Sequence And Series JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Sequence And Series

1. If (21)18 + 20·(21)17 + (20)2 · (21)16 + ... (20)18 = k (2119 − 2019) then k = **(+4, -1)**

[Jan. 11, 2019 (II)]

a. $\frac{21}{20}$

b. 1

c. $\frac{20}{21}$

d. 0

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2. If an be the th term of the GP of positive numbers. Let ∑100 = 100 2 = and ∑100 = 1 2 − 1 = such that ≠ then the common ratio is: **(+4, -1)**

[30-Jan-2024 Shift 2]

a. (A) –

b. (B) –

c. (C) √–

d. (D) √–

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3. For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}, 10$ and $\frac{1}{b}$ are in an arithmetic progression, then $16a + 12b$ is equal to **(+4, -1)**

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4. The 4^{th} term of GP is 500 and its common ratio is $\frac{1}{m}, m \in N$ Let S_n denote the sum of the first n terms of this GP If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is **(+4, -1)**

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5. If the sum and product of four positive consecutive terms of a GP, are 126 and 1296, respectively, then the sum of common ratios of all such GPs is **(+4, -1)**

[28-Jun-2022-Shift-1]

a. $\frac{9}{2}$

b. 3

c. 7

d. 14

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6. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots\ldots\ldots\ldots + 10(11)^9 = k(10)^9$, then k is equal to (+4, -1)

[2014]

a. $\frac{121}{10}$

b. $\frac{441}{100}$

c. 100

d. 110

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7. Three positive numbers form an increasing $G.P.$ If the middle term in this $G.P.$ is doubled, then new numbers are in $A.P.$ Then, the common ratio of the $G.P.$ is (+4, -1)

[2014]

a. $\sqrt{2} + \sqrt{3}$

b. $3 + \sqrt{2}$

c. $2 - \sqrt{3}$

d. $2 + \sqrt{3}$

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8. The sum of first 20 terms of the sequence $0.7, 0.77, 0.777, \ldots.,$ is (+4, -1)

[2013]

a. $\frac{7}{81}(179 - 10^{-20})$

b. $\frac{7}{9}(99 - 10^{-20})$

c. $\frac{7}{81}(179 + 10^{-20})$

d. $\frac{7}{9}(99 + 10^{-20})$

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9. If the $2^{nd}, 5^{th}$ and 9^{th} terms of a non-constant $A.P.$ are in $G.P.$, then the common ratio of this $G.P.$ is : (+4, -1)

[2016]

a. $\frac{8}{5}$

b. $\frac{4}{3}$

c. 1

d. $\frac{7}{4}$

10. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots.,$ is $\frac{16}{5}m$, then m is equal to : **(+4, -1)**

[2016]

a. 102

b. 101

c. 100

d. 99

Answers

1. Answer: b

Explanation:

The correct answer is option (B): 1

$a = (21)^{18}.r = \frac{20}{21}, n = 19$

$S = (21)^{18} = \frac{(1-(\frac{20}{21})^{19})}{1-\frac{20}{21}}$

$\Rightarrow \frac{(21)^{19}}{(21)^{19}}(21^{19} - 20^{19})$

$\Rightarrow (21^{19} - 20^{19}) = 1$

Concepts:

1. Arithmetic Progression:

Arithmetic Progression (AP) is a mathematical series in which the difference between any two subsequent numbers is a fixed value.

For example, the natural number sequence 1, 2, 3, 4, 5, 6,... is an AP because the difference between two consecutive terms (say 1 and 2) is equal to one (2 -1). Even when dealing with odd and even numbers, the common difference between two consecutive words will be equal to 2.

In simpler words, an arithmetic progression is a collection of integers where each term is resulted by adding a fixed number to the preceding term apart from the first term.

For eg:- 4,6,8,10,12,14,16

We can notice Arithmetic Progression in our day-to-day lives too, for eg:- the number of days in a week, stacking chairs, etc.

Read More: Sum of First N Terms of an AP

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2. Answer: a

Explanation:

Explanation:
Given:A geometric progression such that th term is and $\sum^{100}_{=1}$ 2 = , $\sum^{100}_{=1}$ 2 −1 = We have to find the common ratio of given GP.Let a be the first term, r be the common ratio of the given GP (> 0).Then a, ar, ar^2, ar^3, … . , ar^{199} are in GP.We shall use the formula of general term of GP. = $\sum^{100}_{=1}$ 2 = 2 + 4 + … … . + 200 = + 3 + … … . . + 199 = ar(1 + 2 + … … + 198)and = $\sum^{100}_{=1}$ 2 −1 = 1 + 3 + … … … + 199 = + 2 + … … . . + ar^{198} = (1 + 2 + … … . + 198)Clearly, a / = = common ratioHence, the correct option is (A).

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3. **Answer: 3 - 3**

Explanation:

The correct answer is 3.

$a, b, \frac{1}{18} \to GP$

$\frac{a}{18} = b^2$....(i)

$\frac{1}{a}, 10, \frac{1}{b} \to AP$

$\frac{1}{a} + \frac{1}{b} = 20$

$\Rightarrow a + b = 20ab$, from eq. (i); we get

$\Rightarrow 18b^2 + b = 360b^3$

$\Rightarrow 360b^2 - 18b - 1 = 0\{\because b \neq 0\}$

$\Rightarrow b = \frac{18 \pm \sqrt{324+1440}}{720}$

$\Rightarrow b = \frac{18+\sqrt{1764}}{720}\{\because b > 0\}$

$\Rightarrow b = \frac{1}{12}$

$\Rightarrow a = 18 \times \frac{1}{144} = \frac{1}{8}$

Now, $16a + 12b = 16 \times \frac{1}{8} + 12 \times \frac{1}{12} = 3$

Concepts:

1. **Geometric Progression:**

What is Geometric Sequence?

A geometric progression is the sequence, in which each term is varied by another by a common ratio. The next term of the sequence is produced when we multiply a constant to the previous term. It is represented by: a, ar^1, ar^2, ar^3, ar^4, and so on.

Properties of Geometric Progression (GP)

Important properties of GP are as follows:

- Three non-zero terms a, b, c are in GP if $b^2 = ac$
- In a GP,
  Three consecutive terms are as a/r, a, ar
  Four consecutive terms are as a/r3, a/r, ar, ar^3
- In a finite GP, the product of the terms equidistant from the beginning and the end term is the same that means, $t_1.tn = t_2.t_{n-1} = t_3.t_{n-2} = ...$
- If each term of a GP is multiplied or divided by a non-zero constant, then the resulting sequence is also a GP with a common ratio
- The product and quotient of two GP's is again a GP
- If each term of a GP is raised to power by the same non-zero quantity, the resultant sequence is also a GP.

If $a_1, a_2, a_3,...$ is a GP of positive terms then log a_1, log a_2, log a_3,... is an AP (arithmetic progression) and vice versa

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4. **Answer: 12 - 12**

Explanation:

The correct answer is 12.

$T_4 = 500$ where $a =$ first term,
$r = \text{ common ratio } = \frac{1}{m}, m \in N$
$ar^3 = 500$
$\frac{a}{m^3} = 500$
$S_n - S_{n-1} = ar^{n-1}$
$S_6 > S_5 + 1$
and $S_7 - S_6 < \frac{1}{2}$
$S_6 - S_5 > 1 \quad \frac{a}{m^6} < \frac{1}{2}$
$ar^5 > 1 \quad m^3 > 10^3$
$\frac{500}{m^2} > 1 \quad m > 10$...(2)
$m^2 < 500$.........(1)
From (1) and (2)
$m = 11, 12, 13 \ldots \ldots \ldots \ldots, 22$
So number of possible values of m is 12

Concepts:

1. **Sequences:**

A set of numbers that have been arranged or sorted in a definite order is called a sequence. The terms in a series mention the numbers in the sequence, and each term is distinguished or prominent from the others by a common difference. The end of the sequence is frequently represented by three linked dots, which specifies that the sequence is not broken and that it will continue further.

Read More: Sequence and Series

Types of Sequence:

There are four types of sequences such as:

- Arithmetic Sequence
- Fibonacci Sequence
- Geometric Sequence
- Harmonic Sequence

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5. **Answer: c**

Explanation:

$a, ar, ar^2, ar^3 (a, r > 0)$

$a^4r^6 = 1296$

$a^2r^3 = 36$

$a = \frac{6}{r^{3/2}}$

$a + ar + ar^2 + ar^3 = 126$

$\frac{1}{r^{3/2}} + \frac{r}{r^{3/2}} + \frac{r^2}{r^{3/2}} + \frac{r^3}{r^{3/2}} = \frac{126}{6} = 21$

$\left(r^{-3/2} + r^{3/2}\right) + \left(r^{1/2} + r^{-1/2}\right) = 21$

$r^{1/2} + r^{-1/2} = A$

$r^{-3/2} + r^{3/2} + 3A = A^3$

$A^3 - 3A + A = 21$

$A^3 - 2A = 21$

$A = 3$

$\sqrt{r} + \frac{1}{\sqrt{r}} = 3$

$r + 1 = 3\sqrt{r}$

$r^2 + 2r + 1 = 9r$

$r^2 - 7r + 1 = 0$

Concepts:

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What is Geometric Sequence?

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6. Answer: c

Explanation:

$$10^9 + 2 \cdot (11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$$
$$x = 10^9 + 2 \cdot (11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9$$
$$\frac{11}{10}x = 11 \cdot 10^8 + 2 \cdot (11)^2 \cdot (10)^7 + \ldots + 9(11)^9 + 11^{10}$$

$$x\left(1 - \frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2 \times (10)^7 + \ldots + 11^9 - 11^{10}$$
$$\Rightarrow -\frac{x}{10} = 10^9\left(\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1}\right) - 11^{10}$$
$$\Rightarrow -\frac{x}{10} = \left(11^{10} - 10^{10}\right) - 11^{10} = -10^{10}$$

$\Rightarrow x = 10^{11} = k \cdot 10^9$
$\Rightarrow k = 100$

Concepts:

1. Sequence and Series:

Sequence: Sequence and Series is one of the most important concepts in Arithmetic. A sequence refers to the collection of elements that can be repeated in any sort.

Eg: a_1, a_2, a_3, a_4.......

Series: A series can be referred to as the sum of all the elements available in the sequence. One of the most common examples of a sequence and series would be Arithmetic Progression.

Eg: If a_1, a_2, a_3, a_4....... etc is considered to be a sequence, then the sum of terms in the sequence $a_1+a_2+a_3+a_4$....... are considered to be a series.

Types of Sequence and Series:

Arithmetic Sequences

A sequence in which every term is created by adding or subtracting a definite number to the preceding number is an arithmetic sequence.

Geometric Sequences

A sequence in which every term is obtained by multiplying or dividing a definite number with the preceding number is known as a geometric sequence.

Harmonic Sequences

A series of numbers is said to be in harmonic sequence if the reciprocals of all the elements of the sequence form an arithmetic sequence.

Fibonacci Numbers

Fibonacci numbers form an interesting sequence of numbers in which each element is obtained by adding two preceding elements and the sequence starts with 0 and 1. Sequence is defined as, $F_0 = 0$ and $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$

7. **Answer: d**

Explanation:

$a, ar, ar^2 \rightarrow G.P.$
$a, 2ar, ar^2 \rightarrow A.P.$
$2 \times 2ar = a + ar^2$
$4r = 1 + r^2$
$\Rightarrow r^2 - 4r + 1 = 0$
$r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$
$r = 2 + \sqrt{3}$
$r = 2 - \sqrt{3}$ is rejected
$\because (r > 1)$
G.P. is increasing.

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8. Answer: c

Explanation:

Let S = 0.7 + 0.77 + 0.777 +...

$$= \frac{7}{10} + \frac{77}{10^2} + \frac{777}{10^3} + ...upto\ 20\ terms$$

$$= 7\left[\frac{1}{10} + \frac{11}{10^2} + \frac{111}{10^3} + ...upto\ 20\ terms\right]$$

$$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...upto\ 20\ terms\right]$$

$$= \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right)\right.$$

$\hspace70mm +...+ upto\ 20\ terms] $

$$= \frac{7}{9}[(1 + 1 + ... +\ upto\ 20\ terms)$$

$\hspace30mm -\bigg(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3}+...+ upto\ 20\ terms \bigg) \Bigg]$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left\{1 - \left(\frac{1}{10}\right)^{20}\right\}}{1 - \frac{1}{10}}\right]$$

$ \hspace30mm \Bigg[\begin{array}
\ \because \displaystyle\sum_{ i = 1}^{ 20} = 20\ and\ sum\ of\ n\ terms\ of \\
GP, S_n = \frac{a (1 - r^n)}{1 - r } when \ (r < 1) \\
\end{array} \Bigg] $

$$= \frac{7}{9}\left[20 - \frac{1}{9}\left\{1 - \left(\frac{1}{10}\right)^{20}\right\}\right]$$

$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}[179 + (10)^{-20}]$$

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9. **Answer: b**

Explanation:

$t_2 = a + d$
$t_5 = a + 4d$
$t_9 = a + 8d$
Given t_2, t_5, t_9 are in $G.P.$
$(a+4d)^2 = (a+d)(A+8d)$
$a^2 + 16d^2 + 8ad = a^2 + 8d^2 + 9ad$
$8d^2 - ad = 0$
$d(8d - a) = 0$
As given non - constant $AP.$
$\Rightarrow\ d \neq 0$
$\therefore\ d = \frac{a}{8}$
$\Rightarrow a = 8d$
so, $A.P$ is $8d, 9d, 10d$,....
Common ratio of $G.P. = \frac{t_5}{t_2} = \frac{a+4d}{a+d} = \frac{12d}{9d} = \frac{4}{3}$

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10. **Answer: b**

Explanation:

$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots$.. upto 10 terms

$= \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \left(\frac{24}{5}\right)^2 + \ldots$ upto 10 terms.

$(8)^2 + (12)^2 + (16)^2 + \ldots$ up to 10 terms

$T_n\ [4(n+1)]^2$ where n varies from 1 to 10.

$= 16\left(n^2 + 2n + 1\right)$

$$\sum T_n = \sum_{n=1}^{10} 16\left(n^2 + 2n + 1\right)$$

$= 16\,[385 + 55\,(2) + 10]$

$= 16\,(505)$

or

$$\sum_{n=1}^{10} n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10 \times 11 \times 21}{6} = 385$$

$$\sum_{n=1}^{10} n = \frac{n(n+1)}{2} = \frac{10 \times 11}{2} = 55$$

$$\sum_{n=1}^{10} 1 = n = 10$$

$\therefore \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \ldots$.. upto 10 terms $= \frac{16 \times 505}{25}$

It is given that $\frac{16 \times 505}{25} = \frac{16}{5} m$

$\therefore m = \frac{505}{5} = 101$

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