KEAM 2022 Mathematics Solution

Ques 1. Let A = {1, 2, 3, 4, 5} and let B = {1, 2, 3, 4} . If the relation R : A -> B is given by (a, b) in R if and only if a + b is even, then n(R) is equal to
(E) 6
(D) 12
(C) 20
(B) 16
(A) 10

Solu. Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4} .
For a + b to be even, both a and b must be either both even or both odd.
- Even numbers in A : {2, 4}
- Odd numbers in A : {1, 3, 5}
- Even numbers in B : {2, 4}
- Odd numbers in B : {1, 3}

Count the valid pairs:
1. Both even:
   - Pairs: 2 * 2 = 4
2. Both odd:
   - Pairs: 3 * 2 = 6

Total valid pairs n(R) = 4 + 6 = 10 .
Answer: (A) 10

Ques 6. Let f: R R be defined by f(x) = cos x. Then
(A) f is one one and odd
(B) f is odd but not one - one
(C) f is even and onto
(D) f is one one and even
(E) f is even but not onto

Solu. Let f: R -> R be defined by f(x) = cos x.

1. One-one (injective):
   - Not one-one because cos(x) is periodic (e.g., cos(0) = cos(2π) = 1).
2. Odd function:
   - Not odd because cos(-x) = cos(x).
3. Even function:
   - Even because cos(-x) = cos(x).
4. Onto (surjective):
   - Not onto because the range of cos(x) is [-1, 1], not all of R.

Answer: (E) f is even but not onto

Ques 9. The minimum value of |z+1|+|z-2| is equal to

(A) 1

(B) 2

(C) 3

(D) 4

(E) 0

Solu. To find the minimum value of |z + 1| + |z - 2|, where z is a complex number, consider z as a point on the real line, so z = x (where x is a real number).

1. For x ≤ -1:
   |x + 1| + |x - 2| = -(x + 1) + 2 - x = -2x + 1
   As x decreases, -2x + 1 increases, so this cannot give the minimum value.
2. For -1 < x ≤ 2:
   |x + 1| + |x - 2| = (x + 1) + (2 - x) = 3
   This expression is constant in this interval.
3. For x > 2:
   |x + 1| + |x - 2| = (x + 1) + (x - 2) = 2x - 1
   As x increases, 2x - 1 increases, so this cannot give the minimum value.

Thus, the minimum value is obtained when -1 < x ≤ 2, where |x + 1| + |x - 2| = 3.

Answer: (C) 3

Ques 15. If -1+7i, -1+xi and 3+3i are the three vertices of an isosceles triangle which is right angled at -1+xi, then the value of x is equal to
(A) -1
(B) 3
(C) -3
(D) 7
(E) -7

Solu. To find the value of x for the vertices -1 + 7i, -1 + xi, and 3 + 3i to form an isosceles right triangle with a right angle at -1 + xi:

1. Vertices:
   - A = -1 + 7i
   - B = -1 + xi
   - C = 3 + 3i
2. Distances:
   - AB = $|7 - x|$
   - BC = $\sqrt{(16 + (x - 3)^2)}$
   - AC = $4\sqrt{2}$
3. Right-angled isosceles triangle condition:
   - AB = BC
4. Set up the equation:
   - $|7 - x| = \sqrt{(16 + (x - 3)^2)}$
5. Solve for x:
   - $(7 - x)^2 = 16 + (x - 3)^2$
   - $49 - 14x + x^2 = 16 + x^2 - 6x$
   - $49 - 14x = 16 - 6x$
   - $49 - 16 = 14x - 6x$
   - $33 = 8x$
   - $x = 3$

Answer: (B) 3

Ques 16. The sum of the first 24 terms of the series 9+13+17+ is equal to
(A) 1212

(B) 1200
(C) 1440
(D) 1320
(E) 1230

Solu. To find the sum of the first 24 terms of the series 9 + 13 + 17 + ... :
1. First term (a) and common difference (d):
   - a = 9
   - d = 4
2. Sum of the first n terms of an arithmetic series:
   S_n = (n/2) * [2a + (n-1)d]
3. Substitute the values:
   S_24 = (24/2) * [2 * 9 + (24-1) * 4]
   S_24 = 12 * [18 + 23 * 4]
   S_24 = 12 * [18 + 92]
   S_24 = 12 * 110
   S_24 = 1320
Answer: (D) 1320

Ques 17. In an A.P. there are 18 terms and the last three terms of the A.P. are 67, 72, 77. Then the first term of the A.P. is
(A) -7
(B) 9
(C) -9
(D) -8
(E) 7

Solu. To find the first term of the arithmetic progression (A.P.) when the last three terms are given as 67, 72, and 77:
1. Identify the parameters:
   - Number of terms, n = 18
   - Last term, L = 77
   - Common difference, d = 5
2. Use the formula for the nth term of an arithmetic progression:
   L = a + (n - 1)d

3. Substitute the known values into the formula:
   77 = a + (18 - 1) * 5
   77 = a + 17 * 5
   77 = a + 85
   a = 77 - 85
   a = -8
Answer: (D) -8

Ques 18. If the first term of a G.P. is 3 and the sum of second and third terms is 60, then the common ratio of the G.P. is
(A) 4 or-3
(B) 4 only
(C) 4 or 5
(D) 4 or -5
(E) -5 only

Solu. To find the common ratio (r) of the geometric progression (G.P.) when the first term is 3 and the sum of the second and third terms is 60:
1. Identify the parameters:
   - First term, a = 3
   - Sum of the second and third terms, a2 + a3 = 60
2. Use the formula for the terms of a G.P.:
   - The second term is a2 = ar
   - The third term is a3 = ar^2
3. Write the equation for the sum of the second and third terms:
   ar + ar^2 = 60
4. Substitute the values and solve for r:
   3r + 3r^2 = 60
   3r^2 + 3r - 60 = 0
5. Solve the quadratic equation:
   r^2 + r - 20 = 0
   (r + 5)(r - 4) = 0
   From this, r = -5 or r = 4.
Answer: (D) 4 or -5

Ques 23. The number of arrangements containing all the seven letter of the word ALRIGHT that begins with LG is
(A) 720
(B) 120
(C) 600
(D) 540
(E) 760

Solu. To find the number of arrangements containing all seven letters of the word "ALRIGHT" that begin with "LG":
1. Identify the letters in the word "ALRIGHT":
   - The word "ALRIGHT" has 7 letters.
2. Count the number of arrangements:
   - Since we want arrangements that begin with "LG", there are 2 fixed positions.
   - The remaining 5 letters can be arranged in 5! ways.
3. Calculate the total number of arrangements:
   - Total arrangements = 2 * 5! = 240.

Therefore, the correct answer is not provided in the options. It should be (F) 240.

Ques 24. The number of numbers greater than 6000 that can be formed from the digits 3, 5, 6, 7 and 9 (no digit is repeated in a number) is equal to
(A) 264
(B) 720
(C) 192
(D) 132
(E) 544

Solu. To find the number of numbers greater than 6000 formed from the digits 3, 5, 6, 7, and 9 (with no repetition):
1. Consider the possibilities for each place:
   - Thousands place: Only 6 is possible.
   - Hundreds place: 4 options (3, 5, 7, 9).

- Tens and units places: 3 options each.

2. Calculate the total:
   - 1 * 4 * 3 * 2 = 24.

Therefore, the correct answer is not provided in the options. It should be (F) 24.

Ques 25. The number of subsets containing exactly 4 elements of the set
{2, 4, 6, 8, 10, 12, 14, 16, 18} is equal to
(A) 126
(B) 63
(C) 189
(D) 58
(E) 94

Solu. we need to find the number of combinations of 4 elements out of 9. This is the same as calculating 9C4.
Here's the mathematical approach:

1. Formula for Combinations: The number of combinations of n elements taken r at a time is:

nCr = n! / (r! * (n-r)!)

2. where:
   - n: Total number of elements (9 in this case)
   - r: Number of elements to choose (4 in this case)
   - ! : Factorial (n! = n * (n-1) * (n-2) * ... * 1)
3. Apply the formula:

9C4 = 9! / (4! * (9-4)!)
= 9! / (4! * 5!)
= 126

Therefore, the number of subsets containing exactly 4 elements is 126.
This solution demonstrates how to calculate combinations manually using the combination formula and canceling out common factors.

Ques 39. Consider the following statements:
(i) For every positive real number X, x-10 is positive.

(ii) Let n be a natural number. If n^2 is even, then n is even.
(iii) If a natural number is odd, then its square is also odd.
Then
(A) (i) False, (ii) True and (iii) True
(C) (i) True, (11) False and (iii) True
(E) (i) False, (ii) True and (iii) False
(B) (i) False, (ii) False and (iii) True
(D) (i) True, (ii) True and (iii) True

Solu. the answer is (A) (i) False, (ii) True and (iii) True.
Let's revisit statement (i):
(i) For every positive real number X, x-10 is positive.
This statement is False. While many positive real numbers minus 10 will result in a positive number, it's not always true. For example, if X is less than 10 (like X = 5), then X-10 will be negative.
Therefore, only statements (ii) and (iii) are true, making answer choice (A) the correct one.

Ques 42. The range of the function f(x) = 2sin(3x) + 1 is equal to

Solu. Here's how to solve for the range of the function f(x) = 2sin(3x) + 1 using equations:

1. Range of sine function: We know the sine function's range is expressed mathematically as:

$-1 \leq \sin(x) \leq 1$

2. Impact of scaling factor: The function multiplies sin(3x) by 2. This essentially stretches the sine wave vertically. Mathematically, this can be represented as:

$f(x) = 2\sin(3x) + 1 = 2 * [-1 \leq \sin(3x) \leq 1] + 1$

3. Simplifying the range: Now we can replace the range of sin(3x) with its actual values:

$f(x) = 2 * [-1, 1] + 1$

4. This becomes:

$f(x) = [-2, 2] + 1$

5. Effect of vertical shift: Finally, we add 1 to the entire function, which effectively shifts the whole range up by one unit. Mathematically:

$f(x) = [-2, 2] + 1 = [-2 + 1, 2 + 1]$

6. Resulting range: Solving for the new range, we get:

$f(x) \in [-1, 3]$

Therefore, the equations show that the range of the function $f(x) = 2\sin(3x) + 1$ is [-1, 3].