Statistics And Probability JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Statistics And Probability

1. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than or equal to one is : **(+4, -1)**

[Online April 11, 2015]

a. $\frac{1}{16}$

b. $\frac{9}{16}$

c. $\frac{3}{4}$

d. $\frac{15}{16}$

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2. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is : **(+4, -1)**

[2017]

a. 6

b. 4

c. $\frac{6}{25}$

d. $\frac{12}{5}$

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3. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, the $\left(\frac{\text{mean of } X}{\text{standard deviation of } X}\right)$ is equal to : **(+4, -1)**

[Jan. 11, 2019 (II)]

a. 4

b. $\frac{4\sqrt{3}}{3}$

c. $4\sqrt{3}$

d. $3\sqrt{2}$

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4. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the **(+4, -1)**

[27-Jan-2024 Shift2]

drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

a. $\frac{26}{49}$

b. $\frac{32}{49}$

c. $\frac{27}{49}$

d. $\frac{21}{49}$

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5. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is : **(+4, -1)**

[2018]

a. $\frac{3}{10}$

b. $\frac{2}{5}$

c. $\frac{1}{5}$

d. $\frac{3}{4}$

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6. A number x is chosen at random from the set $\{1, 2, 3, 4, ..., 100\}$. Define the event: $A =$ the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \geq 0$ Then $P(A)$ is : **(+4, -1)**

[29-Jan-2024 Shift2]

a. 0.71

b. 0.7

c. 0.51

d. 0.2

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7. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is : **(+4, -1)**

[Online April 10, 2016]

a. $\frac{240}{729}$

b. $\frac{192}{729}$

c. $\frac{256}{729}$

d. $\frac{496}{729}$

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8. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5, otherwise X takes the value ? 1. Then the expected value of X, is : **(+4, -1)**

[Jan. 7, 2020 (I)]

a. $\frac{1}{8}$

b. $\frac{3}{16}$

c. $-\frac{1}{8}$

d. $-\frac{3}{16}$

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9. Maximum value n such that (66)! is divisible by 3^n **(+4, -1)**

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10. Let α be the remainder $(22)^{2022} + (2022)^{22}$ is divided by 3 and β be the remainder when the same is divided by 7 then $\alpha^2 + \beta^2$ is? **(+4, -1)**

[27-Jun-2022-Shift-1]

Answers

1. **Answer: d**

Explanation:

In binomial distribution mean $= np = 2$ and
variance $npq = 1$
$\Rightarrow 2q = 1$
$\Rightarrow q = \frac{1}{2}$
$\Rightarrow p = 1 - q = \frac{1}{2}$
$\Rightarrow n = 4$
$\therefore P(x \geq 1) = 1 - {}^{n}C_0 P^0 (q)^n$
$= 1 - {}^{4}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4$
$= 1 - \frac{1}{16} = \frac{15}{6}$

Concepts:

1. Probability:

Probability is defined as the extent to which an event is likely to happen. It is measured by the ratio of the favorable outcome to the total number of possible outcomes.

The definitions of some important terms related to probability are given below:

Sample space

The set of possible results or outcomes in a trial is referred to as the sample space. For instance, when we flip a coin, the possible outcomes are heads or tails. On the other hand, when we roll a single die, the possible outcomes are 1, 2, 3, 4, 5, 6.

Sample point

In a sample space, a sample point is one of the possible results. For instance, when using a deck of cards, as an outcome, a sample point would be the ace of spades or the queen of hearts.

Experiment

When the results of a series of actions are always uncertain, this is referred to as a trial or an experiment. For Instance, choosing a card from a deck, tossing a coin, or rolling a die, the results are uncertain.

Event

An event is a single outcome that happens as a result of a trial or experiment. For instance, getting a three on a die or an eight of clubs when selecting a card from a deck are happenings of certain events.

Outcome

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2. Answer: d

Explanation:

$n = 10$
p(Probability of drawing a green ball) $= \frac{15}{25}$
$\therefore p = \frac{3}{5}, q = \frac{2}{5}$
var(X) = n.p.q
$= 10.\frac{6}{25} = \frac{12}{5}$

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3. Answer: c

Explanation:

p (probability of getting white ball) = $\frac{30}{40}$
$q = \frac{1}{4}$ and n = 16
mean =np = $16.\frac{3}{4} = 12$

and standard diviation
$= \sqrt{npq} = \sqrt{16.\frac{3}{4}.\frac{1}{4}} = \sqrt{3}$

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4. **Answer: b**

Explanation:

E_1 : Event of drawing a Red ball and placing a green ball in the bag
E_2 : Event of drawing a green ball and placing a red ball in the bag
E : Event of drawing a red ball in second draw

$P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$
$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49}$

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5. **Answer: b**

Explanation:

E_1 : Event that first ball drawn is red.
E_2 : Event that first ball drawn is black.
E : Event that second ball drawn is red.
$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$
$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12} = \frac{2}{5}$

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6. Answer: a

Explanation:

Given $\frac{(x-10)(x-50)}{(x-30)} \geq 0$
Let $x \geq 10, x \geq 50$ equation will be true
$\forall x \geq 50$
as $\left(\frac{x-50}{x-30}\right) \geq 0, \forall x \in [10, 30)$
$\frac{(x-10)(x-50)}{x-30} \geq 0, \forall x \in [10, 30)$

Total value of x between 10 to 30 is 20.
Total values of x between 50 to 100 including 50 and 100 is 51.
Total values of $x = 51 + 20 = 71$
$P(A) = \frac{71}{100} = 0.71$

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7. Answer: c

Explanation:

Given $= p = 2q \,\&$ we know that $p + q = 1$
$\Rightarrow P = 2/3, \quad q = 1/3$
The problem of at least 5 successes
$= {}^6C_5 P^5 q + {}^6C_6 P^6$
$= 6 \times \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + 1 \times \left(\frac{2}{3}\right)^6 = \frac{256}{729}$

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8. **Answer: a**

Explanation:

Expected value $= \sum XP(k)$

$-\frac{1}{32} - \frac{12}{32} - \frac{11}{32} + \frac{15}{32} + \frac{8}{32} + \frac{5}{32}$

$= \frac{28-24}{32} = \frac{4}{32} = \frac{1}{8}$

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9. Answer: 31 - 31

Explanation:

To find the maximum value of n such that (66)! is divisible by 3^n, we need to count the number of factors of 3 in (66)!, since each factor of 3 contributes to the divisibility by 3.
To count the number of factors of 3 in (66)!, we can use the formula:
$[\frac{66}{3}] + [\frac{66}{9}] + [\frac{66}{27}] + [\frac{66}{81}]$
= 22 + 7 + 2 + 0 = 31,

where ⌊x⌋ denotes the greatest integer less than or equal to x.

This means that (66)! is divisible by 3^{31}. Therefore, the maximum value of n such that (66)! is divisible by 3^n is n = 31.

So, the correct answer is 31

Concepts:

1. Mathematical Reasoning:

Mathematical reasoning or the principle of mathematical reasoning is a part of mathematics where we decide the truth values of the given statements. These reasoning statements are common in most competitive exams like JEE and the questions are extremely easy and fun to solve.

Types of Reasoning in Maths:

Mathematically, reasoning can be of two major types such as:

1. **Inductive Reasoning** - In this, method of mathematical reasoning, the validity of the statement is examined or checked by a certain set of rules, and then it is generalized. The principle of mathematical induction utilizes the concept of inductive reasoning.
2. **Deductive Reasoning** - The principle is the opposite of the principle of induction. Contrary to inductive reasoning, in deductive reasoning, we apply the rules of a general case to a provided statement and make it true for particular statements. The principle of mathematical induction utilizes the concept of deductive reasoning.

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10. Answer: 5 - 5

Explanation:

The correct answer is 5.

$$(22)^{2022} + (2022)^{22}$$

For α

$$(21+1)^{2022} + \underbrace{(2022)^{22}}_{\text{divisible by 3}}$$

$$= (3k_1 + 1)$$

For β

$$(21+1)^{2022} + (2023-1)^{22}$$

$$= (7\lambda + 1) + (7\mu + 1)$$

$$= 7k_2 + 2$$

So, $\alpha = 1, \beta = 2$

$$\alpha^2 + \beta^2 = 5$$

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