KCET 2024 Mathematics Question Paper Code D4

1. The value of C in (0, 2) satisfying the mean value theorem for the function $f(x) = x(x-1)^2$, $x \in [0,2]$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Ans. B

Sol. $f(2) = 2, \quad f(0) = 0$

$f'(c) = \frac{2-0}{2-0} = 1$

$f'(c) = 2c(c-1) + (c-1)^2$

option verification

2. $\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ is

(A) $-\frac{3}{4}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

Ans. D

Sol. $x = 2\cos\theta$

$\frac{2+x}{2-x} = \frac{2(1+\cos\theta)}{2(1-\cos\theta)} = \cot^2\frac{\theta}{2}$

$\therefore \frac{d}{dx}\frac{1}{2}(1+\cos\theta) = \frac{1}{2}\frac{d}{dx}\left(1+\frac{x}{2}\right)$

$= \frac{1}{4}$

3. For the function $f(x) = x^3 - 6x^2 + 12x - 3;\ x = 2$ is

(A) a point of minimum

(B) a point of inflexion

(C) not a critical point

(D) a point of maximum

Ans. B

Sol. $f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12$

$f''(2) = 0$

$f'''(2) \neq 0$

4. The function $f(x) = |\cos x|$ is

(A) Everywhere continuous and differentiable

(B) Everywhere continuous but not differentiable at odd multiples of $\frac{\pi}{2}$

(C) Neither continuous nor differentiable at $(2n+1)\frac{\pi}{2}, n \in Z$

(D) Not differentiable everywhere

Ans. B

Sol. Conceptual

5. If $y = 2x^{3x}$, then $\frac{dy}{dx}$ at $x = 1$ is

(A) 2 (B) 6 (C) 3 (D) 1

Ans. B

Sol. $\log y = \log(2.x^{3x})$

$= \log 2 + 3x\log x$

$\frac{1}{y}\frac{dy}{dx} = 3\left[x.\frac{1}{x} + \log x\right]$

$\frac{dy}{dx} = 2.x^{3x}.3(1+\log x)$

$\left(\frac{dy}{dx}\right)_{x=1} = 6$

6. Let the function satisfy the equation $f(x+y) = f(x)f(y)$ for all $x, y \in R$, where $f(0) \neq 0$. If $f(5) = 3$ and $f'(0) = 2$, then $f'(5)$ is

(A) 6 (B) 0 (C) 5 (D) -6

Ans. A

Sol. $f(x) = k^x$

$f(5) = 3$

$k^5 = 3$

$\therefore f(x) = 3^{x/5}$

$f'(x) = 3^{x/5}\log_e 3.\frac{1}{5}$

$f'(0) = 2$

$\frac{\log_e 3}{5} = 2$

$\log_e 3 = 10$

$f'(x) = 2(3)^{x/5}$

$f'(5) = 2 \times 3 = 6$

7. $\int \frac{1}{x\left[6(\log x)^2+7\log x+2\right]}dx =$

(A) $\frac{1}{2}\log\left|\frac{2\log x+1}{3\log x+2}\right|+C$

(B) $\log\left|\frac{2\log x+1}{3\log x+2}\right|+C$

(C) $\log\left|\frac{3\log x+2}{2\log x+1}\right|+C$

(D) $\frac{1}{2}\log\left|\frac{3\log x+2}{2\log x+1}\right|+C$

Ans. **B**

Sol. **Put** $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$I = \int \frac{1}{(3t+2)(2t+1)}dt$

$\frac{1}{(3t+2)(2t+1)} = \frac{A}{3t+2}+\frac{B}{2t+1}$

After solving $A=-3, B=2$

$\therefore I = \int \frac{-3}{3t+2}dt + \int \frac{2}{2t+1}dt$

$= -\log|3t+2| + \log|2t+1| + C$

8. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx =$

(A) $2x+\sin x+2\sin 2x+C$

(B) $x+2\sin x+2\sin 2x+C$

(C) $x+2\sin x+\sin 2x+C$

(D) $2x+\sin x+\sin 2x+C$

Ans. **C**

Sol. $\int \frac{2\sin\left(\frac{5x}{2}\right)\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$

$= \int \frac{\sin 3x+\sin 2x}{\sin x}dx$

$= \int 3-4\sin^2 x+2\cos x dx$

$= \int 1+2\cos 2x+2\cos x\, dx$

$= x+\sin 2x+2\sin x+C$

9. $\int_1^5 \left(|x-3|+|1-x|\right)dx =$

(A) 12 (B) $\frac{5}{6}$ (C) 21 (D) 10

Ans. **A**

Sol. $\int_1^5 \left(|x-3|+|x+1|\right)dx$

$= \int_1^3 2dx + \int_3^5 2x-4dx$

$= 2(2)+\left(x^2-4x\right)_3^5$

$= 4+\left[(25-20)-(9-12)\right] = 4+8 = 12$

10. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+........+\frac{1}{5n}\right) =$

(A) $\frac{\pi}{4}$ (B) $\tan^{-1}3$ (C) $\tan^{-1}2$ (D) $\frac{\pi}{2}$

Ans. **C**

Sol. $\lim_{n\to\infty}\sum_{r=1}^{2n}\left(\frac{n}{n^2+r^2}\right)$

$= \lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2\left(1+\left(\frac{r}{n}\right)^2\right)}$

$= \int_0^2 \frac{1}{1+x^2}dx = \tan^{-1}2$

11. The area of the region bounded by the line $y=3x$ and the curve $y=x^2$ in sq. units is

(A) 10

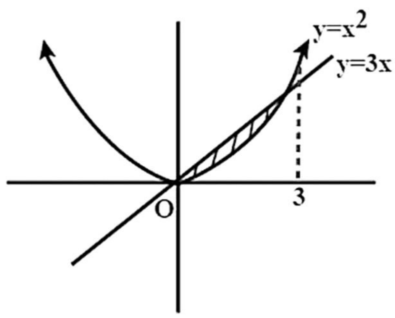
(B) $\frac{9}{2}$

(C) 9

(D) 5

Ans. **B**

Sol. $A = \int_0^3 (3x-x^2)dx = 3\left(\frac{x^2}{2}\right)-\frac{x^3}{3}\Big]_0^3 = \frac{9}{2}$

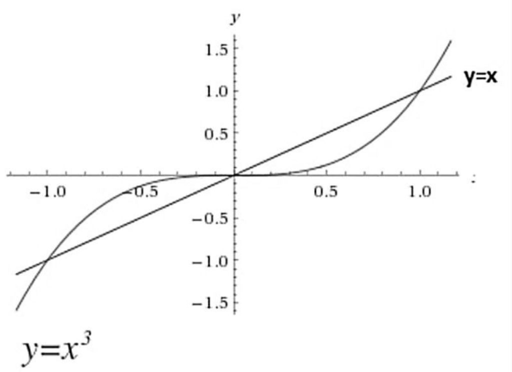


12. The area of the region bounded by the line $y = x$ and the curve $y = x^3$ is

(A) 0.2 sq. units
(B) 0.3 sq. units
(C) 0.4 sq. units
(D) 0.5 sq. units

Ans. D

Sol. $A = 2\int_0^1 (x - x^3)dx = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 0.5\,\text{sq. units}$



13. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and p, q, r are vectors defined by $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, then $(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r}$ is

(A) 0 (B) 1 (C) 2 (D) 3

Ans. D

Sol. **Since** $\vec{a}.(\vec{b} \times \vec{c}) = \left[\vec{a}\ \vec{b}\ \vec{c}\right]$,

$(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r} = 1 + 1 + 1 = 3$

14. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

(A) $-\frac{10}{7}$

(B) $-\frac{7}{10}$

(C) -10

(D) -7

Ans. A

Sol. $(-3)(3k) + (2k)(1) + 2(-5) = 0$

15. The distance between the two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

(A) 2 units (B) 8 units

(C) $\frac{2}{\sqrt{29}}$ units (D) 4 units

Ans. C

Sol. $d = \frac{|6-4|}{\sqrt{4+9+16}} = \frac{2}{\sqrt{29}}$

16. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ and the plane $2x - 2y + z = 5$ is

(A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{2}{5\sqrt{2}}$ (C) $\frac{3}{50}$ (D) $\frac{3}{\sqrt{50}}$

Ans. A

Sol. $\sin\theta = \frac{3(2) + 4(-2) + 5(1)}{\sqrt{9+16+25}\sqrt{4+4+1}}$

17. The equation $xy = 0$ in three-dimensional space represents

(A) a pair of straight lines
(B) a plane
(C) a pair of planes at right angles
(D) a pair of parallel planes

Ans. B

Sol. **Conceptual**

18. The plane containing the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is

(A) $x - y + z = 1$
(B) $x + y + z = 5$
(C) $x + 2y - z = 1$
(D) $2x - y + z = 5$

Ans. A

Sol. $\vec{n} = \begin{vmatrix} i & j & k \\ 0 & 4 & 4 \\ 1 & 5 & 4 \end{vmatrix}$

$= \hat{i}(16 - 20) - \hat{j}(0 - 4) + \hat{k}(0 - 4)$

$= -4\hat{i} + 4\hat{j} - 4\hat{k}$

$\therefore$ Eq. of plane is

$-4(x-3) + 4(y-2) - 4(z-0) = 0$

$\Rightarrow -4x + 12 + 4y - 8 - 4z = 0$

$\Rightarrow x - y + z = 1$

19. Corner points of the feasible region for an LPP are (0, 2), (3,0), (6, 0), (6, 8) and (0, 5). Let $z = 4x + 6y$ be the objective function. The minimum value of z occurs at

(A) Only (0, 2)
(B) Only (3, 0)
(C) The mid-point of the line segment joining the points (0, 2) and (3, 0)
(D) Any point on the line segment joining the points (0, 2) and (3, 0)

Ans. D

Sol. **Conceptual**

20. A die is thrown 10 times. The probability that an odd number will come up at least once is

(A) $\frac{11}{1024}$ (B) $\frac{1013}{1024}$ (C) $\frac{1023}{1024}$ (D) $\frac{1}{1024}$

Ans. C

Sol. $n = 10,\ p = \frac{1}{2};\ q = \frac{1}{2}$

$P(x \geq 1) = 1 - P(x = 0)$

$= 1 - {}^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10}$

21. A random variable X has the following probability distribution :

X	0	1	2
$P(X)$	$\frac{25}{36}$	k	$\frac{1}{36}$

If the mean of the random variable X is $\frac{1}{3}$, then the variance is

(A) $\frac{1}{18}$
(B) $\frac{5}{18}$
(C) $\frac{7}{18}$
(D) $\frac{11}{18}$

Ans. B

Sol. $\sum p_i x_i = \frac{1}{3} \Rightarrow 0 + k + \frac{2}{36} = \frac{1}{3} \Rightarrow K = \frac{1}{3} - \frac{1}{18} = \frac{5}{18}$

$$\sigma^2 + \mu^2 = \frac{\sum p_i x_i^2}{\sum p_i} = \frac{0 + 6 + \frac{4}{36}}{1} = \frac{14}{36}$$

$$\Rightarrow \sigma^2 = \frac{14}{36} - \mu^2 = \frac{14}{36} - \frac{1}{9} = \frac{5}{18}$$

22. If a random variable X follows the binomial distribution with parameters $n = 5$, p and $P(X = 2) = 9P(X = 3)$, then p is equal to

(A) 10 (B) $\frac{1}{10}$ (C) 5 (D) $\frac{1}{5}$

Ans. B

Sol. Given $n = 5$, $P(X = 2) = 9P(X = 3)$

$\Rightarrow {}^nc_2.q^{n-2}.p^2 = 9.{}^nc_3.q^{n-3}.p^3$

$\Rightarrow q = 3(n-2).p$

$\Rightarrow 1 - p = 3(3).p$

$\Rightarrow p = \frac{1}{10}$

23. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are

(A) 7, 6 (B) 5, 1 (C) 6, 3 (D) 8, 7

Ans. C

Sol. $2^m = 56 + 2^n$ then verification $m = 6, n = 3$

24. If $[x]^2 - 5[x] + 6 = 0$, where $[x]$ denotes the greatest integer function, then

(A) $x \in [3, 4]$ (B) $x \in [2, 4)$
(C) $x \in [2, 3]$ (D) $x \in (2, 3]$

Ans. B

Sol. $([x] - 2)([x] - 3) = 0$

$\Rightarrow [x] = 2 \text{ or } [x] = 3$

$\Rightarrow x \in [2, 4)$

25. If in two circles, arcs of the same length subtend angles 30^0 and 78^0 at the centre, then the ratio of their radii is

(A) $\frac{5}{13}$ (B) $\frac{13}{5}$ (C) $\frac{13}{4}$ (D) $\frac{4}{13}$

Ans. B

Sol. $l_1 = l_2, \theta_1 = 30^0, \theta_2 = 78^0$

Then $\frac{l_1}{l_2} = \frac{r_1\theta_1}{r_2\theta_2} \Rightarrow \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{78^0}{30^0} = \frac{13}{5}$

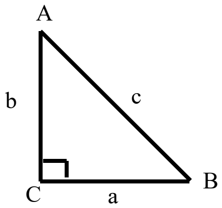
26. If ΔABC is right angled at C, then the value of $\tan A + \tan B$ is

(A) $a+b$ (B) $\frac{a^2}{bc}$

(C) $\frac{c^2}{ab}$ (D) $\frac{b^2}{ac}$

Ans. C

Sol.



Since C is a right angle then

$$\text{Tan}\,A + \text{Tan}\,B = \frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab} = \frac{c^2}{ab}$$

27. The real value of $'\alpha'$ for which $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is

(A) $(n+1)\frac{\pi}{2}, n \in \mathbb{N}$

(B) $(2n+1)\frac{\pi}{2}, n \in \mathbb{N}$

(C) $n\pi, n \in \mathbb{N}$

(D) $(2n-1)\frac{\pi}{2}, n \in \mathbb{N}$

Ans. C

Sol. $z = \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$, after simplify

$z = \frac{(1 - 2\sin^2\alpha) + i(-3\sin\alpha)}{1 + 4\sin^2\alpha}$ and z is purely real

then $\text{Im}(z) = 0$

$\Rightarrow \sin\alpha = 0$

$\Rightarrow \alpha = n\pi, n \in N$

28. The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm, then

(A) Breadth ≤ 15 cm

(B) Breadth ≥ 15 cm

(C) Length ≤ 15 cm

(D) Length = 15 cm

Ans. B

Sol. Given $l = 5b,\ P \ge 180$

$\Rightarrow 2(l + b) \ge 180$

$\Rightarrow b \ge 15$

29. The value of

${}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$ is

(A) ${}^{50}C_4$

(B) ${}^{50}C_3$

(C) ${}^{50}C_2$

(D) ${}^{50}C_1$

Ans. A

Sol. **Since** $n_{c_r} + n_{c_{r-1}} = (n+1)_{c_r}$,

${}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + \left({}^{45}C_3 + {}^{45}C_4\right)$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{46}C_4 ......$

$= {}^{50}C_4$

30. In the expansion of $(1+x)^n$

$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + n\frac{C_n}{C_{n-1}}$ is equal to

(A) $\frac{n(n+1)}{2}$

(B) $\frac{n}{2}$

(C) $\frac{n+1}{2}$

(D) $3n(n+1)$

Ans. A

Sol. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + n\frac{C_n}{C_{n-1}}$

$= n + (n-1) + (n-2) + ... + 1 = \frac{n.(n+1)}{2}$

31. If S_n stands for sum to n-terms of a G.P. with 'a' as the first term and 'r' as the common ratio then $S_n : S_{2n}$ is

(A) $r^n + 1$

(B) $\frac{1}{r^n + 1}$

(C) $r^n - 1$

(D) $\frac{1}{r^n - 1}$

Ans. B

Sol. $\frac{S_n}{S_{2n}} = \frac{\frac{a.(r^n - 1)}{r-1}}{\frac{a.(r^{2n} - 1)}{r-1}} = \frac{r^n - 1}{(r^n - 1)(r^n + 1)} = \frac{1}{(r^n + 1)}$

32. If A.M. and G.M. of roots of a quadratic equation are 5 and 4 respectively, then the quadratic equation is

(A) $x^2-10x-16=0$ (B) $x^2+10x+16=0$

(C) $x^2+10x-16=0$ (D) $x^2-10x+16=0$

Ans. D

Sol. Given A.M $=\frac{\alpha+\beta}{2}=5\Rightarrow\alpha+\beta=10$,

G.M $=\sqrt{\alpha.\beta}=4\Rightarrow\alpha\beta=16$

$\therefore$ The quadratic equation is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

33. The angle between the line $x+y=3$ and the line joining the points $(1,1)$ and $(-3,4)$ is

(A) $\tan^{-1}(7)$ (B) $\tan^{-1}\left(-\frac{1}{7}\right)$

(C) $\tan^{-1}\left(\frac{1}{7}\right)$ (D) $\tan^{-1}\left(\frac{2}{7}\right)$

Ans. C

Sol. Slope of $x+y=3$ is $m_1=-1$ and

Slope of line joining the points $(1,1),(-3,4)$ is $m_2=-\frac{3}{4}$

and $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|\Rightarrow\theta=\tan^{-1}\frac{1}{7}$

34. The equation of parabola whose focus is (6,0) and directrix is $x=-6$ is

(A) $y^2=24x$ (B) $y^2=-24x$

(C) $x^2=24y$ (D) $x^2=-24y$

Ans. A

Sol. Focus = $=F=(a,0)=(6,0)$

Equation of directrix is $x=-6$ then equation of parabola is of the form $y^2=4ax, a=6$

35. $\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1}$ is equal to

(A) 2 (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

Ans. C

Sol. By L.H. Rule

$$\underset{x\to\frac{\pi}{4}}{\text{Lt}}\frac{-\sqrt{2}\times\sin x-0}{-\text{cosec}^2x}=\frac{\sqrt{2}\times\frac{1}{\sqrt{2}}}{\left(\sqrt{2}\right)^2}=\frac{1}{2}$$

36. The negation of the statement

"For every real number x; x^2+5 is positive" is

(A) For every real number x; x^2+5 is not positive.

(B) For every real number x; x^2+5 is negative

(C) There exists at least one real number x such that x^2+5 is not positive

(D) There exists at least one real number x such that x^2+5 is positive

Ans. C

Sol. **Conceptual**

37. Let a, b, c, d and e be the observations with mean m and standard deviation S. The standard deviation of the observations $a+k, b+k, c+k, d+k$ and $e+k$ is

(A) kS (B) S + k (C) $\frac{S}{k}$ (D) S

Ans. D

Sol. adding constant each observation of S.D does not effect.

38. Let $f:R\to R$ be given by $f(x)=\tan x$. Then $f^{-1}(1)$ is

(A) $\frac{\pi}{4}$

(B) $\left\{n\pi+\frac{\pi}{4}:n\in Z\right\}$

(C) $\frac{\pi}{3}$

(D) $\left\{n\pi+\frac{\pi}{3}:n\in Z\right\}$

Ans. B

Sol. $\tan x=1\Rightarrow x=n\pi+\frac{\pi}{4}$

39. Let $f:R\to R$ be defined by $f(x)=x^2+1$. Then the pre images of 17 and -3 respectively are

(A) $\phi,\{4,-4\}$

(B) $\{3,-3\},\phi$

(C) $\{4,-4\},\phi$

(D) $\{4,-4\},\{2,-2\}$

Ans. C

Sol. $f(x)=x^2+1=17 \quad\Rightarrow x=\pm4$

$x^2+1=-3$ is not possible.

No preimage of -3

40. Let $(gof)(x) = \sin x$ and $(fog)(x) = \left(\sin\sqrt{x}\right)^2$. Then

(A) $f(x) = \sin^2 x,\ g(x) = x$

(B) $f(x) = \sin\sqrt{x},\ g(x) = \sqrt{x}$

(C) $f(x) = \sin^2 x,\ g(x) = \sqrt{x}$

(D) $f(x) = \sin\sqrt{x},\ g(x) = x^2$

Ans. C

Sol. $g(f(x)) = g\left(\sin^2 x\right) = \sqrt{\sin^2 x} = \sin x$

$fog(x) = f\left[\sqrt{x}\right] = \left(\sin\sqrt{x}\right)^2$

41. Let $A = \{2,3,4,5,.......16,17,18,\}$. Let R be the relation on the set A of ordered pairs of positive integers defined by (a, b) R (c, d) if and only if ad=bc for all (a, b), (c, d) in $A \times A$. Then the number of ordered pairs of the equivalence class of (8,2) is

(A) 4 (B) 5 (C) 6 (D) 7

Ans. C

Sol. 6 Pairs

$\{(3,2),(6,4),(9,6),(12,8),(18,12),(15,10)\}$

42. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $x(y+z)$ $+y(z+x)+z(x+y)$ equal to

(A) 0 (B) 1 (C) 6 (D) 12

Ans. C

Sol. $x = y = z = -1$

$\Rightarrow x(y+z)+y(z+x)+z(x+y) = 6$

43. If $2\ \sin^{-1} x - 3\cos^{-1} x = 4, x \in [-1,1]$ then $2\sin^{-1}$ $x + 3\cos^{-1} x$ is equal to

(A) $\frac{4-6\pi}{5}$

(B) $\frac{6\pi-4}{5}$

(C) $\frac{3\pi}{2}$

(D) 0

Ans. B

Sol. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\cos^{-1} x = \frac{\pi-4}{5}$

$\sin^{-1} x = \frac{3\pi+8}{10}$

44. If A is square matrix such that $A^2 = A$, then $(I+A)^3$ is equal to

(A) 7A-I (B) 7A (C) 7A+I (D) I-7A

Ans. C

Sol. $(I+A)^3 = I + 3A + 3A + A = 7A + I$

45. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

(A) $2^8 A$ (B) $2^9 A$ (C) $2^{10} A$ (D) $2^{11} A$

Ans. B

Sol. $A^2 = 2^{'} A, A^4 = A^3 A - A^{10} = 2^9 A$

46. If $f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$, then

$f(1).f(3) + f(3).f(5) + f(5).f(1)$ is

(A) -1 (B) 0 (C) 1 (D) 2

Ans. No option

Sol.

47. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then $\lim_{x\to 0} \frac{f(x)}{x^2} =$

(A) -1

(B) 0

(C) 3

(D) 2

Ans. B

Sol. $f(x) = -x^2 \cos x + x\sin x$

$\therefore \lim_{x\to 0} \frac{f(x)}{x^2} = 0$

48. Which one of the following observations is correct for the features of logarithm function to and base b>1?

(A) The domain of the logarithm function is R, the set of real numbers

(B) The range of the logarithm function is R^+, the set of all positive real numbers.

(C) The point (1,0) is always on the graph of the logarithm function

(D) The graph of the logarithm function is decreasing as we move from left to right.

Ans. C

Sol. $\log 1 = 0$

49. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(A) 4 (B) 5 (C) 11 (D) 0

Ans. C

Sol. $|P| = |A - A| = |A|^2 = 16 \Rightarrow 2\alpha - 6 = 16 \Rightarrow \alpha = 11$

50. If $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is

(A) 3A (B) -3B (C) 3B+1 (D) 1-3A

Ans. A

Sol. $A = x^2 - 1, \frac{dB}{dx} = 3(x^2 - 1) = 3A$

51. If $f(x) = xe^{x(1-x)}$ then $f(x)$ is

(A) increasing in $\mathbb{R}$
(B) decreasing in $\mathbb{R}$
(C) decreasing in $\left[-\frac{1}{2}, 1\right]$
(D) increasing in $\left[-\frac{1}{2}, 1\right]$

Ans. D

Sol. $f^1(x) = e^{x-x^2}(x - 2x^2)$

$\Rightarrow$ f is increasing in $\left[-\frac{1}{2}, 1\right]$

52. $\int \frac{\sin x}{3 + 4\cos^2 x} dx =$

(A) $-\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + C$

(B) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\cos x}{3}\right) + C$

(C) $\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{\cos x}{3}\right) + C$

(D) $-\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2\cos x}{3}\right) + C$

Ans. A

Sol. Put

$\cos x = t \Rightarrow \int \frac{-dt}{3 + (2t)^2} = -\frac{1}{\sqrt{3}} \times \frac{1}{2} \times \tan^{-1}\left(\frac{2t}{\sqrt{3}}\right)$

53. $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cdot \cos^2 x dx =$

(A) $\pi - \frac{\pi^2}{3}$ (B) $2\pi - \pi^3$

(C) $\pi - \frac{\pi^3}{2}$ (D) 0

Ans. D

Sol. f(x) is Odd function, then $I = 0$

54. The function $x^x; x > 0$ is strictly increasing at

(A) $\forall x \in \mathbb{R}$ (B) $x < \frac{1}{e}$

(C) $x > \frac{1}{e}$ (D) $x < 0$

Ans. C

Sol. $f^1(x) = x^x(1 + \log x) \Rightarrow f^1(x) > 0 \Rightarrow x > \frac{1}{e}$

55. The maximum volume of the right circular cone with slant height 6 units is

(A) $4\sqrt{3}\pi$ cubic units
(B) $16\sqrt{3}\pi$ cubic units
(C) $3\sqrt{3}\pi$ cubic units
(D) $6\sqrt{3}\pi$ cubic units

Ans. B

Sol. $V = \frac{1}{3}\pi r^2 h,$

$l^2 = r^2 + h^2,$

$r^2 = 36 - h^2,$

$\Rightarrow x > 16\sqrt{3}\pi$

$V_{max} = 16\sqrt{3}\pi$

56. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a ΔABC. The length of the median through A is

(A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{288}$

Ans. C

Sol. $\frac{1}{2}\left|\overline{AB} + \overline{AC}\right| = \frac{1}{2}\left|8\hat{i} - 2\hat{j} + 8\hat{k}\right| - \left|4\hat{i} - \hat{j} + 4\hat{k}\right| = \sqrt{33}$

57. The volume of the parallelopiped whose co-terminous edges are $\hat{j}+\hat{k}, \hat{i}+\hat{k}$ and $\hat{i}+\hat{j}$ is

(A) 6 cu.units (B) 2 cu.units

(C) 4 cu.units (D) 3 cu.units

Ans. B

Sol. $\left[\hat{j}+\hat{k} \quad \hat{i}+\hat{k} \quad \hat{i}+\hat{j}\right]=\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}=2$ cub units

58. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta=\frac{\pi}{4}$ (B) $\theta=\frac{\pi}{3}$

(C) $\theta=\frac{2\pi}{3}$ (D) $\theta=\frac{\pi}{2}$

Ans. C

Sol. $\left|\bar{a}+\bar{b}\right|^2=1 \Rightarrow 1+1+2\left|\bar{a}\right|\left|\bar{b}\right|\cos\theta=1$

$\Rightarrow \cos\theta=-\frac{1}{2} \quad \Rightarrow \theta=\frac{2\pi}{3}$

59. The solution of $e^{\frac{dy}{dx}}=x+1, y(0)=3$ is

(A) $y-2=x\log x-x$

(B) $y-x-3=x\log x$

(C) $y-x-3=(x+1)\log(x+1)$

(D) $y+x-3=(x+1)\log(x+1)$

Ans. D

Sol. $\frac{dy}{dx}=\log(x+1) \Rightarrow \int dy=\int \log(x+1)dx$

And $y(0)=3$ then

$\Rightarrow y+x-3=(x+1)\log(x+1)$

60. The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

(A) $xy=C$ (B) $x^2+y^2=C$

(C) $x^2-y^2=C$ (D) $\frac{y}{x}=C$

Ans. A

Sol. $\frac{x}{2x_1}+\frac{y}{2y_1}=1 \Rightarrow xy=c$