Sequence And Series JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Sequence And Series

1. The number of terms in an $A.P.$ is even; the sum of the odd terms in it is 24 and that the even terms is 30. If the last term exceeds the first term by $10\frac{1}{2}$, then the number of terms in the $A.P.$ is : **(+4, -1)**

[Online April 19, 2014]

a. 4

b. 8

c. 12

d. 16

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2. The sum of the first 16 terms of an AP whose first term and the third term are 5 and 15 respectively is **(+4, -1)**

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3. Let $a_1, a_2, a_3, \ldots$ be an AP If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to : **(+4, -1)**

[29-Jan-2023 Shift 1]

a. $\frac{381}{4}$

b. 9

c. $\frac{33}{4}$

d. 24

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4. Let $a, b, c > 1, a^3, b^3$ and c^3 be in A.P., and $log_a b, log_c a$ and $log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then *abc* is equal to: **(+4, -1)**

[29-Jan-2023 Shift 1]

a. 343

b. 216

c. $\frac{343}{8}$

d. $\frac{125}{8}$

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5. Let $a_1, a_2, \ldots, a_n$ be in AP If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \ldots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to **[29-Jan-2023 Shift2]** **(+4, -1)**

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6. The mean and variance of 7 observations are 8 and 16 respectively If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $a + 3b - 5$ is equal to _____ **(+4, -1)**

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7. Let A_1, A_2, A_3 be the three AP with the same common difference d and having their first terms as $A, A+1, A+2$, respectively, Let a, b, c be the $7^{\text{th}}, 9^{\text{th}}, 17^{\text{th}}$ terms of A_1, A_2, A_3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal to **(+4, -1)**

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8. Let $a, b, c >, a^3, b^3$ and c^3 be in AP, and $\log_a b, \log_c a$ and $\log_b c$ be in GP If the sum of first 20 terms of an AP, whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to: **[30-Jan-2023 Shift2]** **(+4, -1)**

a. 343

b. 216

c. $\frac{343}{8}$

d. $\frac{125}{8}$

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9. Find the sum of series: $2 * 2^2 - 2 * 3^2 + 2 * 4^2 + \ldots . (20\, terms)$ **[2007]** **(+4, -1)**

a. 462

b. -462

c. 460

d. -460

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10. 3, 8, 13, ...,373 are in arithmetic series. The sum of numbers not divisible by three is **(+4, -1)**

[10-Apr-2023 shift 1]

a. 9310

b. 8340

c. 9525

d. 7325

Answers

1. **Answer: b**

Explanation:

Let no. of terms $= 2n$

$a, (a+d), (a+2d), \ldots\ldots\ldots a+(2n-1)d$

sum of even terms

$\frac{n}{2}[2(a+d)+(n-1)2d] = 30 \quad \ldots\ldots(i)$

sum of odd terms

$\frac{n}{2}[2a+(n-1)2d] = 24 \quad \ldots\ldots\ldots(ii)$

$a+(2n-1)d-a = \frac{21}{2} \quad \ldots\ldots(iii)$

e (i)....e (ii)

$\frac{n}{2} \times 2d = 6$

$\Rightarrow nd = 6 \quad \ldots\ldots(iv)$

$(2n-1)d = \frac{21}{2} \quad \ldots\ldots(v)$

$\frac{eq(iv)}{eq(v)} = \frac{n}{2n-1} = \frac{4}{7}$

$\Rightarrow 8n-4 = 7n$

$n = 4$

so no. of terms $= 8$

Concepts:

1. Arithmetic Progression:

Arithmetic Progression (AP) is a mathematical series in which the difference between any two subsequent numbers is a fixed value.

For example, the natural number sequence 1, 2, 3, 4, 5, 6,... is an AP because the difference between two consecutive terms (say 1 and 2) is equal to one (2 -1). Even when dealing with odd and even numbers, the common difference between two consecutive words will be equal to 2.

In simpler words, an arithmetic progression is a collection of integers where each term is resulted by adding a fixed number to the preceding term apart from the first term.

For eg:- 4,6,8,10,12,14,16

We can notice Arithmetic Progression in our day-to-day lives too, for eg:- the number of days in a week, stacking chairs, etc.

Read More: Sum of First N Terms of an AP

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2. Answer: 680 - 680

Explanation:

Explanation:
1st Method1st term = 53rd term = 15 Then, = 516th term = + 15 = 5 + 15 × 5 = 80
Sum = × (+) / 2 = no. of terms × first term + last term / 2 = 16 × (5 + 80) / 2 = 16 × 85/2 = 8 × 85 = 6802nd Method(Thought Process). Sum = number of terms average of that AP
Sum = 16 × (5 + 80) / 2 = 16 × 85/2 = 8 × 85 = 680Hence, the correct answer is 680.

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3. Answer: d

Explanation:

The correct answer is (D) : 24
$a + 6d = 3$.....(1)
$Z = a(a + 3d)$
$= (3 - 6d)(3 - 3d)$
$= 18d^2 - 27d + 9$
Differentiating with respect to d
$\Rightarrow 36d - 27 = 0$
$\Rightarrow d = \frac{3}{4}, \text{ from } (1) a = \frac{-3}{2}, (Z = \text{ minimum })$
Now, $S_a = \frac{n}{2}\left(-3 + (n-1)\frac{3}{4}\right) = 0$
$\Rightarrow n = 5$
Now,
$n! - 4a_{n(n+2)} = 120 - 4\left(a_{35}\right)$
$= 120 - 4(a + (35 - 1)d)$
$= 120 - 4\left(\frac{-3}{2} + 34 \cdot \left(\frac{3}{4}\right)\right)$
$= 120 - 4\left(\frac{-6+102}{4}\right)$
$= 120 - 96 = 24$

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4. Answer: b

Explanation:

As a^3, b^3, c^3 be in A.P. $\rightarrow a^3 + c^3 = 2b^3$.... (1)

$\log_a^b, \log_c^a, \log_b^c$ are in G.P.

$\therefore \frac{\log b}{\log a} \cdot \frac{\log c}{\log b} = \left(\frac{\log a}{\log c}\right)^2$

$\therefore (\log a)^3 = (\log c)^3 \Rightarrow a = c$.....(2)

From (1) and (2)

$a = b = c$

$T_1 = \frac{a+4b+c}{3} = 2a; d = \frac{a-8b+c}{10} = \frac{-6a}{10} = \frac{-3}{5}a$

$\therefore S_{20} = \frac{20}{2}\left[4a + 19\left(-\frac{3}{5}a\right)\right]$

$= 10\left[\frac{20a-57a}{5}\right]$

$= -74a$

$\therefore -74a = -444 \Rightarrow a = 6$

$\therefore abc = 6^3 = 216$

Concepts:

1. Sequence and Series:

Sequence: Sequence and Series is one of the most important concepts in Arithmetic. A sequence refers to the collection of elements that can be repeated in any sort.

Eg: a_1, a_2, a_3, a_4.....

Series: A series can be referred to as the sum of all the elements available in the sequence. One of the most common examples of a sequence and series would be Arithmetic Progression.

Eg: If a_1, a_2, a_3, a_4..... etc is considered to be a sequence, then the sum of terms in the sequence $a_1+a_2+a_3+ a_4$..... are considered to be a series.

Types of Sequence and Series:

Arithmetic Sequences

A sequence in which every term is created by adding or subtracting a definite number to the preceding number is an arithmetic sequence.

Geometric Sequences

A sequence in which every term is obtained by multiplying or dividing a definite number with the preceding number is known as a geometric sequence.

Harmonic Sequences

A series of numbers is said to be in harmonic sequence if the reciprocals of all the elements of the sequence form an arithmetic sequence.

Fibonacci Numbers

Fibonacci numbers form an interesting sequence of numbers in which each element is obtained by adding two preceding elements and the sequence starts with 0 and 1. Sequence is defined as, $F_0 = 0$ and $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$

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5. Answer: 8 - 8

Explanation:

The correct answer is 8.

$2a_5 = a_5(\text{ given })$

$2(a_1 + 6d) = a_1 + 4d$

$a_1 + 8d = 0$....(1)

$a_1 + 10d = 18$...(2)

By (1) and (2) we get $a_1 = -72, d = 9$

$a_{18} = a_1 + 17d = -72 + 153 = 81$

$a_{10} = a_1 + 9d = 9$

$12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d} + \ldots\ldots \frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}\right)$

$12\left(\frac{\sqrt{a_{18}}-\sqrt{a_{10}}}{d}\right) = \frac{12(9-3)}{9} = \frac{12\times 6}{6} = 8$

Concepts:

1. Arithmetic Progression:

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For example, the natural number sequence 1, 2, 3, 4, 5, 6,... is an AP because the difference between two consecutive terms (say 1 and 2) is equal to one (2 -1). Even when dealing with odd and even numbers, the common difference between two consecutive words will be equal to 2.

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Read More: Sum of First N Terms of an AP

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6. Answer: 37 - 37

Explanation:

The correct answer is 37.

$\frac{x_1+x_2+\ldots+x_7}{7} = 8$

$\frac{x_1+x_2+x_3\ldots+x_6+14}{7} = 8$

$\Rightarrow x_1 + x_2 + \ldots. + x_6 = 42$

$\therefore \frac{x_1+x_2\ldots.+x_6}{6} = \frac{42}{6} = 7 = a$

$\frac{\Sigma x_i^2}{7} - 8^2 = 16$

$\Sigma xi^2 = 560$

$\Rightarrow x_1^2 + x_2^2 + \ldots + x_6^2 = 364$

$b = \frac{x_1^2+x_2^2+\ldots.+x_6^2}{6} - 7^2$

$= \frac{364}{6} - 49$

$b = \frac{70}{6}$

$a + 3b - 5 = 7 + 3 \times \frac{70}{6} - 5$

$= 37$

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7. **Answer: 495 - 495**

Explanation:

The correct answer is 495.

$$\begin{vmatrix} A+6d & 7 & 1 \\ 2(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix} + 70 = 0$$

$\Rightarrow A = -7$ and $d = 6$

$\therefore c - a - b = 20$

$S_{20} = 495$

Concepts:

1. **Sequences:**

A set of numbers that have been arranged or sorted in a definite order is called a sequence. The terms in a series mention the numbers in the sequence, and each term is distinguished or prominent from the others by a common difference. The end of the sequence is frequently represented by three linked dots, which specifies that the sequence is not broken and that it will continue further.

Read More: Sequence and Series

Types of Sequence:

There are four types of sequences such as:

- Arithmetic Sequence
- Fibonacci Sequence
- Geometric Sequence
- Harmonic Sequence

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8. **Answer: b**

Explanation:

As a^3, b^3, c^3 be in A.P. $\rightarrow a^3 + c^3 = 2b^3$.... (1)

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$\therefore (\log a)^3 = (\log c)^3 \Rightarrow a = c$.....(2)

From (1) and (2)

$a = b = c$

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$\therefore S_{20} = \frac{20}{2}\left[4a + 19\left(-\frac{3}{5}a\right)\right]$

$= 10\left[\frac{20a-57a}{5}\right]$

$= -74a$

$\therefore -74a = -444 \Rightarrow a = 6$

$\therefore abc = 6^3 = 216$

So, the correct option is (B) : 216

Concepts:

1. Arithmetic Progression:

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Read More: Sum of First N Terms of an AP

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9. Answer: d

Explanation:

The correct option is (D) : -460

$S = 2[2^2 - 3^2 + 4^2.....20^2 - 21^2]$

$= 2[(2^2 + 4^2 + 6^2 + ... + 20^2) - (3^2 + 5^2 + ... + 21^2)]$

$= 2[2(2^2 + 4^2 + 6^2 + ... + 20^2) - (2^2 + 3^2 + 4^2 + ... + 21^2)]$

$= 2[2^3(1^2 + 2^2 + 3^2 + ... + 10^2) - (2^2 + 3^2 + 4^2 + . + 21^2)]$

$= 2[\frac{8\times10\times11\times21}{6} - \frac{21\times22\times43}{6} + 1]$

$= 2[3080 - 3311 + 1]$

$= 2[-230]$

$= -460$

Concepts:

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Read More: Sum of First N Terms of an AP

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10. Answer: c

Explanation:

The correct option is (C): 9525

= 3 + 8 + 13 + 18 + ... 373

$= \frac{75}{2}[3 + 373] = 14100$

Now , $\underbrace{3 + 18 + \ldots}_{25 \text{ terms}}$

$= \frac{25}{2}[6 + 24.15] = 4575$

∴ Required sum = 14100 - 4575 = 9525

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