Matrices And Determinants JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Matrices And Determinants

1. If for a matrix A, |A| = 6 and adj $A = \begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & 1 \\ -1 & k & 0 \end{bmatrix}$, then k is equal to : **(+4, -1)**

[26-Jun-2022-Shift-2]

a. 0

b. 1

c. 2

d. -1

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2. If $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then X is equal to **(+4, -1)**

[27-Jun-2022-Shift-1]

a. $\begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$

b. $\begin{bmatrix} 7 & 0 \\ 1 & 5 \end{bmatrix}$

c. $\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

d. $\begin{bmatrix} 7 & 1 \\ 0 & 4 \end{bmatrix}$

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3. If A = $ \begin {bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end {bmatrix} *isamatrixsatisfyingtheequation* AA^T =9 I,*where*,I*is*3 \times 3 *identitymatrix*, *thentheorderedpair* (a, b)$ is equal to **(+4, -1)**

a. (2,-1)

b. (-2,1)

c. (2,1)

d. (-2,-1)

---

4. The rank of $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is equal to **(+4, -1)**

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5. If a point $P(\alpha, \beta, \gamma)$ satisfying $(\alpha\ \beta\ \gamma)\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0\ 0\ 0)$ lies on the plane **(+4, -1)**

$2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to :

a. $\frac{5}{4}$

b. -1

c. 11

d. $\frac{11}{5}$

---

6. Let the system of linear equations $x + y + kz = 2$ $2x + 3y - z = 1$ $3x + 4y + 2z = k$ have infinitely many solutions Then the system $(k+1)x + (2k-1)y = 7$ $(2k+1)x + (k+5)y = 10$ has: **(+4, -1)**

[Sep. 03, 2020 (I)]

a. infinitely many solutions

b. unique solution satisfying $x - y = 1$

c. no solution

d. unique solution satisfying $x + y = 1$

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7. Let A, B, C be 3×3 matrices such that A is symmetric and B and C are skew-symmetric Consider the statements (S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric (S2) $A^{26}C^{13} - C^{-13}A^{26}$ is symmetric Then, **(+4, -1)**

[24 Feb 2021 Shift 2]

a. Only $S1$ is true

b. Both S1 and S2 are false

c. Both S1 and S2 are true

d. Only S2 is true

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8. The number of square matrices of order 5 with entries from the set $\{0,1\}$, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is **[24-Jan-2023 Shift 2]** **(+4, -1)**

a. 225

b. 120

c. 125

d. 150

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9. The number of real values λ, such that the system of linear equations $2x - 3y + 5z = 9 \ x + 3y - z = -18 \ 3x - y + \left(\lambda^2 - |\lambda|\right) z = 16$ has no solution, is :- **(+4, -1)**

[25-Jul-2022-Shift-2]

a. 0

b. 1

c. 2

d. 4

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10. Let $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b \in R.$ **(+4, -1)**

If for some $n \in N, A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$

then $n + a + b$ is equal to __________.

Answers

1. **Answer: d**

Explanation:

Answer (d) -1

Concepts:

1. **Matrices:**

Matrix:

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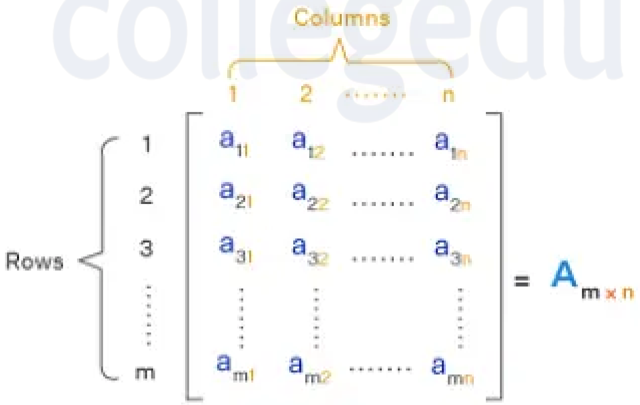

The basic operations that can be performed on matrices are:

1. **Addition of Matrices -** The addition of matrices addition can only be possible if the number of rows and columns of both the matrices are the same.
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5. **Transpose of Matrices -** Interchanging of rows and columns is known as the transpose of matrices.

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2. Answer: c

Explanation:

Given, $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$?.. (i) and

$X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$ ..(ii)

On adding both equation, we get

$2X=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$

$\Rightarrow X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$

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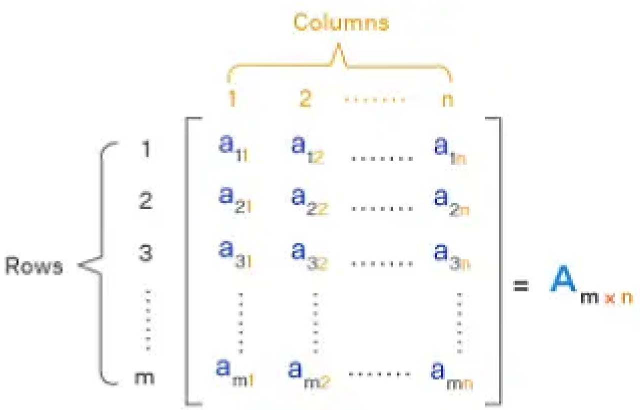

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3. Answer: d

Explanation:

$$AAT = 9I$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$a + 4 + 2b = 0$$

$$\Rightarrow a + 2b = -4 \quad ..., (i)$$

$$2a + 2 - 2b = 0$$

$$\Rightarrow a - b = -1 \quad ...(ii)$$

From i and ii

$$3b = -3$$

$$\Rightarrow b = -1$$

$$a = -2$$

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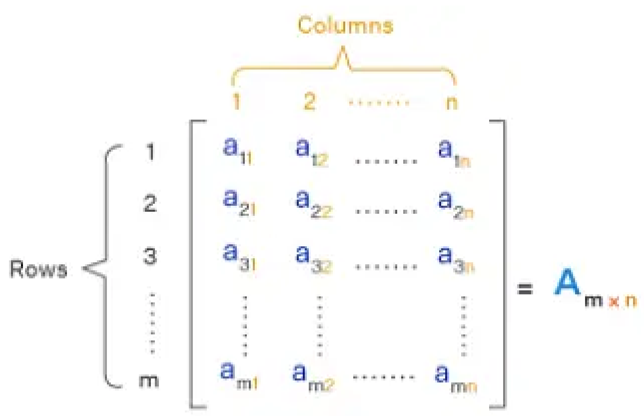

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4. Answer: 3 - 3

Explanation:

Given: A diagonal matrix $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

We have to find the rank of the given matrix. Now, we know that The rank of diagonal matrix = order of matrix = 3

Hence, the answer is 3.00.

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5. Answer: c

Explanation:

$2\alpha + 4\beta + 3\gamma = 5$.........(1)

$2\alpha + 9\beta + 8\gamma = 0$.........(2)

$10\alpha + 3\beta + 4\gamma = 0$.........(3)

$8\alpha + 8\beta + 8\gamma = 0$.........(4)

Subtract (4) from (2)

$-6\alpha + \beta = 0$

$\beta = 6\alpha$.........(5)

From equation (4)

$8\alpha + 48\alpha + 8\gamma = 0$

$\gamma = -7\alpha$.........(6)

From equation (1)

$2\alpha + 24\alpha - 21\alpha = 5$

$5\alpha = 5$

$\alpha = 1$

$\beta = +6, \quad \gamma = -7$

$\therefore 6\alpha + 9\beta + 7\gamma$

$= 6 + 54 - 49$

$= 11$

So, the correct option is (C) : 11

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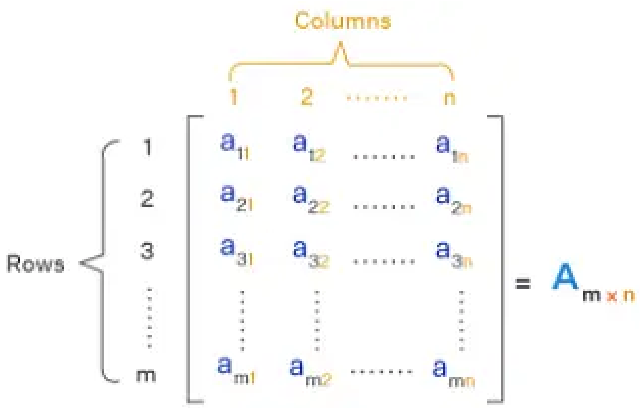

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6. Answer: d

Explanation:

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$\Rightarrow 1(10) - 1(7) + k(-1) - 0$

$\Rightarrow k = 3$

For $k = 3, 2^{md}$ system is

$4x + 5y = 7....$(1)

and $7x + 8y = 10....$(2)

Clearly, they have a unique solution

(2) $-(1) \Rightarrow 3x + 3y = 3$

$\Rightarrow x + y = 1$

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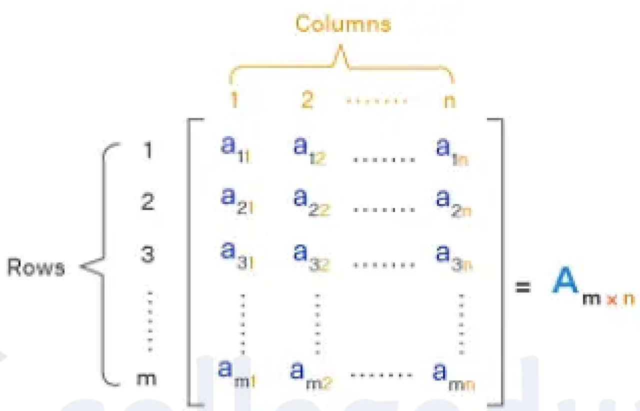

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7. Answer: d

Explanation:

The correct answer is (D) : Only S2 is true

Given, $A^T = A, B^T = -B, C^T = -C$

Let $M = A^{13}B^{26} - B^{26}A^{13}$

Then, $M^T = \left(A^{13}B^{26} - B^{26}A^{13}\right)^T$

$= \left(A^{13}B^{26}\right)^T - \left(B^{26}A^{13}\right)^T$

$= \left(B^T\right)^{26}\left(A^T\right)^{13} - \left(A^T\right)^{13}\left(B^T\right)^{26}$

$= B^{26}A^{13} - A^{13}B^{26} = -M$

Hence, M is skew symmetric

Let, $N = A^{26}C^{13} - C^{13}A^{26}$

then, $N^T = \left(A^{26}C^{13}\right)^T - \left(C^{13}A^{26}\right)^T$

$= -(C)^{13}(A)^{26} + A^{26}C^{13} = N$

Hence, N is symmetric.

$\therefore$ Only S2 is true.

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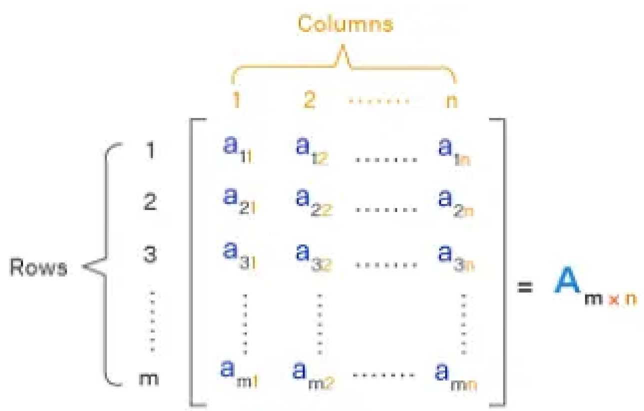

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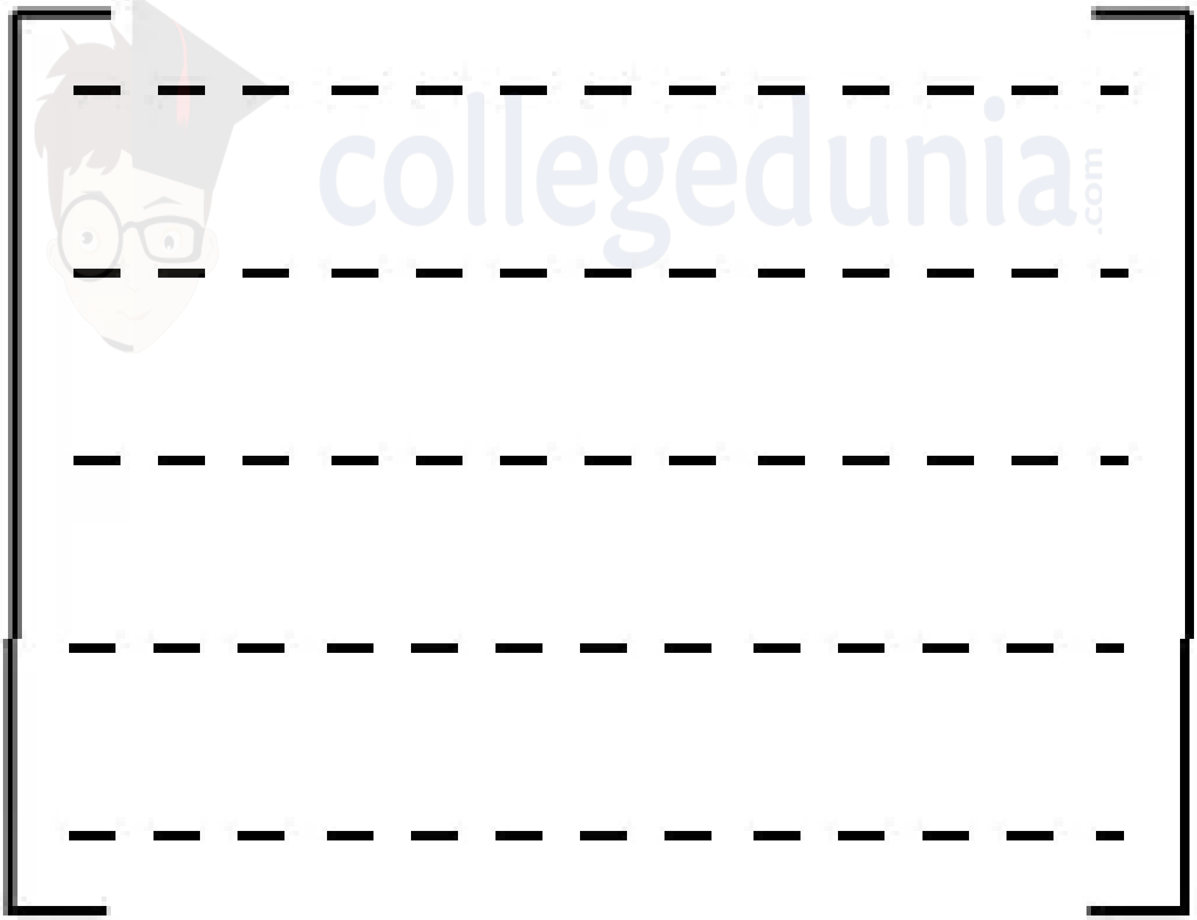
8. Answer: b

Explanation:

The correct answer is (B) : 120



In each row and each column exactly one is to be placed -

$\therefore$ No. of such materials $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Step-1 : Select any 1 place for 1 's in row 1. Automatically some column will get filled

with 0 's.
Step-2 : From next now select 1 place for 1's. Automatically some column will get filled with 0 's. $\Rightarrow$ Each time one less place will be available for putting 1's.
Repeat step-2 till last row.
Req. ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

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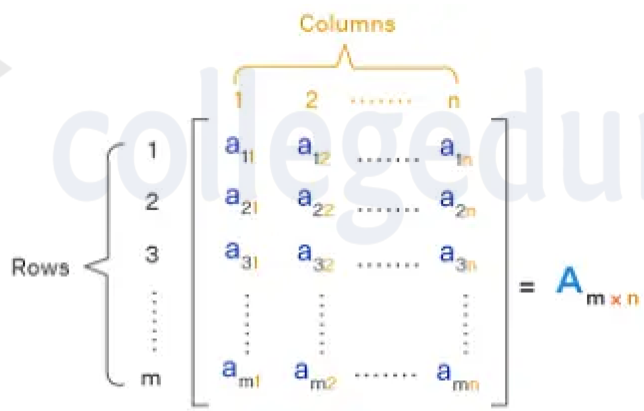

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9. **Answer: a**

Explanation:

$$2x - 3y + 5z = 9$$
$$x + 3y - z = -18$$
$$3x - y + \left(\lambda^2 - |\lambda|\right) z = 16$$
$$D = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - |\lambda \end{vmatrix} = 0$$
$$\Rightarrow 3\lambda^2 - 3|\lambda| - 11 = 0$$

Clearly one negative and one positive root since $|\lambda|$ is there so negative not possible and two values of λ corresponding to positive value

$D_3 = \begin{vmatrix} 2 & -3 & 9 \\ 1 & 3 & -18 \\ 3 & -1 & 16 \end{vmatrix} \neq 0$ so no solution.

Concepts:

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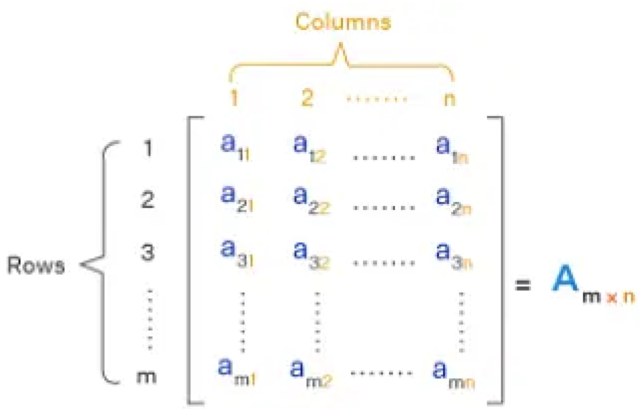

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10. Answer: 24 - 24

Explanation:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = I + B$$

$$B^2 = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$B^3 = 0$

$\therefore A^n = (1 + B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + {}^nC_3 B^3 + \ldots$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & na & na \\ 0 & 0 & nb \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{n(n-1)ab}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)}{2}ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 48 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing we get $na = 48$, $nb = 96$ and

$na + \frac{n(n-1)}{2}ab = 2160$

$\Rightarrow a = 4$, $n = 12$ and $b = 8$

$n + a + b = 24$

Concepts:

1. Matrix Transformation:

The numbers or functions that are kept in a matrix are termed the elements or the entries of the matrix.

Transpose Matrix:

The matrix acquired by interchanging the rows and columns of the parent matrix is termed the Transpose matrix. The definition of a transpose matrix goes as follows - **"A Matrix which is devised by turning all the rows of a given matrix into columns and vice-versa."**