Matrices And Determinants JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Matrices And Determinants

1. The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is : **(+4, -1)**

[25 Jul 2021 Shift 2]

a. 4

b. 3

c. 2

d. 1

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2. For the system of linear equations $\alpha x + y + z = 1, x + \alpha y + z = 1, x + y + \alpha z = \beta$, which one of the following statements is NOT correct ? **(+4, -1)**

[29-Jul-2022-Shift-1]

a. It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$

b. $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$

c. It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

d. It has no solution if $\alpha = -2$ and $\beta = 1$

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3. The system of the equations **(+4, -1)**

$x + y + z = 6$

$x + 2y + \alpha z = 5$

$x + 2y + 6z = \beta$ has

[29-Jul-2022-Shift-2]

a. Infinitely many solution for $\alpha = 6, \beta = 3$

b. Infinitely many solution for $\alpha = 6, \beta = 5$

c. Unique solution for $\alpha = 6, \beta = 5$

d. No solution for $\alpha = 6, \beta = 5$

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4. If $A = \begin{bmatrix} e^t & e^t \cos t & e^{-t} \sin t \\ e^t & -e^t \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\ e^t & 2e^{-t} \sin t & -2e^{-t} \cos t \end{bmatrix}$ Then A is - **(+4, -1)**

[Jan. 09, 2019 (II)]

a. Invertible only if $t = \frac{\pi}{2}$

b. not invertible for any $t \epsilon R$

c. invertible for all $t \epsilon R$

d. invertible only if $t = \pi$

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5. The set of all values of λ for which the system of linear equations **(+4, -1)**

$2x_1 - 2x_2 + x_3 = \lambda x_1$ **[8-Apr-2023 shift 2]**

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$ has a non-trivial solution

a. is an empty set

b. is a singleton

c. contains two elements

d. contains more than two elements

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6. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = AA^T , then $5a + b$ is equal to : **(+4, -1)**

[Jan. 09, 2019 (II)]

a. -1

b. 5

c. 4

d. 13

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7. The system of linear equations $x + \lambda y - z = 0$ $\lambda x - y - z = 0$ $x + y - \lambda z = 0$ has a non-trivial solution for : **(+4, -1)**

[Jan 10, 2019 (I)]

a. infinitely many values of λ

b. exactly one value of λ

c. exactly two values of λ

d. exactly three values of λ

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8. A value of $\theta \in (0, \pi/3)$ for which $\begin{vmatrix} 1+cos^2\theta & sin^2\theta & 4cos6\theta \\ cos^2\theta & 1+sin^2\theta & 4cos6\theta \\ cos^2\theta & sin^2\theta & 1+4cos6\theta \end{vmatrix}$ =0 , is : **(+4, -1)**

[Jan. 12, 2019 (I)]

a. $\frac{7\pi}{24}$

b. $\frac{\pi}{18}$

c. $\frac{\pi}{9}$

d. $\frac{7\pi}{36}$

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9. Let $A = [a_{ij}], a_{ij} \in Z \cap [0,4], 1 \le i, j \le 2$ The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is _____ **(+4, -1)**

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10. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space $[0, 60]$ is less than or equal to a If $P(A) = \frac{11}{36}$, then a is equal to _____ **(+4, -1)**

[27 Aug 2021 Shift 2]

Answers

1. Answer: c

Explanation:

$$\begin{vmatrix} cos\,x & sin\,x & sin\,x \\ sin\,x & cos\,x & sin\,x \\ sin\,x & sin\,x & cos\,x \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} cos\,x - sin\,x & sin\,x - cos\,x & 0 \\ 0 & cos\,x - sin\,x & sin\,x - cos\,x \\ sin\,x & sin\,x & cos\,x \end{vmatrix} = 0$$

$C_2 \to C_2 + C_1$

$$(cos\,x - sin\,x)\begin{vmatrix} 1 & 0 & 0 \\ 0 & cos\,x - sin\,x & sin\,x - cos\,x \\ sin\,x & 2\,sin\,x & cos\,x \end{vmatrix} = 0$$

Expanding using first row

$(2\,sin\,x + cos\,x)\,(sin\,x - cos\,x)^2 = 0$

$tan\,x = \cdot\frac{1}{2}$ or $tan\,x = 1$

Hence two solutions are there in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

Concepts:

1. Applications of Determinants:

What is known as Determinants?

The Determinant of a square Matrix is a value ascertained by the elements of a Matrix. In the 2 × 2 Matrix.

The Determinants are calculated by

Det(a b)

The larger Matrices have more complex formulas.

Determinants have different applications throughout Mathematics. For example, they are used in shoelace formulas for calculating the area which is beneficial as a collinearity condition as three collinear points define a triangle that is equal to 0. The

Determinant is also used in multiple variable calculi and in computing the cross product of vectors.

Read More: Determinant Formula

Second Method to find the determinant:

The second way to define a determinant is to express in terms of the columns of the matrix by expressing an n x n matrix in terms of the column vectors.

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Det [$a_1 + a_2$ .... ba_j+cv ... a_x] = b det (A) + c det [a_1+ a_2 + ... v ... a_x]

Det [$a_1 + a_2$ .... a_j a_{j+1}... a_x] = − det [a_1+ a_2 + ... a_{j+1} a_j ... a_x]

Det (I) = 1

Where the scalars are denoted by b and c, a vector of size x is denoted by v, and the identity matrix of size x is denoted by I.

We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

Properties of Determinant:

- If I_n is the identity matrix of the order nxn, then det(I) = 1
- If the matrix M^T is the transpose of matrix M, then det (M^T) = det (M)
- If matrix M^{-1} is the inverse of matrix M, then det (M^{-1})
- If two square matrices M and N have the same size, then det (MN) = det (M) det (N)
- If matrix M has a size axa and C is a constant, then det (CM) = C^a det (M)
- If X, Y, and Z are three positive semidefinite matrices of equal size, then the following holds true along with the corollary det (X+Y) ≥ det(X) + det (Y) for X,Y, Z ≥ 0 det (X+Y+Z) + det C ≥ det (X+Y) + det (Y+Z)

- In a triangular matrix, the determinant is equal to the product of the diagonal elements.
- The determinant of a matrix is zero if all the elements of the matrix are zero.

Read More: Properties of Determinants

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2. **Answer: a**

Explanation:

$||\alpha 1 1 1 \alpha 1 1 1 \alpha|| = 0$
$\alpha(\alpha^2-1)-1(\alpha-1)+1(1-\alpha)=0$
$\alpha^3-3\alpha+2=0$
$\alpha^2(\alpha-1)+\alpha(\alpha-1)-2(\alpha-1)=0$
$(\alpha-1)(\alpha^2+\alpha-2)=0$
$\alpha=1, \alpha=-2,1$
For $\alpha=1, \beta=1$

$$\left.\begin{aligned} x + y + z &= 1 \\ x + y + z &= b \end{aligned}\right\} \text{infinite solution}$$

$$\text{For } \alpha = 2, \beta = 1$$

$$\Delta = 4$$

$\Delta 1 = ||111121112|| = 3-1-1 \Rightarrow x = 41$
$\Delta 2 = ||211111112|| = 2-1=1 \Rightarrow y = 41$
$\Delta 3 = ||211121111|| = 2-1=1 \Rightarrow z = 41$
For $\alpha=2 \Rightarrow$ unique solution

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Det [a_1 + a_2 .... ba_j+cv ... a_x] = b det (A) + c det [a_1+ a_2 + ... v ... a_x]

Det [a_1 + a_2 .... a_j a_{j+1}... a_x] = – det [a_1+ a_2 + ... a_{j+1} a_j ... a_x]

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3. Answer: b

Explanation:

The correct option is (B): Infinitely many solution for $\alpha = 6, \beta = 5$

Let △ = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & a \\ 1 & 2 & 6 \end{vmatrix}$ = $6 - \alpha$

$\implies$ for $\alpha \neq 6$ system has unique solution

Now, when $\alpha = 6$

$\triangle_1 = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 2 & 6 \\ \beta & 2 & 6 \end{vmatrix} = 0 - (30 - 6\beta) + (10 - 2\beta) = (4\beta - 5)$

$\triangle_2 \begin{vmatrix} 1 & 6 & 2 \\ 1 & 5 & 6 \\ 1 & \beta & 6 \end{vmatrix} = -4(\beta - 5)$

$\triangle_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 2 & \beta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & -1 \\ 0 & 1 & \beta - 6 \end{vmatrix} = \beta - 5$

Clearly at β = 5, $\triangle_i = $ 0 for i = 1, 2, 3

∴ at $\alpha = 6, \beta \ = 5$ system has infinite solutions.

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4. **Answer: c**

Explanation:

$$|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$
$$= e^{-t}\left[5\cos^2 t + 5\sin^2 t\right] \forall t \in R$$
$$= 5e^{-t} \neq 0 \forall t \in R$$

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5. **Answer: d**

Explanation:

The correct answer is D: contains more than 2 elements

$\Delta = (2-\lambda)\left(\lambda^2+3\lambda-4\right)+2(-2\lambda+2)+1$

$(4-3-\lambda)=0$

$-\lambda^3-\lambda^2+6\lambda+8-3-\lambda-8=0$

$-\lambda^3-\lambda^2+5\lambda-3=0$

$\lambda^3+\lambda^2-5\lambda+3=0$

$(\lambda-1)\left(\lambda^2+2\lambda-3\right)=0$

$(\lambda-1)(\lambda+3)(\lambda-1)=0$

$\lambda=1,1,-3$

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$\text{Det}\ [\ a_1 + a_2 \ldots.\ a_j\ a_{j+1} \ldots a_x\ ] = -\det\ [\ a_1+ a_2 + \ldots a_{j+1}\ a_j \ldots a_x\ ]$

$\text{Det}\ (I) = 1$

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---

6. Answer: b

Explanation:

$$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$$

$A.adj\ A = A.A^T$

$$\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$\begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix} = \begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$$

Equate, $10a + 3b = 25a^2 + b^2$

& $10a + 3b = 13$

& $15a - 2b = 0$

$\frac{a}{2} = \frac{b}{15} = k(let)$

Solving $a = \frac{2}{5}, b = 3$

So, $5a + b = 5 \times \frac{2}{5} + 3 = 5$

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---

7. Answer: d

Explanation:

$x + \lambda y - z = 0$
$\lambda x - y - z = 0$
$x + y - \lambda z = 0$
Let's consider that, system of equation has a non trivial solution;
$$\Rightarrow \begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$
$\lambda + 1 - \lambda\{-\lambda^2 + 1\} - (\lambda + 1) = 0$
$\lambda(\lambda^2 - 1) = 0$
$\lambda = 0, 1, -1$
Hence, the system of equation has non-trivial solution for exactly three values of λ.

Answer:

Given that:

$$x + \lambda y + z = 0$$

$$\lambda x - y - z = 0$$

$$x + y - \lambda z = 0$$

Let Consider that the system of equation have a non-trivial solution.

$$\therefore \begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(+\lambda - (-1)) - \lambda(-\lambda^2 + 1) + (-1)(\lambda + 1) = 0$$

$$\Rightarrow \lambda + 1 + \lambda^3 - \lambda - \lambda - 1 = 0$$

$$\Rightarrow \lambda^3 - \lambda = 0$$

$$\Rightarrow \lambda(\lambda^2 - 1) = 0$$

$$\therefore \boxed{\lambda = 0 \text{ or } \lambda = +1 \text{ or } \lambda = -1}$$

Hence the system of eqn has non-trivial solution for exactly three values of λ.

= (Ans)

Concepts:

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Definition of Determinant

A **determinant** can be defined in many ways for a square **matrix**.

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8. **Answer: c**

Explanation:

$R_1 \Rightarrow R_1 - R_2$

$$\begin{vmatrix} 1 & -1 & 0 \\ cos^2\theta & 1+sin^2\theta & 4cos6\theta \\ cos^2\theta & sin^2\theta & 1+4cos6\theta \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ cos^2\theta & sin^2\theta & 1+4cos6\theta \end{vmatrix} = 0$$

$\Rightarrow \quad (1+4cos6\theta) + sin^2\theta + 1(cos^2\theta) = 0$

$1 + 2cos6\theta = 0 \Rightarrow \quad cos6\theta \ = -1/2$

$6\theta \ = \frac{2\pi}{3} \ \Rightarrow \quad \theta = \frac{\pi}{9}$

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Consider the column vectors of matrix A as A = [a_1, a_2, a_3, ...a_n] where any element a_j is a vector of size x.

Then the determinant of matrix A is defined such that

Det [a_1 + a_2 .... ba_j+cv ... a_x] = b det (A) + c det [a_1+ a_2 + ... v ... a_x]

Det [a_1 + a_2 .... a_j a_{j+1}... a_x] = – det [a_1+ a_2 + ... a_{j+1} a_j ... a_x]

Det (I) = 1

Where the scalars are denoted by b and c, a vector of size x is denoted by v, and the identity matrix of size x is denoted by I.

Read More: Minors and Cofactors

We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

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9. Answer: 204 - 204

Explanation:

As given $a+b+c+d=3$ or 5 or 7 or 11
if $\text{sum}=3$
$\left(1+x+x^2+\ldots\ldots+x^4\right)^4 \rightarrow x^3$
$\left(1-x^5\right)^4(1-x)^{-4} \rightarrow x^3$
$\therefore {}^{4+3-1}C_3={}^6C_3=20$
If $\text{sum}=5$
$\left(1-4x^5\right)(1-x)^{-4} \rightarrow x^5$
$\Rightarrow {}^{4+5-1}C_5-4x^{4.4+0-1}C_0={}^8C_5-4=52$
If sum $=7$
$\left(1-4x^5\right)(1-x)^{-4} \rightarrow x^7$
$\Rightarrow {}^{4+5-1}C_4-{}^{4.4+0-1}C_0={}^8C_5-4=52$

$$\text{If sum } = 11$$
$$\left(1-4x^5+6x^{10}\right)(1-x)^{-4} \rightarrow x^{11}$$
$$\Rightarrow {}^{4+11-1}C_{11} - 4 \cdot {}^{4+6-4}C_6 + 6 \cdot {}^{4+1-1}C_1$$
$$= {}^{14}C_{11} - 4 \cdot {}^{9}C_6 + 6.4 = 364 - 336 + 24 = 52$$
$$\therefore \text{ Total matrices } = 20 + 52 + 80 + 52 = 204$$

So, the correct answer is 204.

Concepts:

1. Determinants:

Definition of Determinant

A **determinant** can be defined in many ways for a square **matrix**.

The first and most simple way is to formulate the determinant by taking into account the top-row elements and the corresponding minors. Take the first element of the top row and multiply it by its minor, then subtract the product of the second element and its minor. Continue to alternately add and subtract the product of each element of the top row with its respective min or until all the elements of the top row have been considered.

For example let us consider a 1×1 matrix A.

A=[a_1.......a_n]

Read More: Properties of Determinants

Second Method to find the determinant:

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Then the determinant of matrix A is defined such that

Det [a_1 + a_2 .... ba_j+cv ... a_x] = b det (A) + c det [a_1+ a_2 + ... v ... a_x]

Det [a_1 + a_2 .... a_j a_{j+1}... a_x] = − det [a_1+ a_2 + ... a_{j+1} a_j ... a_x]

Det (I) = 1

Where the scalars are denoted by b and c, a vector of size x is denoted by v, and the identity matrix of size x is denoted by I.

Read More: Minors and Cofactors

We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

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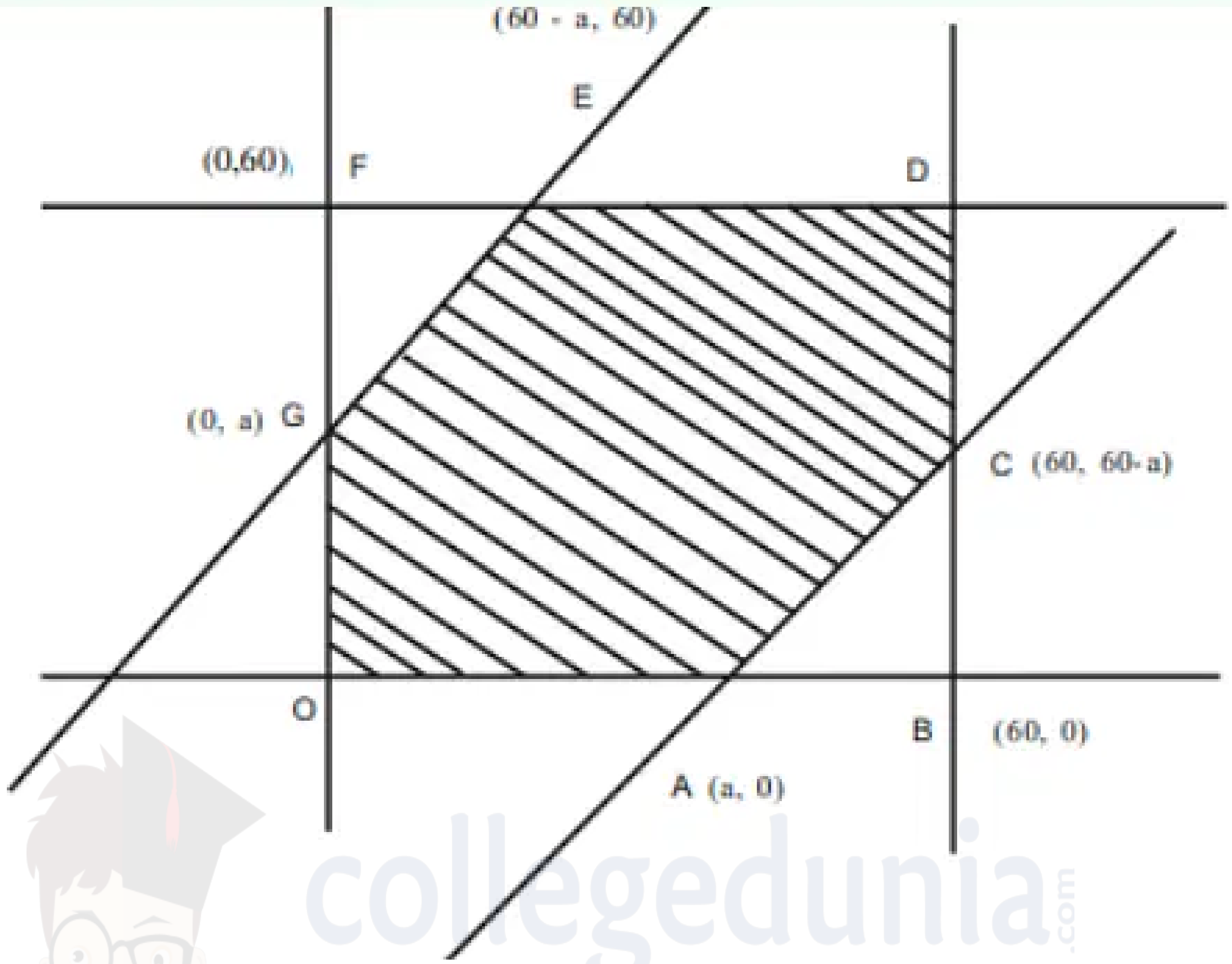
10. **Answer: 10 - 10**

Explanation:

$$|x-y|<a \Rightarrow -a<x-y<a$$
$$\Rightarrow x-y<a \text{ and } x-y>-a$$



$P(A) = \frac{\text{ar}(OACDEG)}{(OBDF)}$

$= \frac{\text{ar}(OBDF) - \text{ar}(ABC) - \text{ar}(EFG)}{\text{ar}(OBDF)}$

$\Rightarrow \frac{11}{36} = \frac{(60)^2 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600}$

$\Rightarrow 1100 = 3600 - (60 - a)^2$

$\Rightarrow (60 - a)^2 = 2500 \Rightarrow 60 - a = 50$

$\Rightarrow a = 10$

So , the correct answer is 10.

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.