Rotational Motion JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Rotational Motion

1. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is : **(+4, -1)**

[2017]

a. $\frac{3g}{2l}\sin\theta$

b. $\frac{2g}{3l}\sin\theta$

c. $\frac{3g}{2l}\cos\theta$

d. $\frac{2g}{2l}\sin\theta$

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2. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by : **(+4, -1)**

[10 Apr. 2019 II]

a. 185

b. 65

c. 285

d. 140

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3. A string is wound around a hollow cylinder of mass $5\,kg$ and radius $0.5\,m$. If the string is now pulled with a horizontal force of $40\,N$, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) : **(+4, -1)**

[11 Jan. 2019 II]

a. $12\,rad/s^2$

b. $16\,rad/s^2$

c. $10\ rad/s^2$

d. $20\ rad/s^2$

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4. A tennis ball (treated as hollow spherical shell) starting from O rolls down a hill. At point A the ball becomes air borne leaving at an angle of 30° with the horizontal. The ball strikes the ground at B. What is the value of the distance AB ? (Moment of inertia of a spherical shell of mass m and radius R about its diameter $= \frac{2}{3}mR^2$) **(+4, -1)**

[Online April 22, 2013]

a. $1.87\,m$

b. $2.08\,m$

c. $1.57\,m$

d. $1.77\,m$

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5. A thin bar of length L has a mass per unit length λ, that increases linearly with distance from one end. If its total mass is M and its mass per unit length at the lighter end is λ_0, then the distance of the centre of mass from the lighter end is : **(+4, -1)**

[Online April 11, 2014]

a. $\frac{L}{2} - \frac{\lambda_0 L^2}{4M}$

b. $\frac{L}{3} + \frac{\lambda_0 L^2}{8M}$

c. $\frac{L}{3} + \frac{\lambda_0 L^2}{4M}$

d. $\frac{2L}{3} - \frac{\lambda_0 L^2}{6M}$

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6. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is : **(+4, -1)**

[8 April 2019 I]

a. $\frac{3}{2}$

b. $\frac{1}{2}$

c. $\frac{3}{5}$

d. $\frac{8}{5}$

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7. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is : (+4, -1)

[27 Jul 2021 Shift 1]

a. $\frac{MR^2}{6}$

b. $\frac{MR^2}{3}$

c. $\frac{2MR^2}{3}$

d. $\frac{MR^2}{2}$

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8. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be : (+4, -1)

[8 Apr. 2019 II]

a. $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

b. $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

c. $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

d. $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

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9. If a solid sphere of mass $5kg$ and a disc of mass $4kg$ have the same radius Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$ The the value of x is ___ (+4, -1)

[25-Jan-2023 Shift 2]

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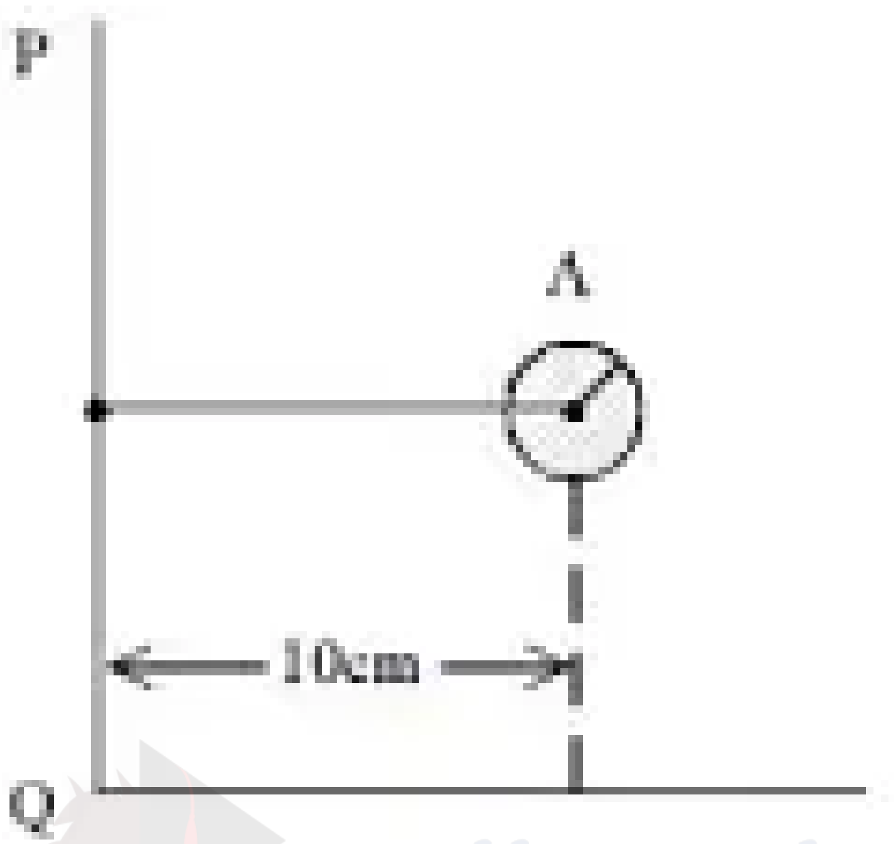
10. Solid sphere A is rotating about an axis PQ If the radius of the sphere is $5cm$ then its radius of gyration about PQ will be $\sqrt{x}cm$ The value of x is __ (+4, -1)

[24-Jan-2023 Shift1]


P
A
10cm
Q

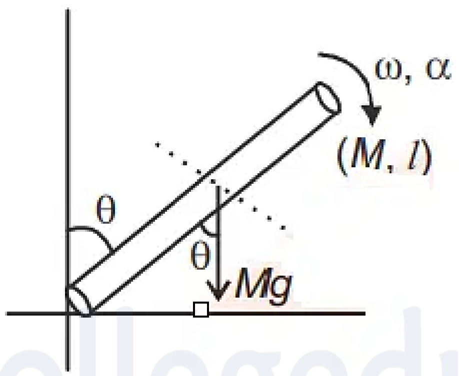
Answers

1. **Answer: a**

Explanation:

Torque at angle θ

$\tau = Mg \sin \theta.$



Concepts:

1. System of Particles and Rotational Motion:

1. The system of particles refers to the extended body which is considered a [rigid b ody] most of the time for simple or easy understanding. A rigid body is a body with a perfectly definite and unchangeable shape.
2. The distance between the pair of particles in such a body does not replace or alter. Rotational motion can be described as the motion of a rigid body originates in such a manner that all of its particles move in a circle about an axis with a common angular velocity.
3. The few common examples of rotational motion are the motion of the blade of a windmill and periodic motion.

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2. **Answer: d**

Explanation:

$I_1 = \frac{\left(\frac{7M}{8}\right)(2R)^2}{2} = \left(\frac{7}{16} \times 4\right) MR^2 = \frac{7}{4} MR^2$
$I_2 = \frac{2}{5}\left(\frac{M}{R}\right) R_1^2 = \frac{2}{5}\left(\frac{M}{8}\right)\frac{R^2}{4} = \frac{MR^2}{80}$
$\frac{4}{3}\pi R^3 = 8\left(\frac{4}{3}\pi R_1^3\right)$
$R^3 = 8R_1^3$
$R = 2R_1$
$\therefore \frac{I_1}{I_2} = \frac{7/4 MR^2}{\frac{MR^2}{80}} = \frac{7}{4} \times 80 = 140$

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3. Answer: b

Explanation:

$40 + f = m(R\alpha)$ .....(i)
$40 \times R - f \times R = mR^2\alpha$
$40 - f = mR\alpha$ ...... (ii)
From (i) and (ii)
$\alpha = \frac{40}{mR} = 16$

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4. Answer: a

Explanation:

Velocity of the tennis ball on the surface of the earth or ground

$v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$ (where k = radius of gyration of spherical shell $= \sqrt{\frac{2}{3}R}$)

Horizontal range $AB = \frac{v^2 \, sin\, 2\theta}{g}$

$= \frac{\left(\frac{\sqrt{2gh}}{1+k^2/R^2}\right) sin\left(2\times 30^{\circ}\right)}{g}$

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5. Answer: d

Explanation:

Mass per unit lengh $\lambda_0 + kx$

$M = \int\limits_0^L (\lambda_0 + kx)\, dx$

$M = \lambda_0 L + \frac{K \times L^2}{2}$

$\frac{2M - \lambda_0 L}{L^2} = K$

$\frac{2M}{L^2} - \frac{\lambda_0}{L} = K$

$\frac{\int dm(r)}{\int dm} = \frac{\int (\lambda dn) x}{M} = \frac{\int_0^L \left(\lambda_0 x + kx^2\right) dx}{M}$

$r_{cm} = \frac{\lambda_0 L + \frac{kL^2}{2}}{M}$

substitute 'k'

$r_{cm} = \frac{2L}{3} - \frac{\lambda_0 \ell^2}{6M}$

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6. Answer: d

Explanation:

$M = \int_0^R \rho_0 r\, (2\pi r dr) = \frac{\rho_0 \times 2\pi \times R^3}{3}$

${$\underset{\text{ (MOI about COM) }}{{ I_0 }}$ = \int^{R}_{0} \rho_{0} r (2 \pi rdr) \times r^{2} = \frac{\rho_{0} \times2\pi R^{5}}{5} } $

by parallel axis theorem

$I = I_0 + MR^2$

$= \frac{\rho_0 \times 2\pi R^5}{5} + \frac{\rho_0 \times 2\pi R^3}{3} \times R^2 = \rho_0 2\pi R^5 \times \frac{8}{15}$
$= MR^2 \times \frac{8}{5}$

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7. Answer: c

Explanation:

$I_{Disc} = \int_0^R (dm)\, r^2 \Rightarrow I_{Disc} = \int_0^R (\sigma 2\pi r dr)\, r^2$
$I_{Disc} = \int_0^R (kr^2 2\pi r dr)\, r^2$
$I_{Disc} = 2\pi k \int_0^R r^5 dr M = \int_0^R 2\pi r dr k r^2$
$I_{Dics} = 2\pi k \left(\frac{r^6}{6}\right)_0^R M = 2\pi k \int_0^R r^3 dr$
$I_{Disc} = 2\pi k \frac{R^6}{6} M = 2\pi k \frac{r^4}{4}\Big|_0^R$
$I_{Disc} = \frac{\pi k R^6}{3} = \left(\frac{\pi k R^4}{2}\right) \frac{R^2 2}{3} M = 2\pi k \frac{R^4}{4}$
$I_{Disc} = \frac{M 2 R^2}{3}$
$I_{Disc} = \frac{2}{3} M R^2$

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8. Answer: d

Explanation:

$$x = \frac{M\frac{a}{2} - \frac{M}{4} \times \frac{3a}{4}}{M - \frac{M}{4}}$$
$$= \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$
$$y = \frac{M\frac{b}{2} - \frac{M}{4} \times \frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

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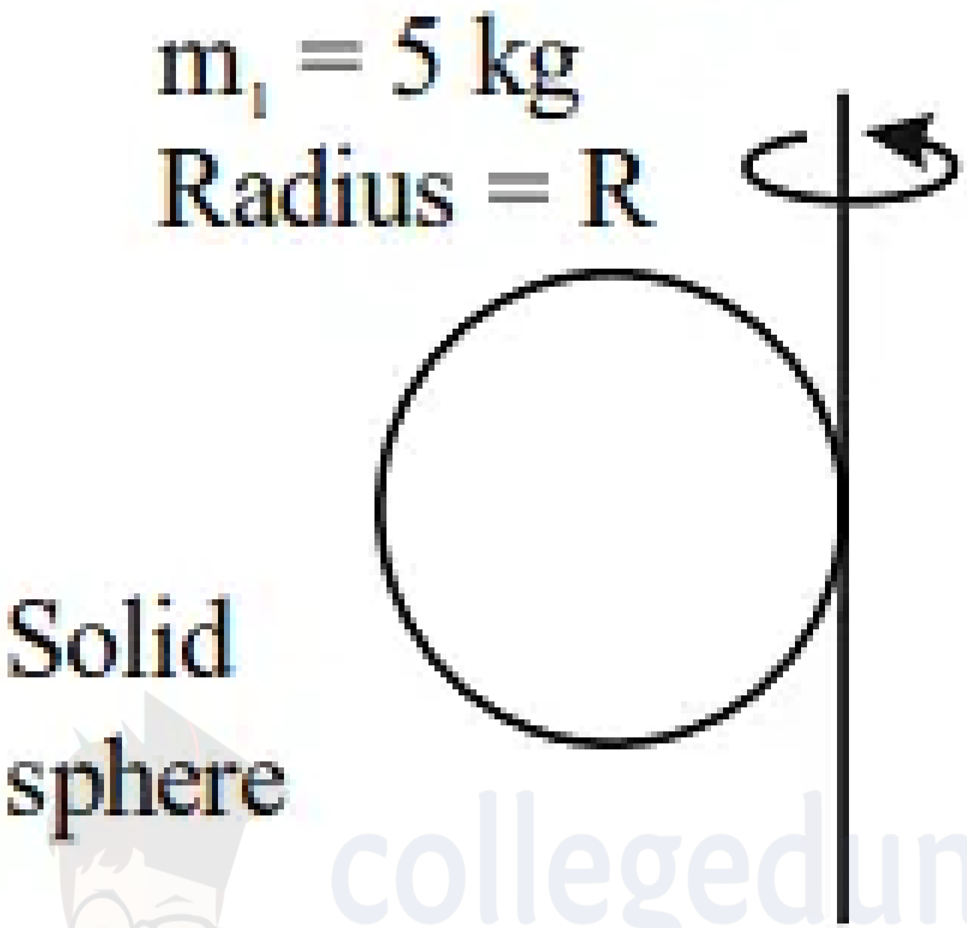
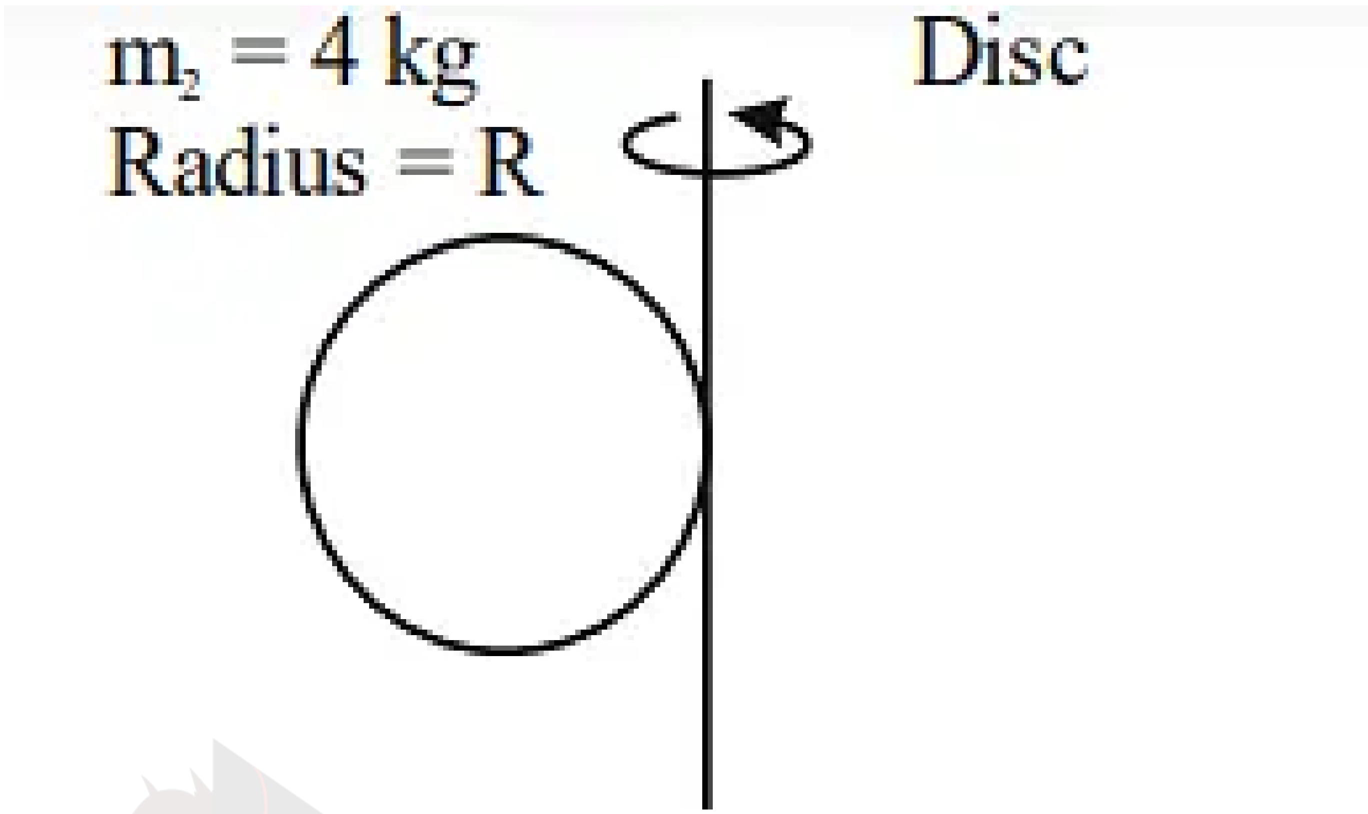
9. Answer: 5 - 5

Explanation:

The correct answer is 5.



$I_1 = \frac{2}{5} m_1 R^2 + m_1 R^2$
$I_1 = m_1 R^2 \left(\frac{7}{5}\right)$
$I_1 = 7R^2$



$$I_2 = \frac{m_2 R^2}{4} + m_2 R^2$$
$$I_2 = \frac{5}{4} m_2 R^2$$
$$I_2 = 5R^2$$
$$\frac{I_2}{I_1} = \frac{5}{7}$$
$$x = 5$$

Concepts:

1. Rotational Motion:

Rotational motion can be defined as the motion of an object around a circular path, in a fixed orbit.

Rotational Motion Examples:

The wheel or rotor of a motor, which appears in rotation motion problems, is a common example of the rotational motion of a rigid body.

Other examples:

- Moving by Bus
- Sailing of Boat
- Dog walking

- A person shaking the plant.
- A stone falls straight at the surface of the earth.
- Movement of a coin over a carrom board

Types of Motion involving Rotation:

1. Rotation about a fixed axis (Pure rotation)
2. Rotation about an axis of rotation (Combined translational and rotational motion)
3. Rotation about an axis in the rotation (rotating axis)

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10. **Answer: 110 - 110**

Explanation:

The correct answer is 110
$I_{cm} = \frac{2}{5}MR^2$
$I_{PQ} = I_{cm} + md^2$
$I_{PQ} = \frac{2}{5}mR^2 + m(10cm)^2$
For radius of gyration
$I_{PQ} = mk^2$
$k^2 = \frac{2}{5}R^2 + (10cm)^2$
$= \frac{2}{5}(5)^2 + 100$
$= 10 + 100 = 110$
$k = \sqrt{110}cm$
$x = 110$

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