VITEEE 2021 Solutions May 28 - Slot 1

Ques - A point is chosen randomly inside the circle of radius r. Let x be the distance of the point from the center of the circle. Then the equation of the random variable is given by? Question - The length of the axis of the conic 25x2+4y2 -10x+4y+1=0 are: A) ⅖ B) ⅘ C) ½, ⅖ D) ½, ⅕

Solution.
1. **For the point inside the circle:**
A point chosen randomly inside the circle of radius r can be anywhere from the center (distance $x = 0$) to the circumference (distance $x = r$). Thus, the range of the random variable x representing the distance from the center is:
$$0 \leq x \leq r$$

2. **For the conic section:**
The given equation of the conic is:
$$25x^2 + 4y^2 - 10x + 4y + 1 = 0$$

To find the length of the axes of the ellipse, we first need to rewrite this equation in standard form.

Group the x terms and the y terms together:
$$25(x^2 - \frac{2}{5}x) + 4(y^2 + y) = -1$$

To complete the square:
For x terms, the square completion term is $\left(\frac{1}{5}\right)^2 = \frac{1}{25}$

For y terms, the square completion term is $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Adding and subtracting these values:
$$25\left(x^2 - \frac{2}{5}x + \frac{1}{25}\right) + 4\left(y^2 + y + \frac{1}{4}\right) = -1 + 1 + 1$$
$$25\left(x - \frac{1}{5}\right)^2 + 4(y + \frac{1}{2})^2 = 1$$

Now, divide by 1 to get the standard form:
$$25\left(x - \frac{1}{5}\right)^2 + 4(y + \frac{1}{2})^2 = 1$$

For the ellipse in the form:
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

The lengths of the major and minor axes are $2a$ and $2b$, respectively.

From our equation:
$$a^2 = \frac{1}{25} \implies a = \frac{1}{5}$$
$$b^2 = \frac{1}{4} \implies b = \frac{1}{2}$$

So, the lengths of the axes are $2a = \frac{2}{5}$ and $2b = 1$.

The correct answer seems to be a mix of the options provided, which might indicate an error in the question or options. However, the lengths derived here are $\frac{2}{5}$ and 1.

Ques - When we push a wooden crate on the concrete floor, then which of the following statements is true?
A) The static friction in this case is more than the kinetic friction
B) It is easier to push the object on a smooth surface than on a rough surface to get it moving.
C) If we keep a heavy weight on the wooden crate we can get it moving easily as compared to when there is no block over it.
D) We need more force to get the crate to move initially compared to keep it moving.

Solution.
Let's analyze each statement:

A) **The static friction in this case is more than the kinetic friction**:
True. Static friction is the friction that keeps the crate from starting to move. Once the crate starts moving, the friction opposing its motion is called kinetic (or sliding) friction. Typically, static friction is higher than kinetic friction. That's why more force is usually required to initiate motion than to maintain it.

B) **It is easier to push the object on a smooth surface than on a rough surface to get it moving**:
True. A smoother surface has less resistance or frictional force compared to a rough surface. As a result, it requires less force to get an object moving on a smooth surface than on a rough one.

C) **If we keep a heavy weight on the wooden crate we can get it moving easily as compared to when there is no block over it**:
False. If you add a heavy weight on the crate, it increases the normal force. Since frictional force is directly proportional to the normal force (friction = μ x normal force), the frictional force will also increase, making it harder to move the crate.

D) **We need more force to get the crate to move initially compared to keep it moving**:
True. As mentioned in option A, static friction (the friction when the crate is at rest) is generally greater than kinetic friction (the friction when the crate is moving). So, more force is required to overcome static friction and start the motion than is needed to overcome kinetic friction and maintain the motion.

Based on the above analysis, the true statements are:
A, B, and D.

Ques - Let P and Q be matrices of size 4X6 and 4X1, respectively which of the following is correct for the system of linear equations Px=Q? A) If the system is consistent then it has infinitely many

solutions. B) If Q=0 then the system is inconsistent. C) If Q≠0 and the system is consistent, then the rank of P must be 6. D) If Q=0, then the system has a unique solution

Solution.
Let's analyze the given system of linear equations $Px = Q$ where P is a 4x6 matrix and Q is a 4x1 matrix.

For the matrix multiplication $Px = Q$ to be defined, x must be a 6x1 matrix (or column vector).

Now, let's evaluate the options:

A) **If the system is consistent then it has infinitely many solutions**:
True. Since the matrix P is a 4x6 matrix, this means there are 4 equations and 6 unknowns. If the system is consistent, then there are more unknowns than equations, so there will be infinitely many solutions.

B) **If $Q = 0$ then the system is inconsistent**:
False. Just because Q is the zero matrix doesn't make the system inconsistent. In fact, if $Q = 0$, it's a homogeneous system, which always has at least the trivial solution ($x = 0$).

C) **If $Q \neq 0$ and the system is consistent, then the rank of P must be 6**:
False. The maximum rank that matrix P can have (being a 4x6 matrix) is 4. The rank cannot exceed the smaller of the two dimensions of the matrix.

D) **If $Q = 0$, then the system has a unique solution**:
False. As mentioned earlier, if $Q = 0$, it's a homogeneous system and will always have the trivial solution ($x = 0$). But because there are more unknowns (6) than equations (4), there will also be non-trivial solutions, making the number of solutions infinite.

Thus, out of the given options, only **A** is correct.

Ques - In which one of the following cases the Rolles Theorem is not applicable? A) f(x) = [x] in [2.5, 2.7] B) f(x) = x2 – 4x + 5 in [1,2] C) f(x) = |x| in [-2,2]

Solution.

To apply Rolle's Theorem, a function must satisfy three conditions on a closed interval $[a, b]$:

1. $f(x)$ must be continuous on $[a, b]$.
2. $f(x)$ must be differentiable on the open interval (a, b).
3. $f(a) = f(b)$.

Let's evaluate the options:

A) $f(x) = [x]$ in $[2.5, 2.7]$

$[f(x)]$ represents the greatest integer function (floor function). In the interval $[2.5, 2.7]$, $f(x)$ has a constant value of 2, which is continuous and satisfies $f(2.5) = f(2.7)$. However, the function is not differentiable at integer points, including at $x = 2.5$ and $x = 2.7$. Thus, Rolle's Theorem is not applicable.

B) $f(x) = x^2 - 4x + 5$ in $[1,2]$

This is a polynomial function, so it's continuous everywhere and differentiable everywhere. Moreover, $f(1) = 2$ and $f(2) = 1$, so $f(a) \neq f(b)$. Rolle's Theorem is not applicable due to this condition.

C) $f(x) = |x|$ in $[-2,2]$

The absolute value function is continuous everywhere. However, it is not differentiable at $x = 0$, which lies inside the interval $[-2,2]$. Moreover, $f(-2) = f(2) = 2$, so the third condition is satisfied. But due to its non-differentiability at $x = 0$, Rolle's Theorem is not applicable.

Out of the given options, Rolle's Theorem is not applicable for all: **A, B, and C**.

Ques - If z1, z2, z3 are the vertices of the equilateral triangle and the z0 be its orthocentre, such that z12+z22+z32 = Kz02, then K equals
A) 6
B) 2
C) 9
D) 3

Solution.
Let's first understand the relationship between the vertices of an equilateral triangle and its orthocenter.

For any equilateral triangle, the centroid, circumcenter, incenter, and orthocenter all coincide. In other words, for an equilateral triangle, the orthocenter is also its centroid.

Given that $z_1, z_2,$ and z_3 are the vertices of the equilateral triangle, the centroid (which is also the orthocenter z_0) is given by:
$$z_0 = \frac{z_1 + z_2 + z_3}{3}$$

Squaring both sides:
$$z_0^2 = \frac{z_1^2 + z_2^2 + z_3^2 + 2(z_1z_2 + z_2z_3 + z_3z_1)}{9}$$

Given:
$$z_1^2 + z_2^2 + z_3^2 = Kz_0^2$$

Substituting for z_0^2 from the above equation:
$$z_1^2 + z_2^2 + z_3^2 = K \cdot \frac{z_1^2 + z_2^2 + z_3^2 + 2(z_1z_2 + z_2z_3 + z_3z_1)}{9}$$

Simplifying:
$$9 = K(3 + 2[z_1z_2 + z_2z_3 + z_3z_1]/[z_1^2 + z_2^2 + z_3^2])$$

Because $z_1, z_2,$ and z_3 are vertices of an equilateral triangle, the product of any two vertices will have the same magnitude. Therefore, $z_1z_2 + z_2z_3 + z_3z_1$ will have a certain magnitude (let's call it M), and this magnitude is divided by the sum of the squares of the vertices, which is again a fixed magnitude.

Thus, K will be a constant value. Comparing the equation to the given expression $z_1^2 + z_2^2 + z_3^2 = Kz_0^2$, we see that:
$$K = 3$$

So, the correct answer is:
D) 3.

Ques - Which of the following fluorides of oxygen do not exist?
A) XeF_4
B) XeF_6
C) XeF_3
D) XeF_2

Solution.
Oxygen doesn't form fluorides by itself. However, the noble gas xenon (Xe) can form a number of compounds with fluorine. Among the given options:

A) XeF_4: Xenon tetrafluoride is a known compound and does exist.

B) XeF_6: Xenon hexafluoride is also a known compound and exists.

C) XeF_3: Xenon trifluoride is not known to exist. The possible fluorides of xenon are XeF_2, XeF_4, and XeF_6.

D) XeF_2: Xenon difluoride is a known compound and does exist.

Therefore, the fluoride of xenon that does not exist is:
C) XeF_3.

Ques - Let f(x) = | |x| – 1|, then the point where f(x)is not differentiable, is / are?
A) 0
B)1

C) ±1
D) 0,±1

Solution.
Given the function:
$$f(x) = | |x| - 1|$$

Let's break down the function step-by-step:

1. For $|x|$:
This function has a cusp (sharp turn) at $x = 0$. So, it's not differentiable at $x = 0$.

2. For $|x| - 1$:
This function is just a transformation of the absolute value function shifted downward by 1 unit. This will create two potential points of non-differentiability at $x = 1$ and $x = -1$ due to the absolute value function having a cusp at these points.

3. For $| |x| - 1|$:
This outer absolute value further creates a cusp at any point where $|x| - 1 = 0$, which are the points $x = 1$ and $x = -1$.

Considering all the above observations, the function $f(x)$ is not differentiable at $x = 0, 1,$ and -1.

Hence, the correct option is:
D) 0,±1.

Ques - The metal ion present in hemoglobin is
A) Zn^{2+}
B) Fe^{2+}
C) Mg^{2+}
D) Mn^{2+}

Solution.
The metal ion present in hemoglobin is:

B) Fe2+ (iron)

Ques - Let G be a group such that (xy)2 = xy, □x, y ∈ G, then which of the following is true? A) xy = x, □x, y ∈ G
B) Xy = y, □x, x ∈ G
C) X2 = e, □x ∈ G
D) xy = yx, □x, y ∈ G

Solution.
Given:
$$ (xy)^2 = xy $$
$$ \implies xyxy = xy $$

Now, multiply both sides by the inverse of x on the left:
$$ \implies x^{-1}xyxy = x^{-1}xy $$
$$ \implies eyxy = y $$
$$ \implies yxy = y $$

Now, multiply both sides by the inverse of y on the right:
$$ \implies yxyy^{-1} = yy^{-1} $$
$$ \implies yx = y $$

Thus, we have:
$$ xyxy = xy $$
$$ yx = y $$
$$ \implies xy = yx $$

So, the answer is:

D) xy = yx, □x, y ∈ G.

Ques- Let a, b be elements of the group G. Assume that A has order 5 and a3b = ba3, then G is:
A) Both abelian and cyclic group
B) Non-abelian group
C) Cyclic group

D) Abelian group

Solution.
Given:
1. The order of a is 5. This means $a^5 = e$ (identity element) and $a^n \neq e$ for $n < 5$.
2. $a^3b = ba^3$.

From the given condition $a^3b = ba^3$, it is evident that $ab = ba$, which implies that the operation is commutative for these elements.

However, just because two elements commute doesn't mean that all elements of the group G commute with each other. So, we can't conclude that the group is abelian based on these two elements alone.

The given information doesn't provide any evidence that G is cyclic either.

Hence, the only conclusion we can make from the provided information is that these specific elements a and b commute, but it does not guarantee anything about the nature of the entire group G.

None of the provided options A, C, and D can be conclusively determined based on the given information. The closest possible answer would be:

B) Non-abelian group.

Ques - The distance of the line x+3 = y+4 = z+5 from the origin is:
A) √12
B) 2
C) √3
D) √2

Solution.
Given the line:
$$x + 3 = y + 4 = z + 5$$

To find a point on the line, set the common value of the three expressions to any number, say 0.

$$x + 3 = 0 \implies x = -3$$

$$y + 4 = 0 \implies y = -4$$

$$z + 5 = 0 \implies z = -5$$

So, one point on the line is $(-3, -4, -5)$.

Now, to find the direction ratios of the line, you can set the common value of the three expressions to another number, say 1.

$$x + 3 = 1 \implies x = -2$$

$$y + 4 = 1 \implies y = -3$$

$$z + 5 = 1 \implies z = -4$$

The direction ratios are obtained by subtracting the coordinates of the two points: (-2 + 3, -3 + 4, -4 + 5) = (1, 1, 1).

Now, the formula for the distance d of a point (x_1, y_1, z_1) from the line $\frac{x - x_2}{a} = \frac{y - y_2}{b} = \frac{z - z_2}{c}$ (where (x2, y2, z2) is a point on the line and (a, b, c) are its direction ratios) is:

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Where the line equation can be represented as $ax + by + cz + d = 0$.

For the given line:

$$x - y + z - 3 = 0$$

So, a = 1, b = -1, c = 1, and d = -3.

Using the point on the line $(-3, -4, -5)$:

$$d = \frac{|1(-3) - 1(-4) + 1(-5) - 3|}{\sqrt{1^2 + (-1)^2 + 1^2}}$$

$$d = \frac{|-3 + 4 - 5 - 3|}{\sqrt{3}}$$

$$d = \frac{|-7|}{\sqrt{3}}$$

$$d = \frac{7}{\sqrt{3}}$$

$$d = \frac{7\sqrt{3}}{3}$$

$$ d = \frac{7\sqrt{3}}{3} \times \frac{\sqrt{3}}{\sqrt{3}} $$
$$ d = \frac{7 \times 3}{3 \times \sqrt{3}} $$
$$ d = \frac{21}{\sqrt{3}} $$
$$ d = 7\sqrt{3} $$

This doesn't match any of the given options. However, if there was a transcription error in the line's equation, then the options might be relevant. As it stands with the provided information, none of the given options are correct.

Ques - Catalytic dehydrogenation of primary alcohol will produce a:
A) Secondary alcohol
B) Ester
C) Aldehyde
D) Ketone

Solution.
The catalytic dehydrogenation of a primary alcohol results in the removal of hydrogen, leading to the formation of an aldehyde.

Thus, the correct answer is:
C) Aldehyde.