to the formation of the corresponding alkyl halide. The time it takes for turbidity to appear can vary, but generally, it happens within a few minutes upon heating. So, the correct option is "Upon heating."

**Ques 11. Which type of linkage is present in nucleotides between base and sugar?**

- A. Peptide linkage**
- B. Glycosidic linkage**
- C. N-Glycosidic linkage**
- D. Amide linkage**

**Ans. C**

**Solution.** The linkage present in nucleotides between the base and the sugar is called a N-glycosidic linkage. This linkage involves the nitrogen atom (N) of the base and the C1 carbon of the sugar in the nucleotide structure. Nucleotides are the building blocks of nucleic acids like DNA and RNA, where they play crucial roles in encoding genetic information.



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## JEE Main Mathematics Questions

**Ques 1.** If  $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ ,  $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$ ,  $\vec{a} \cdot \vec{c} = 3$  and  $\vec{a} \times \vec{c} = \vec{b}$ , then  $\vec{a} \cdot ((\vec{x} \cdot \vec{b}) - \vec{b} \cdot \vec{c}) =$

- A. 24
- B. 38
- C. 10
- D. None of these

**Ans. A**

**Ques 2.** The vertices of a triangle ABC are A(1,2), B(-3,4) and C(5,8) then orthocentre of  $\triangle ABC$  is

- A.  $(\frac{2}{3}, 1)$
- B.  $(-\frac{7}{3}, 2)$
- C. (2, 3)
- D.  $(\frac{3}{2}, 1)$

**Ans. D**

**Solution.** To find the orthocenter of triangle ABC, we need to find the point where the altitudes intersect.

The equation of an altitude of a triangle passing through a vertex is given by the line perpendicular to the opposite side passing through that vertex.

- Altitude from Vertex A: Passing through point A(1, 2), perpendicular to line BC. Equation:  $y - 2 = -2(x - 1)$  or  $y = -2x + 4$ .

- Altitude from Vertex B: Passing through point B(-3, 4), perpendicular to line AC. Equation:  $y-4=-2/3(x+3)$  or  $y=-2/3x+2$ .
- Altitude from Vertex C: Passing through point C(5, 8), perpendicular to line AB. Equation:  $y-8=2(x-5)$  or  $y=2x-2$ .

Now, we find the intersection points of these lines:

Lines 1 and 2: Solve  $y=-2x+4$  and  $y=-2/3x+2$ , we get  $(3/2,1)$ .

Lines 2 and 3: Solve  $y=-2/3x+2$  and  $y=2x-2$ , we get  $(-3/2,3)$ .

Lines 1 and 3: Solve  $y=-2x+4$  and  $y=2x-2$ , we get  $(-3/2,1)$ .

The orthocenter is the point where all three altitudes intersect, which is  $(3/2,1)$ .

**Ques 3.  $S_1 = 3, 9, 15, \dots 25$  terms and  $S_2 = 3, 8, 13, \dots 37$  terms, then the number of common terms in  $S_1, S_2$  is equal to**

- A. 3
- B. 4
- C. 5
- D. 6

**Ans. C**

**Solution.** To find the common terms between the sequences  $S_1$  and  $S_2$ , let's first express the  $n$ th term of each sequence:

For  $S_1$ , the  $n$ th term is given by:  $a_n=3+(n-1)6$

For  $S_2$ , the  $n$ th term is given by:  $b_n=3+(n-1)5$

To find common terms, we equate  $a_n$  and  $b_m$  and solve for  $n$  and  $m$ :

$$3+(n-1)6=3+(m-1)5 \quad 6n-6=5m-5 \quad 6n-5m=1$$

Now, we solve this Diophantine equation for integer solutions of  $n$  and  $m$ .

One approach is to use modular arithmetic. Since  $6n$  and  $5m$  differ by 1, we look for pairs  $(n,m)$  such that  $6n \equiv 1 \pmod{5}$ .

Let's find the modular inverse of 6 modulo 5:  $6 \cdot 1 \equiv 1 \pmod{5}$

So, the modular inverse of 6 modulo 5 is 1.

This means  $n \equiv 1 \cdot 1 \pmod{5}$ , implying  $n \equiv 1 \pmod{5}$ .

Now, we can iterate through  $n$  and find the corresponding  $m$  values using  $6n-5m=1$ .

Starting with  $n=1$ , we find  $m$ :  $6(1)-5m=1$   $6-5m=1$   $5m=5$   $m=1$

So, the first common term is when  $n=m=1$ .

Incrementing  $n$  by 5, we get:  $n=6 \Rightarrow m=7$   $n=11 \Rightarrow m=13$   $n=16 \Rightarrow m=19$

$n=21 \Rightarrow m=25$   $n=26 \Rightarrow m=31$

We find 5 common terms in total.

Therefore, the number of common terms in  $S_1$  and  $S_2$  is 5.

**Ques 4. The value of  $k$  for  $(2k, 3k)$ ,  $(0, 0)$ ,  $(1,0)$  and  $(0,1)$  to be on the circle is:**

- A.  $2/13$
- B.  $5/13$
- C.  $1/13$
- D.  $2/13$

**Ans. B**

**Solution.** The equation of a circle is given by:  $(x-h)^2+(y-k)^2=r^2$

Given points are:

- $(2k,3k)$
- $(0,0)$
- $(1,0)$
- $(0,1)$

We can write the equations for each of these points as they lie on the circle:

For point  $(2k,3k)$ :  $(2k-h)^2+(3k-k)^2=r^2 \Rightarrow (2k-h)^2+(2k)^2=r^2$

For point  $(0,0)$ :  $(0-h)^2+(0-k)^2=r^2 \Rightarrow h^2+k^2=r^2$

For point  $(1,0)$ :  $(1-h)^2+(0-k)^2=r^2 \Rightarrow (1-h)^2+k^2=r^2$

For point  $(0,1)$ :  $(0-h)^2+(1-k)^2=r^2 \Rightarrow h^2+(1-k)^2=r^2$

Now, we have a system of equations with three unknowns ( $h$ ,  $k$ , and  $r^2$ ).

Subtracting equation 2 from equation 1, and equation 2 from equation 3,

we can eliminate  $r^2$ :  $(2k-h)^2+(2k)^2-(h^2+k^2)=0$   $(1-h)^2+k^2-(h^2+k^2)=0$

Expanding and simplifying:  $4k^2-4kh+h^2+4k^2=h^2+k^2 \Rightarrow (1-h)^2-h^2=0$

Expanding and simplifying further:  $1-2h+h^2-h^2=0$   $1-2h=0 \Rightarrow h=1/2$

Now, substituting  $h=1/2$  into equation 2:  $(1/2)^2+k^2=r^2 \Rightarrow 1/4 + k^2=r^2$

From equation 4, we know  $h^2=1/4$ , so we can directly substitute this into equation 4:  $1/4+(1-k)^2=r^2$

Now, we have two equations with two unknowns (k and  $r^2$ ):

$$1/4+k^2=r^2$$

$$1/4+(1-k)^2=r^2$$

Let's solve this system of equations for k. Subtracting the first equation from the second:

$$(1-k)^2-k^2=0$$

$$1-2k=0$$

$$k=1/2$$

Ques 5.  $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} dx = a + b\sqrt{2} + c\sqrt{3}$  then  $2a-3b-4c$  is equal to \_\_\_\_\_.

- A. 10
- B. 0
- C. 12
- D. 20

Ans. C

Ques 6. If  ${}^{n-1}C_r = (k^2 - 8)^n C_{r+1}$  Find k.

- A.  $k \in [-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3]$
- B.  $k \in [-4, -2\sqrt{3}) \cup (2\sqrt{3}, 4]$
- C.  $k \in [2\sqrt{3}, 4]$
- D.  $k \in [3, 2\sqrt{3}]$

Ans. A

Ques 7. If  $f(x) - f(y) = \ln(x/y) + x - y$ , then find  $\sum_{k=1}^{20} f' \left( \frac{1}{k^2} \right)$

- A. 2890
- B. 2390
- C. 1245
- D. None of this

Ans. A

Ques 8. Shortest distance between the parabola  $y^2 = 4x$  and  $x^2 + y^2 - 4x - 16y + 64 = 0$  is equal to

- A.  $2\sqrt{3} - 2$
- B.  $3\sqrt{2} - 3$
- C.  $4\sqrt{5} - 2$
- D.  $2\sqrt{5} - 2$

Ans. D

Ques 9. If  $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Statement I  $\Rightarrow f(x) \cdot f(y) = f(x+y)$

Statement II  $\Rightarrow f(-x) = 0$  is invertible

- A. Statement I is True, Statement II is False
- B. Statement I is True, Statement II is True
- C. Statement I is False, Statement II is True
- D. Statement I is False, Statement II is False

Ans. B

Ques 10. If  $\lim_{x \rightarrow 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4} = A$  and  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}} = B$ , then  $AB^3 = \underline{\hspace{2cm}}$ .

- A. 8
- B. 32
- C. 6
- D. None of these

Ans. B

Ques 11. Two lines  $L_1$  &  $L_2$  passing through origin trisecting the line segment intercepted by the line  $4x + 5y = 20$  between the coordinate axes. Then the tangent of angle between the lines  $L_1$  and  $L_2$  is:

- A.  $\sqrt{3}$
- B.  $1/\sqrt{3}$
- C. 1
- D. 30/41

Ans. D

Ques 12. If  $\cos 2x - a \sin x = 2a - 7$  then range of  $a$  is:

- A.  $-2 \leq a \leq 0$
- B.  $2 \leq a \leq 6$
- C.  $a \geq 6$
- D.  $6 \leq a \leq 8$

Ans. B