Rotational Motion JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Rotational Motion

1. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by : **(+4, -1)**

[8 Apr. 2019 II]

a. $\frac{14}{15}$

b. $\frac{4}{5}$

c. 1

d. $\frac{2}{\sqrt{5}}$

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2. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is : **(+4, -1)**

[10 Jan. 2019 I]

a. $\frac{3F}{2mR}$

b. $\frac{F}{3mR}$

c. $\frac{2F}{3mR}$

d. $\frac{F}{2mR}$

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3. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \leq x \leq L$. The value of x for the centre of mass of the rod is a t : **(+4, -1)**

[9 Jan. 2020 II]

a. $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

b. $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

c. $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

d. $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

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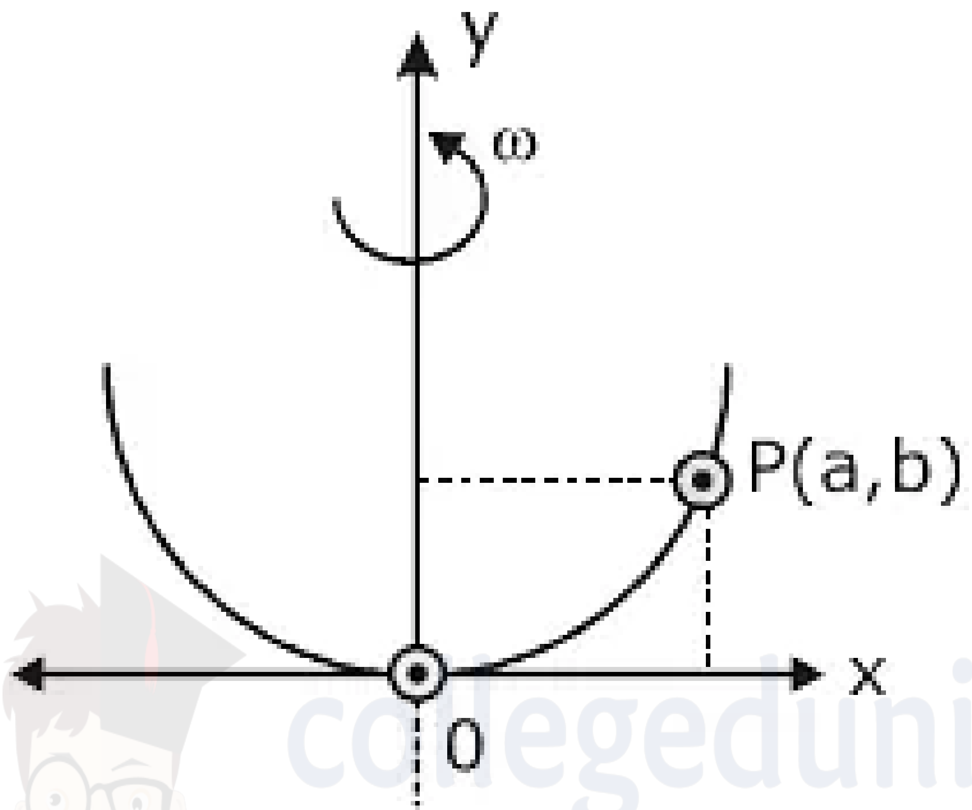
4. A bead of mass m stays at point $P(a, b)$ on a wire bent in the shape of a parabola $y = 4cx^2$ and rotating with angular speed ω (see figure). **(+4, -1)**



The value of ω is (neglect friction) :

[Sep. 02, 2020 (I)]

a. $\sqrt{\frac{2gC}{ab}}$

b. $2\sqrt{2gC}$

c. $\sqrt{\frac{2g}{C}}$

d. $2\sqrt{gC}$

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5. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed $\omega\, rad/s$ about the vertical support. About the point of suspension **(+4, -1)**

[2014]

a. Angular momentum is conserved

b. Angular momentum changes in magnitude but not in direction

c. Angular momentum changes in direction but not in magnitude

d. Angular momentum changes both in direction and magnitude

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6. A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z' axes will be : (+4, -1)

[Online April 16, 2018]

a. 1:03

b. 1:04

c. 1:05

d. 1:02

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7. A thin uniform bar of length L and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ are moving in the same horizontal plane from opposite sides of the bar with speeds $2v$ and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be : (+4, -1)

[Online April 15, 2018]

a. $\frac{v}{5L}$

b. $\frac{6v}{5L}$

c. $\frac{3v}{5L}$

d. $\frac{v}{6L}$

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8. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to : (Take the radius of the drum to be $1.25\,m$ and its axle to be horizontal) : (+4, -1)

[Online April 10, 2016]

a. 0.4

b. 1.3

c. 8

d. 27

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9. A solid sphere and a ring have equal masses and equal radius of gyration. If the sphere is rotating about its diameter and ring about an axis passing through and perpendicular to its plane, then the ratio of radius is $\sqrt{\frac{x}{2}}$ then find the value of x. **(+4, -1)**

[15-Apr-2023 shift 1]

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10. Moment of inertia of a disc of mass M and radius 'R' about any of its diameter is $\frac{M^2}{4}$ The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $\frac{x}{2} MR^2$ The value of x is __ **(+4, -1)**

Answers

1. Answer: a

Explanation:

for solid sphere

$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mR^2.\frac{v^2}{R^2} = mgh_{sph}$

for solid cylinder

$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{1}{2}mR^2.\frac{v^2}{R^2} = mgh_{cyl}$

$\Rightarrow \frac{h_{sph}}{h_{cyl}} = \frac{7/5}{3/2} = \frac{14}{15}$

Concepts:

1. System of Particles and Rotational Motion:

1. The system of particles refers to the extended body which is considered a rigid body most of the time for simple or easy understanding. A rigid body is a body with a perfectly definite and unchangeable shape.
2. The distance between the pair of particles in such a body does not replace or alter. Rotational motion can be described as the motion of a rigid body originates in such a manner that all of its particles move in a circle about an axis with a common angular velocity.
3. The few common examples of rotational motion are the motion of the blade of a windmill and periodic motion.

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2. Answer: c

Explanation:

$FR = \frac{3}{2}MR^2\alpha$

$\alpha = \frac{2F}{3MR}$

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3. Answer: c

Explanation:

$$x_{cm} = \frac{\int x dm}{\int dm} = \frac{\int (\lambda dx) x}{\int dm}$$
$$= \frac{\int_0^L \left(a + \frac{bx^2}{L^2}\right) x dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right) dx}$$
$$= \frac{\frac{aL^2}{2} + \frac{b}{L^2} \cdot \frac{L^4}{4}}{aL + \frac{b}{L^2} \cdot \frac{L^3}{3}}$$
$$= \frac{\left(\frac{4a+2b}{8}\right) L}{\frac{(3a+b)}{3}} = \frac{3}{4} \frac{(2a+b)L}{(3a+b)}$$

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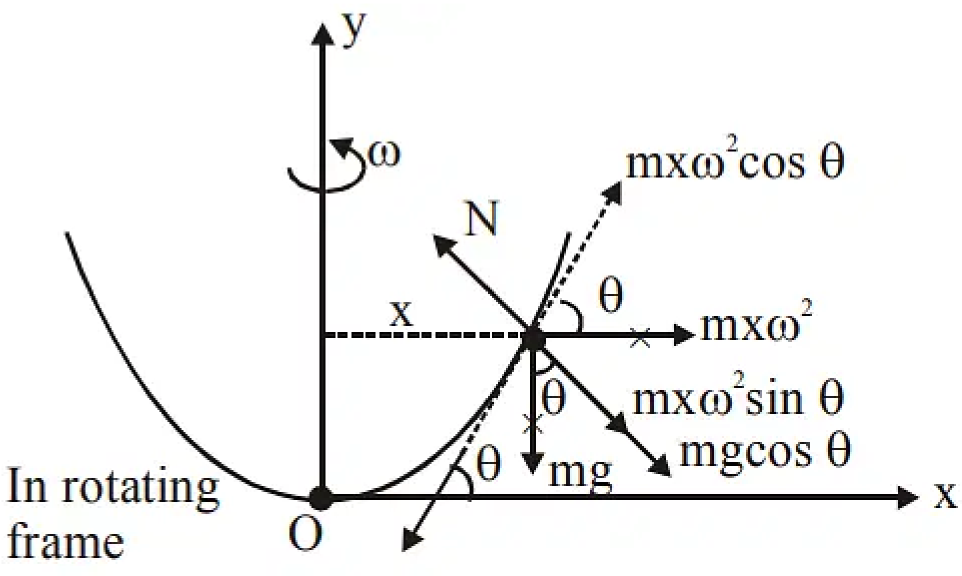
4. Answer: b

Explanation:



For Particle to be in equilibrium,

$mg\,sin\theta = mx\,\omega^2 cos\theta$

$\Rightarrow tan\theta = \frac{\omega^2 x}{g}$

Also, $y = 4cx^2$

$\Rightarrow \frac{dy}{dx} = 8cx =$ Slope at point P = $tan\theta$

Equating both values of $tan\theta$ we get,

$\frac{\omega^2 x}{g} = 8cx$

$\Rightarrow \omega^2 = 8cg$

$\Rightarrow \omega = \sqrt{8cg} = 2\sqrt{2cg}$

Therefore, The correct option is (B): $2\sqrt{2cg}$

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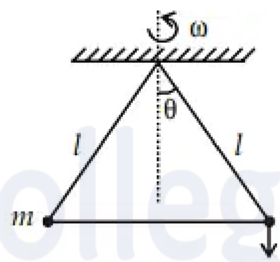
5. **Answer: c**

Explanation:

$\tau = mg \times l \sin\theta$ (Direction parallel to plane of rotation of particle)



as τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same

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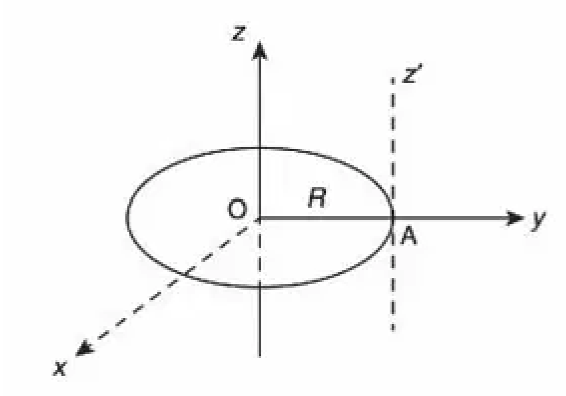
6. **Answer: a**

Explanation:

The moment of inertia of a disc about central axis is



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7. **Answer: b**

Explanation:

Centre of mass after the collision will be at O. Therefore, $2m\frac{L}{6} = m\frac{L}{3}$

Now, using conservation of angular momentum, we get $2m\frac{L}{6} \times v + m\frac{L}{3} \times 2v = I\omega$

Therefore, $2m\frac{L}{6} \times v + m\frac{L}{3} \times 2v = \left[8m\frac{L^2}{12} + 2m\left(\frac{L}{6}\right)^2 + m\left(\frac{L}{3}\right)^2\right]\omega$

$\Rightarrow \left(\frac{L}{6} + \frac{1}{3}\right)2mv = \left[8m\frac{L^2}{12} + \frac{2mL^2}{36} + \frac{mL^2}{9}\right]\omega$

$\Rightarrow \frac{3L}{6} \times 2mV = \frac{5}{6}mL^2\omega$

$\Rightarrow Lmv = \frac{5}{6}mL^2\omega$

$\Rightarrow \omega = \frac{6Lmv}{5mL^2}$

$\Rightarrow \omega = \frac{6}{5}\frac{v}{L}$

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8. Answer: d

Explanation:

$\zeta \leq \sqrt{5\,gr}$

$\omega = \frac{v}{r} \leq \sqrt{\frac{5\,g}{r}} = \sqrt{\frac{50\times 8}{10}} = \sqrt{\frac{400}{10}}$

$\omega = \sqrt{40}\,rad/s$

$= \frac{60\sqrt{10}}{\pi}$ rpm

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9. Answer: 5 - 5

Explanation:

$$(\frac{2}{5})mR_1^2 = mK_1^2 and\ R_2^2 = K_2$$
$$K_1 = \sqrt{(\frac{2}{5})R_1}$$
$$K_2 = R_2$$
$$K_1 = K_2$$
$$\sqrt{(\frac{2}{5})R_1} = R_2$$
$$\frac{R_1}{R_2} = \sqrt{\frac{5}{2}}$$

Therefore, the value of x is 5.

Concepts:

1. Moment of Inertia:

Moment of inertia is defined as the quantity expressed by the body resisting angular acceleration which is the sum of the product of the mass of every particle with its square of a distance from the axis of rotation.

Moment of inertia mainly depends on the following three factors:

1. The density of the material
2. Shape and size of the body
3. Axis of rotation

Formula:

In general form, the moment of inertia can be expressed as,

$I = m \times r^2$

Where,

I = Moment of inertia.

m = sum of the product of the mass.

r = distance from the axis of the rotation.

$M^1 L^2 T^0$ is the dimensional formula of the moment of inertia.

The equation for moment of inertia is given by,

$I = I = \sum m_i r_i^2$

Methods to calculate Moment of Inertia:

To calculate the moment of inertia, we use two important theorems-

- Perpendicular axis theorem
- Parallel axis theorem

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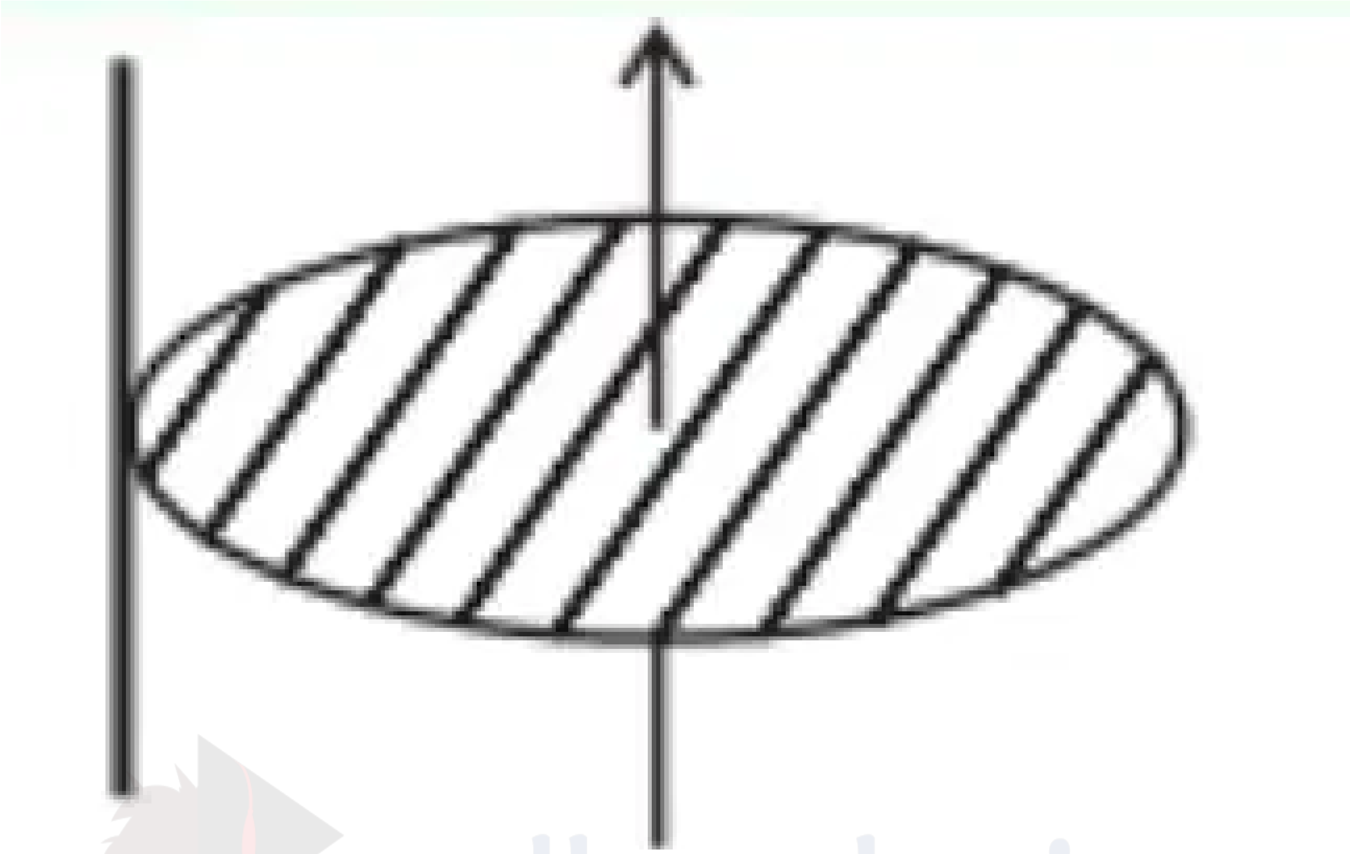
10. **Answer: 3 - 3**

Explanation:

The correct answer is 3.



$I = I_{cm} + Md^2$
$= \frac{MR^2}{2} + MR^2$
$= \frac{3}{2}MR^2$
$x = 3$

Concepts:

1. Rotational Motion:

Rotational motion can be defined as the motion of an object around a circular path, in a fixed orbit.

Rotational Motion Examples:

The wheel or rotor of a motor, which appears in rotation motion problems, is a common example of the rotational motion of a rigid body.

Other examples:

- Moving by Bus
- Sailing of Boat
- Dog walking

- A person shaking the plant.
- A stone falls straight at the surface of the earth.
- Movement of a coin over a carrom board

Types of Motion involving Rotation:

1. Rotation about a fixed axis (Pure rotation)
2. Rotation about an axis of rotation (Combined translational and rotational motion)
3. Rotation about an axis in the rotation (rotating axis)