VITEEE 2021 Solutions May 31 - Slot 2

Q: The value of c for which the mean value of the theorem hols for the function f(x) = 2x - x 2 on the interval [0,1] is:

1. 0
2. ¼
3. ½
4. ⅓

Solution.

The Mean Value Theorem states that if f is continuous on $[a, b]$ and differentiable on (a, b), then there exists some number c in (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Given:

$$f(x) = 2x - x^2$$

on the interval [0,1].

First, find $f'(x)$:

$$f'(x) = 2 - 2x$$

Next, evaluate the function at the endpoints of the interval:

$$f(1) = 2(1) - (1)^2 = 1$$

$$f(0) = 0$$

Substitute the given values into the formula from the Mean Value Theorem:

$$ 2 - 2c = \frac{1 - 0}{1 - 0} = 1 $$

Solving for c:

$$ 2 - 2c = 1 $$
$$ -2c = -1 $$
$$ c = \frac{1}{2} $$

So, the correct answer is:
3. $c = \frac{1}{2}$

Q: If a x b and c x d are perpendicular satisfying a.c = λ, b.d = λ (λ>0) and a.d = 4, b.c = 9, then λ is equal to:

1. 3
2. 6
3. 36
4. 2

Solution.
Given that vectors a and b are perpendicular, as well as vectors c and d.

From this, we can conclude:

$$ a \cdot b = 0 $$ and $c \cdot d = 0$ because the dot product of two perpendicular vectors is zero.

We're also given:
1) $a \cdot c = \lambda$
2) $b \cdot d = \lambda$
3) $a \cdot d = 4$
4) $b \cdot c = 9$

From the properties of the dot product:
$$ (a + b) \cdot (c + d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d $$

Given that $a \times b$ and $c \times d$ are perpendicular, the vectors $a + b$ and $c + d$ are also perpendicular to each other. Thus:

$$(a + b) \cdot (c + d) = 0$$

Substituting in the known dot products:

$$\lambda + 4 + 9 + \lambda = 0$$

$$2\lambda + 13 = 0$$

$$2\lambda = -13$$

$$\lambda = -\frac{13}{2}$$

This result seems contradictory because we're given $\lambda > 0$. It's possible that the problem may have an error or it might require additional context or constraints to yield a positive value for λ.