VITEEE 2021 Solutions May 29 - Slot 3

Ques. Bakelite is a thermosetting resin. It is formed by the condensation of

Solution.
Bakelite is a thermosetting resin and is one of the earliest synthetic polymers. It is formed by the condensation reaction of:

phenol and **formaldehyde**.

The process of making Bakelite involves the reaction of phenol and formaldehyde under heat and pressure, in the presence of a catalyst, to produce a polymer structure. This resulting polymer is hard, heat-resistant, and an excellent electrical insulator, which made Bakelite an ideal material for various applications, including the manufacturing of electrical and automotive components.

Ques. Which one of the following has zero dipole moment?

Solution.
molecules like $\text{CO}_2, \text{BF}_3, \text{CCl}_4, \text{BeCl}_2$ etc., have zero dipole moments due to their symmetric geometry.

Ques. The intermediate obtained in Reimer Tiemann reaction is

Solution.

In the Reimer-Tiemann reaction, phenols are converted into ortho-hydroxy benzaldehyde derivatives when treated with chloroform ($CHCl_3$) in the presence of a strong base.

The key intermediate in the Reimer-Tiemann reaction is the carbanion formed from chloroform under the effect of a strong base, which is known as the **dichlorocarbene** ($:CCl_2$) intermediate.

The mechanism involves the generation of this dichlorocarbene, which then reacts with the aromatic ring to form an intermediate chloromethyl derivative. Hydrolysis of this intermediate then gives the ortho-hydroxy benzaldehyde derivative.

Ques. The rest mass of photon is

Solution.
A photon is a massless particle, and it always moves at the speed of light in a vacuum (c), approximately 3×10^8 m/s).

Whether it is related to protons, electrons, heating, cooling, or any other process, the rest mass of a photon is always zero.

Ques. A cylindrical wire is stretched to increase its length by 10%. Percentage increase in its resistance is:

Solution.
When a wire is stretched, its length increases and its cross-sectional area decreases. The resistance R of a wire is given by:

$$R = \frac{\rho L}{A}$$

Where:
ρ = resistivity of the material (a constant)
L = length of the wire
A = cross-sectional area of the wire

Given that the wire is stretched to increase its length by 10%, the new length is:

$$L' = 1.10L$$

Since volume remains constant upon stretching:

$$A \times L = A' \times L'$$

$$A' = \frac{A}{1.10} = 0.909A$$

The new resistance R' is:

$$R' = \frac{\rho L'}{A'}$$

$$R' = \frac{\rho (1.10L)}{0.909A}$$

$$R' = 1.21 \frac{\rho L}{A}$$

$$R' = 1.21R$$

The percentage increase in resistance is:

$$\frac{R' - R}{R} \times 100\%$$

$$\frac{1.21R - R}{R} \times 100\%$$

$$= 21\%$$

So, the resistance increases by 21%.

Ques.If you place 0°C ice into 0°C water in an insulated container, what will the net result be? Will there be less ice and more liquid water, or more ice and less liquid water, or will the amounts stay the same?

Solution.
If you place $0°C$ ice into $0°C$ water in an insulated container, there will be no net change in the amounts of ice and water. Here's why:

At $0°C$, both ice and water can coexist in equilibrium. This means that the rate of melting of the ice is equal to the rate of freezing of the water. Since the container is insulated, there's no heat exchange with the surroundings. So, in this closed system, the amount of ice melting to become water is equal to the amount of water freezing to become ice.

Therefore, the amounts of ice and liquid water will stay the same as long as the temperature remains constant at $0°C$ and no other external factors come into play.

Ques. 1 atomic mass unit is

Solution.
1 atomic mass unit (amu), or simply 1 unified atomic mass unit (u), is defined as one twelfth (1/12) of the mass of an atom of carbon-12.

The value, in terms of kilograms, is approximately:

$$1 \text{ amu (or u)} \approx 1.660539040(20) \times 10^{-27} \text{ kg}$$

This unit is used to express the weights of atoms and molecules.

Ques. Two identical metal block with charges +2Q and -Q are separated by some distance, and exert a force F on each other. The force between them then will be.

Solution.
Given:
One block has a charge of $+2Q$ and the other block has a charge of $-Q$.

Using Coulomb's law, the force between two charges is given by:
$$F = \frac{k|q_1q_2|}{r^2}$$
Where:
F = force between the charges
k = Coulomb's constant (approximately $8.9875 \times 10^9 \, \text{N.m}^2/\text{C}^2$)
q_1 and q_2 = the two charges
r = distance between the charges

For our problem:
$q_1 = +2Q$ and $q_2 = -Q$

The force between them:
$$F' = \frac{k(2Q)(-Q)}{r^2}$$
$$F' = \frac{-2kQ^2}{r^2}$$

Given that they initially exert a force F on each other:
$$F = \frac{k|q_1q_2|}{r^2} = \frac{2kQ^2}{r^2}$$

Comparing the two forces:
$$F' = -F$$

The negative sign indicates that the direction of the force is reversed. But since the magnitude of the force remains the same, the force between them will still be F, but in the opposite direction.

Ques. A 12V battery, a 12 Ω resistor and a 4 Ω resistor are connected. The voltage across 12Ω resistor is that across

Solution.
To determine the voltage across the 12Ω resistor, we first need to figure out how the resistors are connected: in series or parallel. Since the configuration isn't specified, let's solve for both situations:

1. If the resistors are connected in series:
The total resistance R_{total} is:
$$R_{total} = R_1 + R_2 = 12\Omega + 4\Omega = 16\Omega$$

Using Ohm's Law, the current I through the circuit is:
$$I = \frac{V}{R_{total}} = \frac{12V}{16\Omega} = 0.75A$$

The voltage across the 12Ω resistor $V_{12\Omega}$ is:
$$V_{12\Omega} = I \times 12\Omega = 0.75A \times 12\Omega = 9V$$

2. If the resistors are connected in parallel:
The inverse of the total resistance $1/R_{total}$ is:
$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12\Omega} + \frac{1}{4\Omega}$$
$$\frac{1}{R_{total}} = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}$$

$$ R_{total} = 3\Omega $$

The current $ I_{total} $ from the battery is:
$$ I_{total} = \frac{V}{R_{total}} = \frac{12V}{3\Omega} = 4A $$

But for parallel resistors, the voltage across each resistor is the same as the battery's voltage. Thus, the voltage across the 12Ω resistor $ V_{12\Omega} $ is:
$$ V_{12\Omega} = 12V $$

Summary:
- If the resistors are in series, the voltage across the 12Ω resistor is 9V.
- If the resistors are in parallel, the voltage across the 12Ω resistor is 12V.

Ques. A straight section PQ of a circuit lies along the x-axis from x = -a/2 to x = +a/2 and carries a steady current i. The magnetic field due to the section PQ at a point x = +a will be

Solution.
To find the magnetic field at point $ x = +a $ due to a current-carrying section PQ on the x-axis, we can use Biot-Savart's law. According to Biot-Savart's law, the infinitesimal magnetic field $ dB $ due to an infinitesimal section $ dx $ carrying a current $ i $ at a distance $ r $ from the section is:

$$ dB = \frac{\mu_0 \cdot i \cdot dx \cdot \sin(\theta)}{4\pi \cdot r^2} $$

Where:
- $ \mu_0 $ is the permeability of free space.
- $ \theta $ is the angle between $ dx $ and the line joining $ dx $ to the point where we want to find the field. In this scenario, since the current is along the x-axis and the radial line from $ dx $ to the point $ x = +a $ is also along the x-axis, $ \theta = 90^\circ $ and $ \sin(90^\circ) = 1 $.

Considering a small section $ dx $ at a distance $ x $ from the origin, the distance $ r $ of $ dx $ from the point $ x = +a $ is $ a - x $.

Now, integrating over the entire section PQ, which is from $x = -a/2$ to $x = +a/2$:

$$B = \int \frac{\mu_0 \cdot i \cdot dx}{4\pi \cdot (a - x)^2}$$
$$= \frac{\mu_0 \cdot i}{4\pi} \int_{-a/2}^{a/2} \frac{dx}{(a - x)^2}$$

On solving this integral, the magnetic field at point $x = +a$ due to the current in section PQ can be determined.

Ques. If p is the hole concentration, n is the electron concentration and ni is the intrinsic concentration then at thermal equilibrium, then the mass action in semiconductors states that

Solution.
The mass action law for intrinsic semiconductors at thermal equilibrium states that the product of the electron concentration (n) and the hole concentration (p) is always equal to the square of the intrinsic carrier concentration (n_i).

Mathematically, this is expressed as:

$$n \times p = n_i^2$$

This relationship holds true for intrinsic as well as extrinsic (both p-type and n-type) semiconductors at thermal equilibrium, given that the temperature and the semiconductor material remain constant.

Ques. Area of the greatest rectangle that can be inscribed in the ellipse x2 /a2 + y 2 /b2 = 1 is

Solution.
For a rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, let's assume the coordinates of one of the corners of the rectangle to be (x,y). The opposite corner will be at $(-x,-y)$ because of the symmetry of the rectangle and the ellipse.

The sides of the rectangle are $2x$ and $2y$, as the lengths are doubled due to symmetry.

Given the equation of the ellipse, if $x = a \cos \theta$, then $y = b \sin \theta$, where θ is the parameter.

The area A of the rectangle is:
$$A = 2x \times 2y = 4xy$$
Substitute the parametric coordinates for x and y:
$$A = 4a \cos \theta \times b \sin \theta = 4ab \cos \theta \sin \theta$$
Using the double angle identity, $\sin 2\theta = 2\sin \theta \cos \theta$:
$$A = 2ab \sin 2\theta$$

To find the maximum area, differentiate A with respect to θ and set it equal to zero:
$$\frac{dA}{d\theta} = 2ab \cos 2\theta$$

For maximum area, $\frac{dA}{d\theta} = 0$, so $\cos 2\theta = 0$. This means $2\theta = 90^\circ$ or $\theta = 45^\circ$.

Plug $\theta = 45^\circ$ into the area function:
$$A_{max} = 2ab \sin 90^\circ = 2ab$$

Thus, the area of the greatest rectangle that can be inscribed in the ellipse is $2ab$.

Ques. The number integral values in the range of the function f(x) = sin-1 x - cot-1 x + x 2 + 2x + 6 is

Solution.
To find the range of the function $f(x) = \sin^{-1}(x) - \cot^{-1}(x) + x^2 + 2x + 6$, we need to find the range of each component and combine them.

1. $\sin^{-1}(x)$: The range of the arcsin function, or $\sin^{-1}(x)$, is $[-\pi/2, \pi/2]$.

2. $\cot^{-1}(x)$: The range of the arccot function, or $\cot^{-1}(x)$, is $(0, \pi)$.

3. x^2: This is a parabolic function, and its range is $[0, \infty)$.

4. $2x$: This is a linear function, and its range is $-\infty, \infty$.

5. 6: This is a constant.

Combining the ranges:

For $\sin^{-1}(x) - \cot^{-1}(x)$, the minimum value occurs when $\sin^{-1}(x)$ is at its minimum and $\cot^{-1}(x)$ is at its maximum. This would be $-\pi/2 - 0 = -\pi/2$. The maximum value occurs when $\sin^{-1}(x)$ is at its maximum and $\cot^{-1}(x)$ is at its minimum, which is $\pi/2 - \pi = -\pi/2$. So, the range of $\sin^{-1}(x) - \cot^{-1}(x)$ is $[- \pi/2, -\pi/2]$, which is a constant value of $-\pi/2$.

Ques. If |a + b| < |a - b|, then the angle between a and b can lie in interval

Solution.
Given:
$$|a + b| < |a - b|$$

Now, we know that the magnitude of the dot product of two vectors $\vec{a}$ and $\vec{b}$ is:
$$\vec{a} \cdot \vec{b} = |a| \times |b| \times \cos(\theta)$$
where θ is the angle between $\vec{a}$ and $\vec{b}$.

Let's compute the square of the magnitudes of $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$:

1) $|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$
$$= \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + 2 \vec{a} \cdot \vec{b}$$
$$= |a|^2 + |b|^2 + 2|a||b|\cos(\theta)$$

2) $|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$

$$= \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} - 2 \vec{a} \cdot \vec{b}$$

$$= |a|^2 + |b|^2 - 2|a||b|\cos(\theta)$$

From the given condition, $|a + b| < |a - b|$, squaring both sides, we get:

$$|a|^2 + |b|^2 + 2|a||b|\cos(\theta) < |a|^2 + |b|^2 - 2|a||b|\cos(\theta)$$

Simplifying:

$$4|a||b|\cos(\theta) < 0$$

$$\cos(\theta) < 0$$

Now, $\cos(\theta)$ is negative in the 2nd and 3rd quadrants. So:

$$\pi/2 < \theta < 3\pi/2$$

Thus, the angle between $\vec{a}$ and $\vec{b}$ can lie in the interval $(\pi/2, 3\pi/2)$.

Ques. Perpendicular distance of the point P (3,5,6) from Y-axis is

Solution.

To find the perpendicular distance of a point from an axis in 3D space, we consider the coordinates of the point in relation to that specific axis.

For the y-axis in a 3D coordinate system, all points lie on the line where $x = 0$ and $z = 0$.

Given the point P(3,5,6):

The coordinates on the y-axis that would be directly opposite (or 'under') point P would be (0,5,0).

The perpendicular distance from point P to the y-axis would then be the distance between the points P(3,5,6) and (0,5,0). This distance is along the xz-plane and is essentially the magnitude of the x-coordinate of P, since the z-coordinate doesn't change.

Thus, the perpendicular distance is 3 units.

Ques. The solution of the differential equation dy/dx = ex-y = 1 is

Solution.
To solve the differential equation, we can start by rearranging it:

$\frac{dy}{dx} - e^{-y} = e^x$

This is a first-order linear differential equation in the form:
$$\frac{dy}{dx} + P(x) y = Q(x)$$

Where:
$P(x) = -e^{-y}$
and
$Q(x) = e^x$

The integrating factor $\mu(x)$ for this differential equation is given by:
$$\mu(x) = e^{\int P(x) \, dx}$$
$$= e^{\int -e^{-y} \, dy}$$

The integral of $-e^{-y}$ with respect to y is tricky because it involves y in the exponent. However, it seems there may have been a typographical error or misrepresentation in the provided differential equation. Typically, for a first-order linear differential equation, $P(x)$ should be a function of x alone, not y.

Ques. The equation of the curve passing through (3, 9) satisfies dy/dx = x + 1/x2

Solution.
To solve the differential equation,

$\frac{dy}{dx} = x + \frac{1}{x^2}$

Separate variables and integrate:

$\int dy = \int (x + \frac{1}{x^2})\, dx$

Integrating both sides:

$y = \frac{x^2}{2} - \frac{1}{x} + C$

Where C is the integration constant.

Now, using the point (3, 9) to find the value of C:

$9 = \frac{3^2}{2} - \frac{1}{3} + C$
$9 = 4.5 - 0.3333 + C$

$C = 4.8333$

Thus, the equation of the curve is:

$y = \frac{x^2}{2} - \frac{1}{x} + 4.8333$.

Ques. The largest positive form of the harmonic progression whose first two terms are 2/5 and 12/23

Solution.
A harmonic progression (HP) is the reciprocals of an arithmetic progression (AP).

Given two terms of an HP as $\frac{2}{5}$ and $\frac{12}{23}$, the terms of the corresponding AP will be the reciprocals of these terms, i.e., $\frac{5}{2}$ and $\frac{23}{12}$.

Now, let's find the common difference of the AP.
Using the formula:
$$a_2 = a_1 + d$$

Where:
a_1 is the first term of the AP.
a_2 is the second term of the AP.

d is the common difference.

Given:
$a_1 = \frac{5}{2}$
$a_2 = \frac{23}{12}$

Substituting in the formula:
$$\frac{23}{12} = \frac{5}{2} + d$$
$$d = \frac{23}{12} - \frac{5}{2}$$
$$d = \frac{23 - 30}{12}$$
$$d = -\frac{7}{12}$$

Now, the n-th term of an AP is given by:
$$a_n = a_1 + (n-1)d$$
For the term to be the largest positive term of the HP, a_n must be the smallest positive term of the AP.

Since d is negative, the terms of the AP will keep getting smaller. To ensure a_n is positive and smaller than the preceding terms, $a_1 + (n-1)d$ must be positive.

$$\frac{5}{2} + (n-1)(-\frac{7}{12}) > 0$$
$$5n - \frac{7n-7}{12} > 10$$
$$60n - 7n + 7 > 120$$
$$53n > 113$$
$$n < 2.132$$

The largest possible integer value for n is 2. But since n=2 corresponds to our second term, the HP only has two terms that are positive, and thus the largest positive term is $\frac{12}{23}$.