Matrices And Determinants JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Matrices And Determinants

1. If P is a 3×3 real matrix such that $P^T = aP + (a-1)I$, where $a >$, then **(+4, -1)**

[30-Jan-2023 Shift 2]

a. $|\text{Adj } P| > 1$

b. $|AdjP| = \frac{1}{2}$

c. P is a singular matrix

d. $|AdjP| = 1$

---

2. Let $M = [a_{ij}]_{2*2}$, $0 \leq i, j \leq 2$, where $[a_{ij}] \in \{0,1,2\}$ and A be the event such that M is invertible then P(A) is? **(+4, -1)**

[25-Jul-2022-Shift-2

a. $\frac{49}{81}$

b. $\frac{16}{27}$

c. $\frac{47}{81}$

d. $\frac{46}{81}$

---

3. If the order of matrix A is 3×3 and $|A| = 2$, then the value of $|3adj(|3A|A2)|$ is? **(+4, -1)**

[28-Jun-2022-Shift-2]

a. $3^{10}.2^{21}$

b. $2^{10}.3^{21}$

c. $2^{12}.3^{15}$

d. $3^{12}.2^{15}$

---

4. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$ Then the sum of the diagonal elements of the matrix $(A+I)^{11}$ is equal to : **(+4, -1)**

[27-Jan-2024 Shift 2]

a. 2050

b. 4094

c. 6144

d. 4097

---

5. If A and B are two non-zero $n \times n$ matrics such that $A^2 + B = A^2B$, then **(+4, -1)**

[24-Jan-2023 Shift 1]

a. $A^2 = I$ or $B = I$

b. $A^2B = BA^2$

c. $AB = I$

d. $A^2B = I$

---

6. Let $A = [a_{ij}], a_{ij} \in Z \cap [0, 4], 1 \leq i, j \leq 2$. **(+4, -1)**
The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is _____.

[31-Jan-2023 Shift 2]

---

7. Let $\lambda \in R$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x$ is an integer solution of $E\}$ is **(+4, -1)**

---

8. If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then : **(+4, -1)**

a. $A^{30} - A^{25} = 2I$

b. $A^{30} = A^{25}$

c. $A^{30} + A^{25} - A = I$

d. $A^{30} + A^{25} + A = I$

---

9. Let $A = [a_{ij}]_{2\times2}$ be a matrix and $A^2 = I$ where $a_{ij} \neq 0$. If a sum of diagonal elements and b=det(A), then $3a^2 + 4b^2$ is **(+4, -1)**

[6-Apr-2023 shift 2]

a. 10

b. 12

c. 4

d. 8

---

10. If the system of linear equations : $x+3y+7z=0$ $-x+4y+7z=0$ $(\sin 3\theta)x+(\cos 2\theta)y+2z=0$ has a non-trivial solution, then the number of values of 6 lying in the interval $[0,\pi]$, is : **(+4, -1)**

[26-Jun-2022-Shift-1]

a. two

b. three

c. more than three

d. one

Answers

1. **Answer: d**

Explanation:

$$P^T = aP + (a-1)I$$
$$\Rightarrow P = aP^T + (a-1)I$$
$$\Rightarrow P^T - P = a\left(P - P^T\right)$$
$$\Rightarrow P = P^T, \text{ as } a \neq -1$$
$$\text{Now, } P = aP + (a-1)I$$
$$\Rightarrow P = -I \Rightarrow |P| = 1$$
$$\Rightarrow |\text{Adj}P| = 1$$

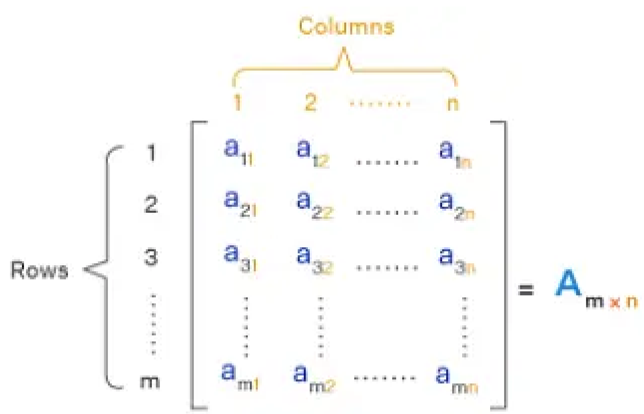
So, the correct option is (D) : $|AdjP| = 1$

Concepts:

1. **Matrices:**

Matrix:

A matrix is a rectangular array of numbers, variables, symbols, or expressions that are defined for the operations like subtraction, addition, and multiplications. The size of a matrix is determined by the number of rows and columns in the matrix.



The basic operations that can be performed on matrices are:

1. **Addition of Matrices -** The addition of matrices addition can only be possible if the number of rows and columns of both the matrices are the same.
2. **Subtraction of Matrices -** Matrices subtraction is also possible only if the number of rows and columns of both the matrices are the same.
3. **Scalar Multiplication -** The product of a matrix A with any number 'c' is obtained by multiplying every entry of the matrix A by c, is called scalar multiplication.
4. **Multiplication of Matrices -** Matrices multiplication is defined only if the number of columns in the first matrix and rows in the second matrix are equal.
5. **Transpose of Matrices -** Interchanging of rows and columns is known as the transpose of matrices.

---

2. Answer: b

Explanation:

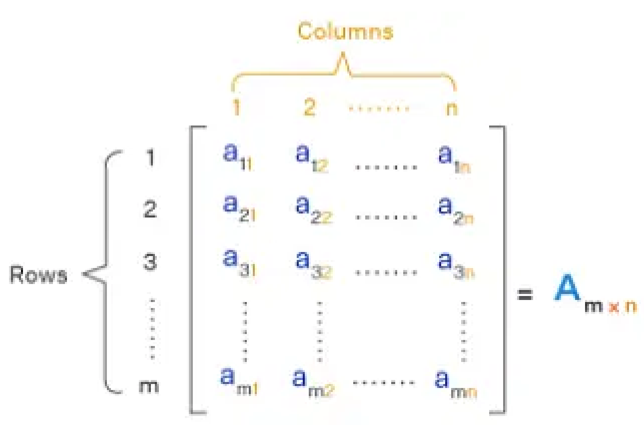
The correct option is (B): $\frac{16}{27}$

Concepts:

1. Matrices:

Matrix:

A matrix is a rectangular array of numbers, variables, symbols, or expressions that are defined for the operations like subtraction, addition, and multiplications. The size of a matrix is determined by the number of rows and columns in the matrix.



The basic operations that can be performed on matrices are:

1. **Addition of Matrices -** The addition of matrices addition can only be possible if the number of rows and columns of both the matrices are the same.
2. **Subtraction of Matrices -** Matrices subtraction is also possible only if the number of rows and columns of both the matrices are the same.
3. **Scalar Multiplication -** The product of a matrix A with any number 'c' is obtained by multiplying every entry of the matrix A by c, is called scalar multiplication.
4. **Multiplication of Matrices -** Matrices multiplication is defined only if the number of columns in the first matrix and rows in the second matrix are equal.
5. **Transpose of Matrices -** Interchanging of rows and columns is known as the transpose of matrices.

---

3. **Answer: b**

Explanation:

The correct option is (B): $2^{10}.3^{21}$

$$|3A| = 3^3 \cdot |A| = 2 \cdot 3^3$$

$$\text{adj}\,(|3A|A^2) = \text{adj}\,(2 \cdot 3^3 \cdot A^2)$$
$$= (2 \cdot 3^3)^2\,(\text{adj}A)^2$$
$$= 2^2 \cdot 3^6(\text{adj}A)^2$$

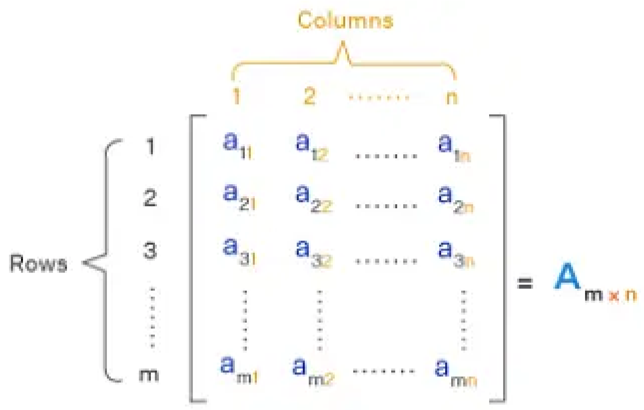
$$|3\text{adj}\,(|3A|A^2)| = |2^2 \cdot 3^7\,(\text{adj}A)^2|$$
$$= (2^2 \cdot 3^7)^3 \cdot |\text{adj}A|^2$$
$$= 2^6 \cdot 3^{21} \cdot (|A|^2)^2$$
$$= 2^6 \cdot 3^{21} \cdot 2^4 = 2^{10} \cdot 3^{21}$$

Concepts:

1. **Matrices:**

Matrix:

A matrix is a rectangular array of numbers, variables, symbols, or expressions that are defined for the operations like subtraction, addition, and multiplications. The size of a matrix is determined by the number of rows and columns in the matrix.



The basic operations that can be performed on matrices are:

1. **Addition of Matrices -** The addition of matrices addition can only be possible if the number of rows and columns of both the matrices are the same.
2. **Subtraction of Matrices -** Matrices subtraction is also possible only if the number of rows and columns of both the matrices are the same.
3. **Scalar Multiplication -** The product of a matrix A with any number 'c' is obtained by multiplying every entry of the matrix A by c, is called scalar multiplication.
4. **Multiplication of Matrices -** Matrices multiplication is defined only if the number of columns in the first matrix and rows in the second matrix are equal.
5. **Transpose of Matrices -** Interchanging of rows and columns is known as the transpose of matrices.

---

4. Answer: d

Explanation:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$$\Rightarrow A^3 = A^4 = \ldots\ldots = A$$

$$(A+I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \ldots . {}^{11}C_{10} A + {}^{11}C_{11} I$$

$$= \left({}^{11}C_0 + {}^{11}C_1 + \ldots . {}^{11}C_{10}\right) A + I$$

$$= \left(2^{11} - 1\right) A + I = 2047A + I$$

$\therefore$ Sum of diagonal elements $= 2047(1 + 4 - 3) + 3$

$= 4094 + 3 = 4097$

Concepts:

1. Matrix Transformation:

The numbers or functions that are kept in a matrix are termed the elements or the entries of the matrix.

Transpose Matrix:

The matrix acquired by interchanging the rows and columns of the parent matrix is termed the Transpose matrix. The definition of a transpose matrix goes as follows - **"A Matrix which is devised by turning all the rows of a given matrix into columns and vice-versa."**

---

5. **Answer: b**

Explanation:

$A^2 + B = A^2B$

$(A^2 - I)(B - I) = I$.......(1)

$A^2 + B = A^2B$

$A^2(B - I) = B$

$A^2 = B(B - I)^{-1}$

$A^2 = B(A^2 - I)$

$A^2 = BAA^2 - B$

$A^2 + B = BA^2$

$A^2B = BA^2$

Concepts:

1. Determinants:

Definition of Determinant

A determinant can be defined in many ways for a square matrix.

The first and most simple way is to formulate the determinant by taking into account the top-row elements and the corresponding minors. Take the first element of the top row and multiply it by its minor, then subtract the product of the second element and its minor. Continue to alternately add and subtract the product of each element of the top row with its respective min or until all the elements of the top row have been considered.

For example let us consider a 1×1 matrix A.

$A=[a_1.......a_n]$

Read More: Properties of Determinants

Second Method to find the determinant:

The second way to define a determinant is to express in terms of the columns of the matrix by expressing an n x n matrix in terms of the column vectors.

Consider the column vectors of matrix A as $A = [\ a_1, a_2, a_3, ...a_n]$ where any element a_j is a vector of size x.

Then the determinant of matrix A is defined such that

$\text{Det}\ [\ a_1 + a_2 \ .... \ ba_j+cv \ ... \ a_x\ ] = b\ \det\ (A) + c\ \det\ [\ a_1+ a_2 + ...\ v\ ...\ a_x\ ]$

$\text{Det}\ [\ a_1 + a_2 \ ....\ a_j\ a_{j+1}...\ a_x\ ] = -\det\ [\ a_1+ a_2 + ...\ a_{j+1}\ a_j\ ...\ a_x\ ]$

$\text{Det}\ (I) = 1$

Where the scalars are denoted by b and c, a vector of size x is denoted by v, and the identity matrix of size x is denoted by I.

Read More: Minors and Cofactors

We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

---

6. **Answer: 204 - 204**

Explanation:

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $a_{11}, a_{12}, a_{21}, a_{22} \in \{0, 1, 2, 3, 4\}$

Now, $a_{11} + a_{12} + a_{23} + a_{22} = \text{Prime no.} = 3,5,7,11$

$\Rightarrow (x^0 + x^1 + x^2 + x^3 + x^4)^4$

$\Rightarrow \left(\frac{1-x^5}{1-x}\right)^4$

$\Rightarrow (1 - x^5).(1 - x)^{-4}$

$\Rightarrow (4C_{r_1}.(-x^5)^{r_1}).({}^{4+r_2-1}C_{r_2}.x^{r_2})$

$\Rightarrow 4C_{r_1}.{}^{3+r_2}C_{r_2}.(-1)^{r_1}.(x^{5r_1+r_2})$

$\therefore 5r_1 + r_2 = 3, 5, 7, 11$

When, $5r_1 + r_2 = 3$

$\Rightarrow r_1 = 0,\ r_2 = 3$

$5r_1 + r_2 = 5$

$\Rightarrow r_1 = 0,\ r_2 = 5$ Or $r_1 = 1,\ r_2 = 0$

$5r_1 + r_2 = 7$

$\Rightarrow r_1 = 1,\ r_2 = 2$ Or $r_1 = 0,\ r_2 = 7$

$5r_1 + r_2 = 11$

$\Rightarrow r_1 = 0,\ r_2 = 11$ Or $r_1 = 1,\ r_2 = 6$ Or $r_1 = 2,\ r_2 = 1$

$\text{Sum of all coefficients} =$
${}^4C_0 \times^6 C_3 \times^4 C_0 \times^8 C_5 \times^4 C_1 \times^3 C_0 + {}^4C_0 \times^{10} C_7 -^4 C_1 \times^5 C_2 +^4 C_0 \times^{14} C_{11} -^4 C_1 \times^9 C_6 +^4 C_2 \times^4 C_1$

$= \frac{4!}{4!} \times \frac{6!}{3!\times3!} + \frac{4!}{4!} \times \frac{8!}{5!\times3!} - \frac{4!}{3!} \times \frac{3!}{3!} + \frac{4!}{4!} \times \frac{10!}{7!\times3!} - \frac{4!}{3!} \times \frac{5!}{3!\times2!} + \frac{4!}{4!} \times \frac{14!}{11!\times3!} - \frac{4!}{3!} \times \frac{9!}{6!\times3!} + \frac{4!}{2!\times2!} \times \frac{4!}{3!}$

$= 20 + 56 - 4 + \left(\frac{10\times9\times8}{3\times21}\right) - 4 \times 10 + \frac{14\times13\times12}{6} - \frac{4\times9\times8\times7}{3\times2} + 24$

$= 204$

Concepts:

1. Determinants:

Definition of Determinant

A determinant can be defined in many ways for a square matrix.

The first and most simple way is to formulate the determinant by taking into account the top-row elements and the corresponding minors. Take the first element of the top row and multiply it by its minor, then subtract the product of the second element and its minor. Continue to alternately add and subtract the product of each element of the top row with its respective min or until all the elements of the top row have been considered.

For example let us consider a 1×1 matrix A.

$A=[a_1 \ldots\ldots a_n]$

Read More: Properties of Determinants

Second Method to find the determinant:

The second way to define a determinant is to express in terms of the columns of the matrix by expressing an n x n matrix in terms of the column vectors.

Consider the column vectors of matrix A as A = [a_1, a_2, a_3, ...a_n] where any element a_j is a vector of size x.

Then the determinant of matrix A is defined such that

Det [$a_1 + a_2 \ldots . ba_j+cv \ldots a_x$] = b det (A) + c det [$a_1+ a_2 + \ldots v \ldots a_x$]

Det [$a_1 + a_2 \ldots . a_j \, a_{j+1} \ldots a_x$] = – det [$a_1+ a_2 + \ldots a_{j+1} \, a_j \ldots a_x$]

Det (I) = 1

Where the scalars are denoted by b and c, a vector of size x is denoted by v, and the identity matrix of size x is denoted by I.

Read More: Minors and Cofactors

We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

---

7. **Answer: 5 - 5**

Explanation:

The correct answer is 5.

$|x|^2-2|x|+|\lambda-3|=0$

$|x|^2-2|x|+|\lambda-3|-1=0$

$(|x|-1)^2+|\lambda-3|=1$

At $\lambda=3, x=0$ and 2 ,

at $\lambda=4$ or 2 ,

then $x=1$ or -1

So maximum value of $x+\lambda=5$

Concepts:

1. **Determinants:**

Definition of Determinant

A determinant can be defined in many ways for a square matrix.

The first and most simple way is to formulate the determinant by taking into account the top-row elements and the corresponding minors. Take the first element of the top row and multiply it by its minor, then subtract the product of the second element and its minor. Continue to alternately add and subtract the product of each element of the top row with its respective min or until all the elements of the top row have been considered.

For example let us consider a 1×1 matrix A.

A=[a_1.......a_n]

Read More: Properties of Determinants

Second Method to find the determinant:

The second way to define a determinant is to express in terms of the columns of the matrix by expressing an n x n matrix in terms of the column vectors.

Consider the column vectors of matrix A as A = [a_1, a_2, a_3, ...a_n] where any element a_j is a vector of size x.

Then the determinant of matrix A is defined such that

Det [a_1 + a_2 .... ba_j+cv ... a_x] = b det (A) + c det [a_1+ a_2 + ... v ... a_x]

Det [a_1 + a_2 .... a_j a_{j+1}... a_x] = − det [a_1+ a_2 + ... a_{j+1} a_j ... a_x]

Det (I) = 1

Where the scalars are denoted by b and c, a vector of size x is denoted by v, and the identity matrix of size x is denoted by I.

Read More: Minors and Cofactors

We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

---

8. Answer: c

Explanation:

$$A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$$

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ Here $\alpha = \frac{\pi}{3}$

$$A^2 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$$

$$A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{25} = \begin{bmatrix} \cos 25\alpha & \sin 25\alpha \\ -\sin 25\alpha & \cos 25\alpha \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$A^{25} = A$

$A^{25} - A = 0$

Concepts:

1. Matrix Transformation:

The numbers or functions that are kept in a matrix are termed the elements or the entries of the matrix.

Transpose Matrix:

The matrix acquired by interchanging the rows and columns of the parent matrix is termed the Transpose matrix. The definition of a transpose matrix goes as follows - **"A Matrix which is devised by turning all the rows of a given matrix into columns and vice-versa."**

---

9. **Answer: c**

Explanation:

The correct answer is (C) : 4

Let A $= \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$A^2 = \begin{bmatrix} p^2 + qr & pq + qs \\ pr + rs & qs + s^2 \end{bmatrix}$

⇒ p^2 +qr=1 (1) pq + qs = 0

⇒ q(p+s) = 0 (3)

⇒ s^2 + qr =1 (2) pr + rs = 0

⇒ r(p+s) = 0 (4)

From , eqn (1) - eqn (2)

$p^2 = s^2$ ⇒ p+s=0

Now $3a^2 + 4b^2$

$= 3(p+s)^2 + 4(ps-qr)$

$= 3.0 + 4(-p^2-qr)^2$

$= 4(p^2 + qr)^2$
$= 4$

Concepts:

1. Matrix Transformation:

The numbers or functions that are kept in a matrix are termed the elements or the entries of the **matrix**.

Transpose Matrix:

The matrix acquired by interchanging the rows and columns of the parent matrix is termed the **Transpose matrix**. The definition of a transpose matrix goes as follows - **"A Matrix which is devised by turning all the rows of a given matrix into columns and vice-versa."**

---

10. **Answer: d**

Explanation:

Answer (d) one

Concepts:

1. Applications of Determinants:

What is known as Determinants?

The Determinant of a square Matrix is a value ascertained by the elements of a Matrix. In the 2 × 2 Matrix.

The Determinants are calculated by

Det(a b)

The larger Matrices have more complex formulas.

Determinants have different applications throughout Mathematics. For example, they are used in shoelace formulas for calculating the area which is beneficial as a collinearity condition as three collinear points define a triangle that is equal to 0. The Determinant is also used in multiple variable calculi and in computing the cross product of vectors.

Read More: Determinant Formula

Second Method to find the determinant:

The second way to define a determinant is to express in terms of the columns of the matrix by expressing an n x n matrix in terms of the column vectors.

Consider the column vectors of matrix A as A = [a_1, a_2, a_3, ...a_n] where any element a_j is a vector of size x.

Then the determinant of matrix A is defined such that

Det [a_1 + a_2 .... ba_j+cv ... a_x] = b det (A) + c det [a_1+ a_2 + ... v ... a_x]

Det [a_1 + a_2 .... a_j a_{j+1}... a_x] = − det [a_1+ a_2 + ... a_{j+1} a_j ... a_x]

Det (I) = 1

Where the scalars are denoted by b and c, a vector of size x is denoted by v, and the identity matrix of size x is denoted by I.

We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

Properties of Determinant:

- If I_n is the identity matrix of the order nxn, then det(I) = 1
- If the matrix M^T is the transpose of matrix M, then det (M^T) = det (M)
- If matrix M^{-1} is the inverse of matrix M, then det (M^{-1})
- If two square matrices M and N have the same size, then det (MN) = det (M) det (N)
- If matrix M has a size axa and C is a constant, then det (CM) = C^a det (M)

- If X, Y, and Z are three positive semidefinite matrices of equal size, then the following holds true along with the corollary det $(X+Y) \geq \det(X) + \det(Y)$ for X,Y, $Z \geq 0$ det $(X+Y+Z) + \det C \geq \det(X+Y) + \det(Y+Z)$
- In a triangular matrix, the determinant is equal to the product of the diagonal elements.
- The determinant of a matrix is zero if all the elements of the matrix are zero.

Read More: Properties of Determinants