PART : MATHEMATICS

1. The coefficient of $x^{2012}$ in $(1 - x)^{2006} (1 + x + x^2)^{2007}$ is

   (1) 0
   (2) 1
   (3) 2
   (4) 3

Ans. (1)

Sol. $(1 - x)^{2006} (1 + x + x^2)^{2007} = (1 - x)^{2006} (1 - x^3)^{2007}$

$= (1-x) \sum_{r=0}^{2007} \binom{2007}{r} (-x^3)^r$
2. If the equation \(2\tan^2 \theta - 5 \sec \theta = 1\) has 7 solutions in \(\theta \in [0, \frac{n\pi}{2}]\) for least value of \(n \in \mathbb{N}\) then value of \(\sum_{k=1}^{n} \frac{k}{2^k}\) is equal to

\[
\begin{align*}
(1) \quad \frac{9}{2^3} & \quad (2) \quad \frac{91}{2^{13}} & \quad (3) \quad \frac{11}{2^3} & \quad (4) \quad \frac{7}{2^7} \\
\end{align*}
\]

Ans. (2)

Sol. \(2(\sec^2 \theta - 1) - 5\sec \theta - 1 = 0\)
\(2\sec^2 \theta - 5\sec \theta - 3 = 0\)
\((2\sec \theta + 1)(\sec \theta - 3) = 0\)

\(\sec \theta = -\frac{1}{2}\) or \(\sec \theta = 3\)

Rejected the solution are in 1st and 4th quadrant only

the least value of \(n\) is 13 for which equation has 7 solutions in \(0, \frac{13\pi}{2}\)

now \(\sum_{k=1}^{n} \frac{k}{2^k} = \sum_{k=1}^{13} \frac{k}{2^k} = \frac{91}{2^{13}}\)

3. \(\lim_{x \to 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1+x)}{3\tan^2 x} = \frac{1}{3}\), then the value of \((2\alpha - \beta)\) is equal to

Ans. (1)

Sol. \(\lim_{x \to 0} \frac{3 + \alpha \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right) + \beta \left(1 - \frac{x^2}{2!}\right) + \log_e(1+x)}{3\tan^2 x} = \frac{1}{3}\)

\[\Rightarrow \lim_{x \to 0} \frac{(3 + \beta) + (\alpha + 1)x - \left(\frac{\beta + 1}{2}\right)x^2}{3\tan^2 x} = \frac{1}{3}\]

4. The position vector of the vertices A, B, C of a triangle are \(2\hat{i} + 3\hat{j} + 3\hat{k}, 2\hat{i} - 2\hat{j} + 3\hat{k}, \hat{i} - \hat{j} - \hat{k}\) respectively. Let \(r\) denotes the length of the angle bisector AD of \(\angle BAC\). Where 'D' is on the line segment BC, then \(2r^2\) equals?

Ans. (45)

Sol.

\(A(2, 3, 3)\)

\(B(2, -2, 3)\)

\(C(-1, -1, 3)\)

\[r^2 = \left(2 - \frac{1}{2}\right)^2 + \left(3 + \frac{3}{2}\right)^2 + 0\]

\(= \frac{1}{4} + \frac{9}{4} = \frac{5}{2}\)

\(2r^2 = 5\)
5. \[ \int_{1}^{1+\alpha^2 - 2\alpha \cos x} \frac{1}{a^2 - 2\alpha \cos x} \, dx \] is equal (where \(|a| > 1\))

\begin{align*}
(1) \quad & \frac{\pi}{1-a^2} \\
(2) \quad & \frac{\pi}{\alpha^2 - 1} \\
(3) \quad & \frac{\pi}{2(a^2 - 1)} \\
(4) \quad & \frac{\pi}{2(1-a^2)}
\end{align*}

Ans. (2)

Sol. \[ \text{Let } I = \int_{1}^{1+\alpha^2 - 2\alpha \cos x} \frac{1}{a^2 - 2\alpha \cos x} \, dx \]

\[ I = \frac{1}{2\alpha} \int_{1}^{1+\alpha^2 - 2\alpha \cos x} \frac{1}{a^2 - 2\alpha \cos x} \, dx \]

\[ = \frac{1}{2\alpha} \int_{1}^{1+\alpha^2 - 2\alpha \cos x} \frac{1}{\frac{a^2 - 1}{2\alpha}} \, dx \]

Let \( a = \frac{1 + \alpha^2}{2\alpha} \) then \(|a| > 1\)

\[ I = \frac{1}{2\alpha} \int_{a - \cos x}^{a + \cos x} \frac{1}{\cos x} \, dx \]

\[ = \frac{1}{2\alpha} \ln \left| \frac{a + \cos x}{a - \cos x} \right| \]

\[ = \frac{1}{2\alpha} \ln \left| \frac{a + 1}{a - 1} \right| \]

\[ = \frac{1}{2\alpha} \ln \left| \frac{\tan \frac{\pi}{2}}{\tan \frac{\pi}{2}} \right| \]

\[ = \frac{1}{2\alpha} \ln \left| \frac{\tan \frac{\pi}{2}}{\tan \frac{\pi}{2}} \right| \]

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6. Number of solution of equation \( \tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4} \) is

Ans. (2)

Sol. \( \tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4} \)

\[ \tan^{-1} 2x = \tan^{-1} 1 - \tan^{-1} x \]

\[ 2x = \frac{1}{1-x} \]

\[ 2x^2 + 3x - 1 = 0 \]

\[ x = \frac{-3 \pm \sqrt{9 + 8}}{4} \]

\[ x = \frac{-3 \pm \sqrt{9 + 8}}{4} \]
4
x = \frac{-3 \pm \sqrt{17}}{4}

Number of solution is 1.

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7. 20th term from the end of the progression 20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, 129\frac{1}{4} is ________

(1) -120
(2) -115
(3) -125
(4) -110

Ans. (2)

Sol. 20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, 129\frac{1}{4}

are in A.P. with common difference

d = 19\frac{1}{4} - 20 = -\frac{3}{4}

20\text{th term from end } = -129\frac{1}{4} + (20 - 1)\left(\frac{3}{4}\right)

= -129\frac{1}{4} + 19\frac{3}{4}

= -51\frac{1}{4}

= -115

8. A is the area of region 0 \leq y \leq \min (2x, 6x - x^2) then find 12A

(304)

Sol.

y = 2x

y = 6x - x^2

Solving 2x = 6x - x^2

x^2 - 4x = 0

y = x = 0, x = 4, y = 8

A = \frac{1}{2} \cdot 8 \cdot \frac{6}{4} \cdot \left(6x - x^2\right) dx

= 16 + \left(3x^2 - \frac{x^3}{3}\right)_{\frac{1}{2}}

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9. \(a\) and \(b\) are the roots of \(x^2 - x - 1 = 0\) and \(S_n = 2023\ a^n + 2024\ b^n\) then

\begin{align*}
(1) \quad & S_{12} = S_{11} + S_{10} \\
(2) \quad & 2S_{12} = S_{11} + S_{12} \\
(3) \quad & S_{11} = 2S_{10} + S_{12} \\
(4) \quad & S_{10} = S_{11} + S_{12}
\end{align*}

Ans. (1)

Sol.
\begin{align*}
x^2 - x - 1 &= 0 \\
S_0 &= S_{-1} = S_{-2} = 0 \\
S_n &= S_{n-1} + S_{n-2} \\
S_{12} &= S_{11} + S_{10}
\end{align*}

\(n = 12\)

10. If circle \((x-\alpha)^2 + (y-\beta)^2 = 50\) and line \(x + y = 0\) intersect at only one point \(P\) distance of \(P\) from origin is \(4\sqrt{2}\) then \(\alpha^2 + \beta^2\) is

\begin{align*}
(1) \quad & 81 \\
(2) \quad & 82 \\
(3) \quad & 85 \\
(4) \quad & 169
\end{align*}

Ans. (2)

Sol.
\[x + y = 0\] is tangent to the circle

\[(x-\alpha)^2 + (y-\beta)^2 = 50\]

\[OP = 4\sqrt{2}\]

\[PC = \sqrt{50} = 5\sqrt{2}\]

\[OC^2 = \alpha^2 + \beta^2 = (OP)^2 + (PC)^2\]

\[\alpha^2 + \beta^2 = 16 \times 2 + 50 = 82\]

11. Two finite set \(A\) and \(B\) have \(m\) and \(n\) elements respectively. If subset of \(A\) is 56 more than that of \(B\) then the distance between \((m, n)\) and \((2n-2, -3)\) is

\begin{align*}
(1) \quad & 8 \\
(2) \quad & 10 \\
(3) \quad & 11 \\
(4) \quad & 15
\end{align*}

Ans. (2)

Sol.
\[2n - 2 = 56\]

we know that \(64 - 8 = 56\)

\[m = 6, \quad n = 3\]

\[A = \{6, 3\}, \quad B = \{2, -3\}\]

\[AB = \sqrt{64 + 36} = 10\]

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13. If mean and standard deviation of 15 observation are 12 and 3 respectively. But an error is found that 10 is written in place of 12. If the correct mean is \( \mu \) and the correct variance \( \sigma^2 \) then find the value of \( 15(\mu + \mu^2 + \sigma^2) \)

**Ans. (2521)**

**Sol.** Since \( \frac{\sum x_i}{15} = 12 \Rightarrow \sum x_i = 180 \)

but when data’s are corrected the new mean is

\[
\mu = \frac{\sum x_i - 10 + 12}{15} = \frac{180 - 10 + 12}{15} = \frac{182}{15}
\]

also given SD = \( \sqrt{\frac{\sum x_i^2}{15} - (12)^2} \)

\[
\Rightarrow 9 + 144 = \frac{\sum x_i^2}{15} - (12)^2
\]

\[
\Rightarrow \frac{\sum x_i^2}{15} = 153 + 18 = 2295
\]

new variance \( \sigma^2 = \frac{\sum x_i^2 - 100 + 144}{15} - (\mu)^2 \)

\[
\sigma^2 + \mu^2 = \frac{2295 + 44}{15} = 2521
\]
\[ \tan^{-1} \left( \frac{y-1}{x-1} \right) - \frac{1}{2} \ln \left( \frac{y-1}{x-1} \right)^2 = c \]
\[ x = 0, \ y = 2 \]
\[ \frac{\pi}{4} = \frac{1}{2} \ln + \frac{c}{2n2} \]
\[ c = \frac{\pi}{4} - \frac{1}{2} \ln + \frac{c}{2n2} \]

hence at \( x = 2 \), \( \tan^{-1} \left( \frac{y-1}{x-1} \right) - \frac{1}{2} \ln \left( \frac{y-1}{x-1} \right)^2 = c = \frac{\pi}{4} - \frac{1}{2} \ln + \frac{c}{2n2} \)
\( y = 0 \)

15. \( A \) is a \( 2 \times 2 \) matrix, \( I \) is \( 2 \times 2 \) identity matrix, \( |A - xI| = 0 \) has the roots \( -1, 3 \). Then find the sum of diagonal elements of \( A^2 \).

Ans. \( 10 \)

Sol. Let \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \)
\( |A - xI| = 0 \Rightarrow \begin{bmatrix} a - x & b \\ c & d - x \end{bmatrix} = 0 \)

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\[ |a - x| (d - x) - b(c - x) = 0 \]
\[ x^2 - (a + d) x + ad - bc = 0 \]
has roots \( -1, 3 \)
\[ a + d = 2 \]
\[ ad - bc = -3 \]
\[ A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \]
\[ = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} \]
Sum of diagonal element of \( A^2 \)
\[ = a^2 + d^2 + 2bc \]
\[ = (a + d)^2 + 2(bc - ad) \]
\[ = 4 + 6 = 10 \]

16. For \( x \in (0, 3) \), \( g(x) = 3f \left( \frac{x}{3} \right) + f(3 - x) \) and \( f'(x) > 0 \) for \( x \in (0, 3) \).

If \( g(x) \) is increasing in \( (a, 3) \) and decreasing in \( (0, a) \) then find \( a' \)
\[ \begin{align*}
(1) & \\
(2) & \\
(3) & \\
(4) & \\
\end{align*} \]

Ans. \( 2 \)

Sol. Since \( f'(x) > 0 \) then \( f(x) \) is increasing in \( (0, 3) \).
Now \( g'(x) = f \left( \frac{x}{3} \right) + f(3 - x) \)
For \( g(x) \) to be increasing
\[ g'(x) = f \left( \frac{x}{3} \right) - f(3 - x) > 0 \]
\[ \Rightarrow f \left( \frac{x}{3} \right) > f(3 - x) \]
\[ \Rightarrow \frac{x}{3} > 3 - x \Rightarrow x > 9 - 3x \Rightarrow x > \frac{9}{4} \]
For \( g(x) \) to be decreasing
\[ g'(x) < 0 \Rightarrow x < \frac{9}{4} \]
\[ a = \frac{9}{4} \]

17. Let \( f(x) = \int_0^x f(t) \left( \frac{1 - t}{1 + t} \right)^{1/2} dt \). If \( g \) is odd continuous function and \( \int_0^a f(x) \left( \frac{x^2 \cos x}{1 + e^x} \right) dx = \frac{\pi^2}{12} - \alpha \), then
value of $\alpha$ is ________.

An. (1) 1  (2) 2  (3) 3  (4) 4

Sol. $f(x) = \frac{5}{x} g(t) \ln \left(\frac{1}{1+t} \right) dt$ is an odd function.

18. Let $R$ be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of $\alpha$ for which the points $(\alpha^2, \alpha + 1)$ lies in $R$ is

(1) $(-1, -\infty) \cup (3, \infty)$
(2) $(-3, 0) \cup \left(\frac{1}{3}, 1\right)$
(3) $(-\infty, -1) \cup \left(0, \frac{1}{3}\right) $
(4) $(-\infty, -2) \cup \left(0, \frac{1}{3}\right) $

An. (2)

Sol. $(\alpha^2, \alpha + 1)$ and $(0,0)$ lies on the same side of $3x - y + 1 = 0$ and $x + 2y - 5 = 0$

$\Rightarrow 3a^2 - a + 1 > 0$ and $a^2 + 2a + 2 - 5 < 0$

$a (3a - 1) > 0$ and $(a + 3)(a - 1) < 0$

$\Rightarrow (a > \frac{1}{3})$}

An. $(-3, 0) \cup \left(\frac{1}{3}, 1\right) $

19. If $1 + \frac{a}{3} \leq 0$ then $\alpha$ lies in the interval

(1) $(0, 3)$
(2) $(-3, 0)$
(3) $(-2, 1)$
(4) $(-2, 0)$

An. (2)

Sol. $C_2 \rightarrow C_2 - (c_1 + c_2)$

$\begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 0 \\ 2a + 3 & 3a + 1 & -2a^2 + 6a + 1 \end{bmatrix}$

$\Rightarrow (2\alpha - 6\alpha + 1) = 0$

$\Rightarrow \alpha = \frac{3 + \sqrt{7}}{2}, \frac{3 - \sqrt{7}}{2}$
20. If \( f(x) = 6x - x^2 \) for \( x \in [0,2] \) and \( g(x) = \begin{cases} \min(g(t)); & 0 \leq t \leq x, 0 \leq x < 1 \\ 3 + x; & x \in [1,2] \end{cases} \), then number of points where \( g(x) \) is not differentiable is:

\[ \begin{align*}
(1) & \quad 1 \\
(2) & \quad 0 \\
(3) & \quad 2 \\
(4) & \quad 3 \\
\end{align*} \]

Ans. (1)

Sol. \( f(x) = 6x - x^2 \)

\( g(x) = \begin{cases} 0; & 0 \leq x < 1 \\
3 + x; & 1 \leq x \leq 2 \end{cases} \)

\( g(x) \) is not differentiable at \( x = 1 \)

21. Three lines \( 2x - y - 3 = 0, 6x + 3y + 4 = 0, \alpha x + 2y + 4 = 0 \) does not form a triangle then find \( \sum \alpha^2 \)

where \( \lfloor \cdot \rfloor \) denotes the greatest integer function.

Ans. (32)

Sol. Triangle will not form if either at least two lines are parallel or lines are concurrent.

If two line are parallel

\[ \begin{align*}
& \frac{2}{\alpha} = \frac{-1}{2} \\
& \frac{6}{3} = \frac{4}{2} \\
& \frac{\alpha}{2} = \frac{4}{4} = 1
\end{align*} \]

If lines are concurrent

\[ \begin{align*}
& \alpha = 4 \\
& \sum \alpha^2 = 16 + 16 + 16 = 5 \times 16 = 32
\end{align*} \]

22. Let \( S_1 = \binom{4}{2} \) and \( S_2 = \frac{\binom{5}{2}}{\binom{6}{2}} \) then

\[ \begin{align*}
(1) & \quad S_1 \in \mathbb{N} \text{ and } S_2 \notin \mathbb{N} \\
(2) & \quad S_1 \notin \mathbb{N} \text{ and } S_2 \in \mathbb{N} \\
(3) & \quad S_1 \in \mathbb{N} \text{ and } S_2 \in \mathbb{N} \\
(4) & \quad S_1 \notin \mathbb{N} \text{ and } S_2 \notin \mathbb{N}
\end{align*} \]

Ans. (2)

Sol. 24 different objects into 6 persons of 4 each

Number of ways of making groups = \( \frac{24!}{(4!)^6} = I_1 \)

\( S_1 \in \mathbb{N} \)

divide 120 different objects into 24 persons of 5 each

\( \frac{5!}{(5!)^{24}} = I_2 \)

Hence \( S_2 \notin \mathbb{N} \)
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