Sequence And Series JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Sequence And Series

1. The sum of the common terms of the following three arithmetic progressions $3, 7, 11, 15, \ldots, 399$, $2, 5, 8, 11, \ldots, 359$ and $2, 7, 12, 17, \ldots, 197$, is equal to _____ **(+4, -1)**

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2. The 8^{th} common term of the series **(+4, -1)** [1-Feb-2023 Shift 2]
$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots\ldots$
$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots$ is ______

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3. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then : **(+4, -1)**
[2017]

a. b, c and a are in $A.P$

b. a, b and c are in $A.P$

c. a, b and c are in $G.P$

d. b, c and a are in $G.P$

---

4. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy, \forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to : **(+4, -1)**
[Online April 15, 2018]

a. 165

b. 190

c. 255

d. 330

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5. Given an $A.P.$ whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4^{th} term is : **(+4, -1)**
[Online April 9, 2014]

a. 8

b. 16

c. 20

d. 24

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6. If b is the first term of an infinite $G.P.$ whose sum is five, then b lies in the interval : (+4, -1)

[Online April 15, 2018]

a. $(-\infty, -10]$

b. $(-10, 0)$

c. $(0, 10)$

d. $[10, \infty)$

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7. If the sum of the series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \ldots$ upto n^{th} term is 488 and the n^{th} term is negative, then : (+4, -1)

[Sep. 03, 2020 (II)]

a. n^{th} term is $-4\frac{2}{5}$

b. $n = 41$

c. n^{th} term is -4

d. $n = 60$

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8. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? (+4, -1)

[April 08, 2019 (II)]

a. d, e, f are in $A.P$.

b. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in $G.P.$

c. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in $A.P.$

d. d, e, f are in $G.P.$

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9. Let $\frac{1}{x_1}, \frac{1}{x^2}, ..., \frac{1}{x_n} (x_1 \neq 0$ for $i = 1, 2, ..., n)$ be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n} \left(\frac{1}{x_i}\right)$ is equal to **(+4, -1)**

[Online April 16, 2018]

a. $\frac{1}{8}$

b. 3

c. $\frac{13}{8}$

d. $\frac{13}{4}$

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10. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals : **(+4, -1)**

[Online April 15, 2018]

a. $2(5^2)$

b. $4(5^2)$

c. 5^4

d. 5^3

Answers

1. **Answer: 321 - 321**

Explanation:

The correct answer is 321

$3, 7, 11, 15, \ldots\ldots\ldots\ldots, 399 \quad d_1 = 4$

$2, 5, 8, 11, \ldots\ldots\ldots\ldots, 359 \quad d_2 = 3$

$2, 7, 12, 17, \ldots\ldots, 197 \quad d_3 = 5$

$\text{LCM}\,(d_1, d_2, d_3) = 60$

$\text{Common terms are } 47, 107, 167$

$\text{Sum } = 321$

Concepts:

1. Arithmetic Progression:

Arithmetic Progression (AP) is a mathematical series in which the difference between any two subsequent numbers is a fixed value.

For example, the **natural number** sequence 1, 2, 3, 4, 5, 6,... is an AP because the difference between two consecutive terms (say 1 and 2) is equal to one (2 -1). Even when dealing with odd and even numbers, the common difference between two consecutive words will be equal to 2.

In simpler words, an arithmetic progression is a collection of integers where each term is resulted by adding a fixed number to the preceding term apart from the first term.

For eg:- 4,6,8,10,12,14,16

We can notice Arithmetic Progression in our day-to-day lives too, for eg:- the number of days in a week, stacking chairs, etc.

Read More: Sum of First N Terms of an AP

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2. **Answer: 151 - 151**

Explanation:

$$T_8 = 11 + (8-1) \times 20$$
$$= 11 + 140 = 151$$

So, the correct answer is 151.

Concepts:

1. Sequence and Series:

Sequence: Sequence and Series is one of the most important concepts in Arithmetic. A sequence refers to the collection of elements that can be repeated in any sort.

Eg: a_1, a_2, a_3, a_4.......

Series: A series can be referred to as the sum of all the elements available in the sequence. One of the most common examples of a sequence and series would be Arithmetic Progression.

Eg: If a_1, a_2, a_3, a_4....... etc is considered to be a sequence, then the sum of terms in the sequence $a_1+a_2+a_3+a_4$....... are considered to be a series.

Types of Sequence and Series:

Arithmetic Sequences

A sequence in which every term is created by adding or subtracting a definite number to the preceding number is an arithmetic sequence.

Geometric Sequences

A sequence in which every term is obtained by multiplying or dividing a definite number with the preceding number is known as a geometric sequence.

Harmonic Sequences

A series of numbers is said to be in harmonic sequence if the reciprocals of all the elements of the sequence form an arithmetic sequence.

Fibonacci Numbers

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3. Answer: a

Explanation:

$9\left(25a^2+b^2\right)+25\left(c^2+3ac\right)=15b\left(3a+c\right)$
$\Rightarrow (15a)^2+(3b)^2+(5c)^2-45ab-15bc-75ac=0$
$\Rightarrow (15a-3b)^2+(3b-5c)^2+(15a-5c)^2=0$
It is possible when
$15a-3b=0$ and $3b-5c=0$ and $15a-5c=0$
$15a=3b=5c$
$\frac{a}{1}=\frac{b}{5}=\frac{c}{3}$
∴ b, c, a are in A.P.

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4. Answer: d

Explanation:

As, $f(x+y) = f(x) + f(y) + xy$
Given, $f(1) = 3$
Putting, $x = y = 1$
$\Rightarrow f(2) = 2f(1) + 1 = 7$
Similarly, $x = 1, y = 2$
$\Rightarrow f(3) = f(1) + f(2) + 2 = 12$
Now, $\sum_{n=1}^{10} f(n) = f(1) + f(2) + f(3) + \ldots + f(10)$
$= 3 + 7 + 12 + 18 + \ldots = S$ (let)
Now, S_n = 3 + 7 + 12 + 18 + ... + t_n
Again, S_n = 3 + 7 + 12 + ... + $t_{n-1} + t_n$
We ge, t_n = 3 + 4 + 5+ ... n terms
$= \frac{n(n+5)}{2}$
i.e., $S_n = \sum_{n=1}^{n} t_n = \frac{1}{2}\left\{\sum n^2 + 5\sum n\right\}$
$= \frac{n(n+1)(n+8)}{6}$
So, $S_{10} = \frac{10\times11\times18}{6} = 330$

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5. **Answer: c**

Explanation:

$(12-d)+12+(12+d)+(12+2d)+\ldots.12+7d$

$=12?9+27d=108+27d$

now according to question

$200<108+27d<220$

$92<27d<112$

$\frac{92}{27}<d<\frac{112}{27}$

$\Rightarrow d=4$ only integer

$\Rightarrow$ 4th term = $12+2d=12+8=20$

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6. **Answer: c**

Explanation:

If b is the first term and r is the common ratio of an infinite G.P. then sum is 5

$5 = \frac{b}{1-r}]$

$1 - r = \frac{b}{5}$

$r = 1 - \frac{b}{5}$

$r = \frac{5-b}{5}$

? -1 < r < 1

$\therefore-1

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7. Answer: c

Explanation:

$$S = \frac{100}{5} + \frac{98}{5} + \frac{96}{5} + \frac{94}{5} + \ldots n$$
$$S_n = \frac{n}{2}\left(2 \times \frac{100}{5} + (n-1)\left(-\frac{2}{5}\right)\right) = 188$$
$$n(100 - n + 1) = 488 \times 5$$
$$n^2 - 101n + 488 \times 5 = 0$$
$$n = 61, 40$$
$$T_n = a + (n-1)d = \frac{100}{5} - \frac{2}{5} \times 60$$
$$= 20 - 24 = -4$$

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8. **Answer: c**

Explanation:

$b^2 = ac$

Also roots of $ax^2 = 2bx + c = 0$ are equal

$\Rightarrow x = \frac{-b}{a}$, common root

$\Rightarrow d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + \int = 0$

$db^2 - 2eab + fa^2 = 0, b^2 = ac$

$\Rightarrow dac - 2eab + fa^2 = 0$

$\Rightarrow dc - 7eb + fa = 0$

Dividing by ac

$\Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\Rightarrow \frac{d}{a} + \frac{f}{c} 2.\frac{e}{b}$

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9. **Answer: d**

Explanation:

Given:
$AP : \frac{1}{x_1}, \frac{1}{x_2}, \ldots \frac{1}{x_n}$
And $x_1 = 4, x_{21} = 20$
So, $\frac{1}{4} + 20d = \frac{1}{20}$
$20d = \frac{1}{20} - \frac{1}{4} = \frac{1-5}{20}$
$\Rightarrow 20d = \frac{-4}{20}$
$\Rightarrow d = \frac{-4}{20\times2}$
$\Rightarrow d = \frac{-1}{100}$
Now, $\frac{1}{x_n} 24$
$n = 25$
Therefore, $\sum_{i=1}^{25}\left(\frac{1}{x_i}\right) = \frac{25}{2}\left(2 \times \frac{1}{4} - \frac{1}{100} \times 24\right)$
$= \frac{13}{4}$

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10. **Answer: c**

Explanation:

$a_1, a_2, ..., a_{10}$ are in $G.P.$,
Let the common ratio be r

$\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1 r^2}{a_1} = 25 \Rightarrow r^2 = 25$
$\frac{a_9}{a_5} = \frac{a_1 r^8}{a_1 r^4} = r^4 = 5^4$

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