VITEEE 2021 Solutions May 31 - Slot 1

Q: Perpendicular distance of the point P (3, 5, 6) from y-axis is: 1. √7 2. 7 3. 6 4. √45

Solution.
The perpendicular distance from a point to the y-axis in a three-dimensional coordinate system is simply the x-coordinate of that point, as the y-axis is represented by the line where x = 0 and z can be any value.

For the point $P(3, 5, 6)$, the x-coordinate is 3.

Therefore, the perpendicular distance of point P from the y-axis is 3.

Q: For what values of 'a' the function F (x) = -x 3 + 4ax 2 + 2x - 5 is decreasing for all real x 1. (3, 4) 2. (-1, 1) 3. No value of a 4. (1, 2)

Solution.
For a function $f(x)$ to be decreasing for all real x, its first derivative $f'(x)$ must be less than zero for all real x.

Given:
$$f(x) = -x^3 + 4ax^2 + 2x - 5$$

First, we need to find the first derivative:
$$f'(x) = \frac{d}{dx}(-x^3 + 4ax^2 + 2x - 5)$$
$$f'(x) = -3x^2 + 8ax + 2$$

For $f'(x)$ to be less than zero for all real x, the discriminant of this quadratic equation must be negative. The discriminant Δ of a quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac$.

Here, for \(-3x^2 + 8ax + 2 \):
a = -3
b = 8a
c = 2

The discriminant \(\Delta \) is:
\[\Delta = (8a)^2 - 4(-3)(2) \]
\[\Delta = 64a^2 + 24 \]

For the function to be decreasing for all real x, \(\Delta \) must be negative:
\[64a^2 + 24 < 0 \]
\[64a^2 < -24 \]
\[a^2 < -\frac{3}{8} \]

However, \(a^2 \) (the square of a real number) is always non-negative. Thus, there's no real value of a for which \(a^2 \) can be less than \(-\frac{3}{8} \).

So, the correct answer is:
3. No value of a.

Q.Find the independent solution of the differential equation:

Q: p→(p →q) logically equivalent to?

Solution.
Let's analyze the given expression:
\[p \rightarrow (p \rightarrow q) \]

Using the definition of the material conditional, the statement \(p \rightarrow q \) is equivalent to \(\neg p \vee q \).

So the expression becomes:
\[p \rightarrow (\neg p \vee q) \]

Now, let's apply the definition of the material conditional again to the entire expression:

$$ p \rightarrow (\neg p \vee q) $$

$$ \neg p \vee (\neg p \vee q) $$

Using the distributive law, we can simplify the expression:

$$ \neg p \vee \neg p \vee q $$

This is equivalent to:

$$ \neg p \vee q $$

So, $p \rightarrow (p \rightarrow q)$ is logically equivalent to $\neg p \vee q$, which is also the material conditional $p \rightarrow q$.

Q: The truth table shown below is which gate?

A B Y
1 1 0
1 0 1
0 1 1
0 0 1

1. NAND
2. XOR
3. AND

Solution.

Let's analyze the given truth table:

A B Y
1 1 0
1 0 1
0 1 1
0 0 1

From the truth table:

- When both A and B are 1, the output Y is 0.
- For all other combinations of A and B, the output Y is 1.

This behavior corresponds to the NAND gate.

So, the correct answer is:
1. NAND.

Q: Find cos8 o cos10 o cos12 o - sin8 o sin10 o cos12 o = sin18 o sin 12 o

Solution.
We have to find the value of:

$$ \frac{cos 8^\circ cos 10^\circ cos 12^\circ - sin 8^\circ sin 10^\circ cos 12^\circ}{sin 18^\circ sin 12^\circ} $$

To simplify, we'll use a number of trigonometric identities:

1) Product-to-sum formula:

$$ cosA cosB = \frac{1}{2} [cos(A - B) + cos(A + B)] $$

$$ sinA sinB = \frac{1}{2} [cos(A - B) - cos(A + B)] $$

2) The relationship between sine and cosine:

$$ sin(90^\circ - A) = cosA $$

Using (1), let's expand the terms in the numerator:

$$ cos 8^\circ cos 10^\circ = \frac{1}{2} [cos(10^\circ - 8^\circ) + cos(10^\circ + 8^\circ)] $$

$$ = \frac{1}{2} [cos 2^\circ + cos 18^\circ] $$

$$ sin 8^\circ sin 10^\circ = \frac{1}{2} [cos(10^\circ - 8^\circ) - cos(10^\circ + 8^\circ)] $$

$$ = \frac{1}{2} [cos 2^\circ - cos 18^\circ] $$

Substituting these into our original expression, we get:

$$ \frac{\frac{1}{2} [cos 2^\circ + cos 18^\circ] cos 12^\circ - \frac{1}{2} [cos 2^\circ - cos 18^\circ] cos 12^\circ}{sin 18^\circ sin 12^\circ} $$

Combining the terms, we get:

$$ \frac{cos 12^\circ cos 2^\circ}{sin 18^\circ sin 12^\circ} $$

Using the identity $cos(90^\circ - A) = sinA$, we have:

$$sin(90^\circ - 12^\circ) = sin 78^\circ = cos 12^\circ$$

So,

$$\frac{cos 12^\circ cos 2^\circ}{sin 18^\circ sin 12^\circ} = \frac{sin 78^\circ cos 2^\circ}{sin 18^\circ sin 12^\circ}$$

Using the product-to-sum formula again, we get:

$$\frac{\frac{1}{2} [sin(78^\circ + 2^\circ) + sin(78^\circ - 2^\circ)]}{sin 18^\circ sin 12^\circ}$$

$$= \frac{\frac{1}{2} [sin 80^\circ + sin 76^\circ]}{sin 18^\circ sin 12^\circ}$$

Now, using the fact that:

$$sin 2A = 2sinAcosA$$

We can express $sin 76^\circ$ as:

$$sin 76^\circ = sin(2 \times 38^\circ) = 2sin 38^\circ cos 38^\circ$$

And $sin 80^\circ$ can be expressed as:

$$sin 80^\circ = sin(2 \times 40^\circ) = 2sin 40^\circ cos 40^\circ$$

Plugging these in, we can simplify the expression further, but it will become quite complex. However, just from the transformations and simplifications we've made so far, it's clear that the given expression does not easily simplify to $sin 18^\circ sin 12^\circ$ based on standard trigonometric identities.